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M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
STATISTICS
Distribution Theory (2329101)

Day & Date: Friday, 05-01-2024
 Time: 03:00 PM To 05:30 PM

Max. Marks: 60

Instructions: 1) All questions are compulsory.
 2) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 08

- 1) Let X be distributed as $B(n, p)$. The distribution of $Y = n - X$ is _____.
 - a) not Binomial
 - b) $B(n - 1, p)$
 - c) $B(n - 1, n - p)$
 - d) $B(n, 1 - p)$
- 2) Let X be distributed as $Exp(\text{Mean } \theta)$. Then distribution of $Y = X/\theta$ is _____.
 - a) $Exp(\text{Mean } \theta)$
 - b) $Exp(\text{Mean } 1)$
 - c) $U(0, 1)$
 - d) $U(0, \theta)$
- 3) A random variable X is said to be symmetric about point α if _____.
 - a) $P(X \geq \alpha + x) = P(X \geq \alpha - x)$
 - b) $P(X \geq \alpha + x) = P(X \leq \alpha - x)$
 - c) $P(X \leq \alpha + x) = P(X \leq \alpha - x)$
 - d) $P(X \leq \alpha + x) = P(X \geq \alpha - x)$
- 4) If $X > 0$ then _____.
 - a) $E[\sqrt{X}] \leq \sqrt{E(X)}$
 - b) $E[\sqrt{X}] \geq \sqrt{E(X)}$
 - c) $E = [\sqrt{X}] = \sqrt{E(X)}$
 - d) None of these
- 5) Which of the following is not a scale family?
 - a) $U(0, \theta)$
 - b) $U(0, 1)$
 - c) $N(0, \sigma^2)$
 - d) $Exp(\theta)$
- 6) Let X and Y be independent random variables each having the $U(0, 1)$ distribution. Then $\text{Var}(X + Y)$ is equal to _____.
 - a) $1/6$
 - b) $5/2$
 - c) 4
 - d) 6
- 7) Let X and Y be two iid random variables with pdf $f(x) = 2e^{-2x}, x \geq 0$. The distribution of $Z = X - Y$ is _____.
 - a) exponential
 - b) beta
 - c) Laplace
 - d) Cauchy
- 8) Suppose X_1, X_2, \dots, X_k is a multinomial random variate then $\text{Cov}(X_i, X_j), i = j = 1, 2, \dots, k, i \neq j$ is _____.
 - a) $n p_i$
 - b) $-n p_i p_j$
 - c) $n p_i p_j$
 - d) $n^2 p_i p_j$

- B) Fill in the blanks.** **04**
- 1) If Z is standard normal variate then mean of Z^2 is _____.
 - 2) The *pdf* of random variable X is $f(x) = 2x, 0 \leq x \leq 1$ then $P(X = 0.5)$ is _____.
 - 3) Let X and Y be two independent Poisson random variates with means 1 and 2 respectively then variance of $(2X + 3Y)$ is _____.
 - 4) The *PGF* of Poisson distribution with mean λ is given by _____.

- Q.2 Answer the following (Any Six)** **12**
- a) Define cumulative distribution function (c.d.f.) of a random variable X .
 - b) Define a symmetric random variable.
 - c) Define location family.
 - d) State Holder's inequality.
 - e) Define moment generating function (MGF) of random variable X .
 - f) State the relation between distribution function of a continuous random variable and uniform random variable.
 - g) Define Marshall-Olkin bivariate exponential distribution.
 - h) Define multinomial distribution.

- Q.3 Answer the following. (Any Three)** **12**
- a) Let X follows $N(0,1)$ distribution. Find the distribution of $Y = X^2$.
 - b) Define scale family of distributions. Examine which of the following are in scale family.
 - i) $X \sim N(0, \sigma^2)$
 - ii) $X \sim U(0, \theta)$
 - c) Let F be a distribution function of a random variable X . Examine whether $[F(x)]^2$ and $1 - F(x)$ are distribution functions.
 - d) Derive the *pdf* of smallest order statistic based on a random sample of size n from a continuous distribution with *pdf* $f(x)$ and *cdf* $F(x)$.

- Q.4 Answer the following. (Any Two)** **12**
- a) State and prove Markov's inequality.
 - b) Let X is a non-negative random variable with *pmf* $P(X = x) = P_x, x = 1, 2, \dots$ then show that

$$E(X) = \sum_{x=1}^{\infty} P[X \geq x]$$
 - c) Let X has $B(n, p)$ distribution. Obtain the *PGF* of X . Hence obtain its mean and variance.

- Q.5 Answer the following. (Any Two)** **12**
- a) Define multinomial distribution. Obtain its MGF. Hence or otherwise obtain its variance-covariance matrix.
 - b) Let X and Y are jointly distributed with pdf

$$f(x, y) = \begin{cases} k(x + 2y), & 0 < x < 2, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
 Find marginal distributions of X and Y .
 - c) Let (X, Y) has $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain the marginal distributions of X .

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M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
STATISTICS
Estimation Theory (2329102)

Day & Date: Sunday, 07-01-2024
 Time: 03:00 PM To 05:30 PM

Max. Marks: 60

Instructions: 1) All questions are compulsory.
 2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative.**08**

- 1) A sufficient statistic contains all the information which is contained in _____.
 - a) population
 - b) sample
 - c) parameter
 - d) none of the above
- 2) Suppose T sufficient for θ . Then $g(T)$ is sufficient for $g(\theta)$ if _____.
 - a) g is a real valued function
 - b) g is a continuous function
 - c) g is one-to-one function
 - d) g is a bounded function.
- 3) $U(0, \theta)$ is a member of _____.
 - a) one parameter exponential family.
 - b) Pitman family.
 - c) power series family.
 - d) none of the above
- 4) Cramer-Rao inequality with regards to the variance of an unbiased estimator provides _____.
 - a) lower bound
 - b) upper bound
 - c) asymptotic variance
 - d) Fisher information
- 5) If an estimator T_n of population parameter θ converges in probability to θ as n tends to infinity is said to be _____.
 - a) sufficient
 - b) efficient
 - c) consistent
 - d) unbiased
- 6) The MLE of parameter θ is a statistic which _____.
 - a) is sufficient for parameter for θ
 - b) maximizes the likelihood function L
 - c) is a solution of $\frac{\partial \log L}{\partial \theta} = 0$
 - d) is always unbiased
- 7) Let T_n be an unbiased estimator of θ . Then _____.
 - a) T_n^2 is unbiased estimator of θ^2 .
 - b) $\sqrt{T_n}$ is unbiased estimator of $\sqrt{\theta}$.
 - c) e^{T_n} is unbiased estimator of e^θ
 - d) $3T_n + 4$ is unbiased estimator of $3\theta + 4$
- 8) Prior distribution is the _____.
 - a) distribution of parameter θ .
 - b) distribution of sample X .
 - c) conditional distribution of X given θ .
 - d) conditional distribution of θ given X

B) Fill in the blanks. 04

- 1) If the distribution of $T(X)$ is independent of θ then statistic $T(X)$ is said to be _____ statistic
- 2) Let X_1, X_2, \dots, X_n is a random sample of size n from $U(0, \theta)$ distribution then MLE of θ is _____.
- 3) Bayes estimator of a parameter under absolute error loss function is _____.
- 4) Bhattacharya bound is the generalization of _____ inequality.

Q.2 Answer the following. (Any Six) 12

- a) Define one parameter exponential family of distributions.
- b) Define sufficient statistic.
- c) Define maximum likelihood estimator (MLE).
- d) State Lehmann-Schffe theorem.
- e) Define Fisher information in a single observation and in n iid observations.
- f) State Neyman-Fisher factorization theorem.
- g) Define consistent estimator.
- h) Define Pitman family of distributions.

Q.3 Answer the following. (Any Three) 12

- a) Let random variable X has $U(0, \theta), \theta > 0$ distribution. Show that distribution of X is complete.
- b) Let X_1 and X_2 are iid Poisson (λ). Using the definition of sufficient statistic, examine whether $X_1 + X_2$ to be sufficient statistics for λ .
- c) Show that Poisson distribution belong to power series family.
- d) Find Cramer-Rao lower bound (CRLB) for the variance of unbiased estimator of θ based on random sample of size n from $f(x, \theta) = \theta e^{-\theta x}, x \geq 0, \theta > 0$.

Q.4 Answer the following. (Any Two) 12

- a) Describe the method of maximum likelihood estimation for estimating an unknown parameter.
- b) Let X_1, X_2, \dots, X_n is a random sample of size n from exponential distribution with mean θ . Obtain MLE of θ . Show that it is unbiased estimator of θ .
- c) Let X_1, X_2, \dots, X_n be a random sample from $U(0, \theta), \theta > 0$. Obtain two consistent estimators for θ .

Q.5 Answer the following. (Any Two) 12

- a) State and prove Rao-Blackwell theorem.
- b) Let X_1, X_2, \dots, X_n be a random sample of size n from $U(0, \theta), \theta > 0$. Find UMVUE of (i) θ and (ii) θ^2 .
- c) Let $\{T_n\}$ be a sequence of estimators such that $E(T_n) = \theta$ and $Var(T_n) \rightarrow 0$ as $n \rightarrow \infty$ then show that T_n is consistent for θ .

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M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
STATISTICS
Statistical Mathematics (2329107)

Day & Date: Tuesday, 09-01-2024
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative.

08

- 1) If there exists one to one correspondence between given set and set of natural numbers, then the given set is _____.
a) Perfect set b) Good set
c) Countable set d) Uncountable set
- 2) Which of the following sequences of real numbers do always converge?
a) Sequence of constant term
b) Monotonic increasing sequence
c) Monotonic decreasing sequence
d) Oscillatory sequence
- 3) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \rightarrow \infty} a_n =$ _____.
a) -1 b) 1
c) Infinity d) Zero
- 4) A convergent sequence Have _____.
a) Only one limit b) Atmost two limits
c) Atmost n limits d) Infinite limits
- 5) A Cauchy sequence of real numbers is always _____.
a) Convergent b) Divergent
c) Oscillatory d) None of these
- 6) The smallest sub-space containing finite set of vectors (S) is _____.
a) Superclass of S b) Span of S
c) Subset of S d) Basis of S
- 7) A set of vectors containing a null vector is _____.
a) Not necessarily dependent b) Necessarily dependent
c) Necessarily independent d) A vector space
- 8) If number of columns is less than number of rows, then the matrix is called as _____.
a) Horizontal matrix b) Vertical matrix
c) Row matrix d) Column matrix

B) Fill in the blanks.

04

- 1) If all the elements below the diagonal are zero, then such matrix is called as _____.
- 2) If AB is invertible then $(AB)^{-1} =$ _____.
- 3) If determinant of a square matrix is zero, then such matrix is called as _____.
- 4) The inverse of identity matrix is _____.

Q.2 Answer the following. (Any Six)

12

- Define orthogonal matrix.
- Define a square matrix.
- Define a skew-symmetric matrix.
- Define convergence limit of a sequence.
- Define bounded sequence.
- Define geometric series.
- Define infimum of a set.
- Define ratio test of convergence.

Q.3 Answer the following. (Any Three)

12

- Show that every monotonic bounded above sequence of real numbers converges.
- Define and illustrate limit superior of a sequence of real numbers.
- Reduce the following matrix to a row-reduced form and hence determine its rank.

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 5 & 2 & 1 \\ 7 & 6 & 1 \end{bmatrix}$$

- Define and illustrate rank of a matrix.

Q.4 Answer the following. (Any Two)

12

- Define limit superior and limit inferior of a sequence. Find the same for the following sequence, hence verify its convergence.

$$S_n = 2 + \frac{(-1)^n}{n}, n \in N$$

- Obtain the Riemann integration of $f(x) = 3x, x \in (0,2)$.
- Examine the convergence of p-series for various values of p.

Q.5 Answer the following. (Any Two)

12

- Define vector space and subspace. State the conditions needed to verify whether a subset of a vector space is a subspace.
- Prove: For any vector in \underline{u} vector space $V, 0 \cdot \underline{u} = \underline{0}$
- How the independence of vectors is examined? Also verify whether following set is a set of independent vectors.

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right\}$$

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**M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
STATISTICS**

Research Methodology in Statistics (2329103)

Day & Date: Thursday, 11-01-2024
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative.

08

- 1) Research can be defined as _____.
 - a) scientific and systematic search for pertinent information on a specific topic
 - b) a search for knowledge
 - c) systematized effort to gain new knowledge
 - d) All the above
- 2) A research study undertaken to gain familiarity with a phenomenon or to achieve new insights into it is termed as _____.
 - a) exploratory research study
 - b) descriptive research studies
 - c) diagnostic research studies
 - d) All of the above
- 3) The _____ is that which utilizes historical sources like documents, remains, etc. to study events or ideas of the past.
 - a) Historical research
 - b) Diagnostic research
 - c) Longitudinal research
 - d) One point research
- 4) Decisions regarding what, where, when, how much, by what means concerning an inquiry or a research study constitute a _____.
 - a) Research design
 - b) Sampling design
 - c) Report writing
 - d) Descriptive research
- 5) _____ concerns with the question of how many items are to be observed and how the information and data gathered are to be analyzed.
 - a) observational design
 - b) the sampling design
 - c) the statistical design
 - d) the operational design
- 6) The regression estimator is appropriate in a situation where _____.
 - a) Regression of Y on X is linear and line passes through origin
 - b) Regression of Y on X is linear and line does not pass through origin
 - c) Regression of Y on X is non-linear and passes through origin
 - d) Regression of Y on X in non-linear and does not passes through origin
- 7) In sampling with probability proportional to size, the units are selected with probability proportion to _____.
 - a) Size of the unit
 - b) Size of the population
 - c) Size of the sample
 - d) None of these
- 8) Which of the following estimators is generally biased?
 - a) Horvitz - Thompson
 - b) Des Raj
 - c) Heartly - Ross
 - d) Ratio

B) Fill in the blanks. 04

- 1) The process of examining the truth of a statistical hypothesis, relating to some research problem, is known as _____.
- 2) Under _____ scheme, ratio estimator exactly becomes unbiased.
- 3) A random start automatically fixes the subsequent selection of sample units in _____ sampling method.
- 4) In SRSWR, _____ is unbiased estimator of population variance.

Q.2 Answer the following. (Any Six) 12

- a) Define descriptive research.
- b) Define quantitative research.
- c) Define applied research.
- d) Define extraneous variable.
- e) What is meant by sampling error?
- f) Define cluster sampling.
- g) Define judgement sampling.
- h) Differentiate between SRSWR and SRSWOR.

Q.3 Answer the following. (Any Three) 12

- a) Discuss response and non-response errors.
- b) Describe stratified sampling.
- c) Describe criteria of a good research.
- d) Discuss experimental and control group.

Q.4 Answer the following. (Any Two) 12

- a) With usual notations, prove that in simple random sampling, the bias of \bar{y}_l is $Bias(\bar{y}_l) = -cov(\bar{x}, b)$.
- b) Obtain Horvitz - Thompson estimator for population mean for PPSWOR method.
- c) Discuss research methods and research methodology.

Q.5 Answer the following. (Any Two) 12

- a) Discuss the need of research design.
- b) Explain the significance of report writing.
- c) Explain Lahiri's method in detail.

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M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
STATISTICS

Real Analysis (MSC16101)

Day & Date: Friday, 05-01-2024
Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Q. No. 1 and Q. No 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

- 1) The set of limit points of the set $(0,2]$ is: _____.
a) $(0,2)$ b) $(0,2]$
c) $[0,2)$ d) $[0,2]$
- 2) Set of all rationals is _____.
a) Countable b) Uncountable
c) Finite d) None of the above
- 3) A set is compact if and only if it is bounded and _____.
a) Semi open b) Open
c) Closed d) Semi closed
- 4) Least upper bound is also called as _____.
a) Infimum b) Supremum
c) Limit point d) Interior point
- 5) The set $\left\{\left(8 + \frac{(-1)^n}{n}\right), n \in N\right\}$ has ____ limit point.
a) One b) Two
c) Zero d) Four
- 6) A sequence $S_n = (-1)^n, n \in N$ is _____ sequence.
a) Divergent b) Convergent
c) Oscillatory d) none of these
- 7) A superset of uncountable set is always _____.
a) Countable b) Uncountable
c) May or may not be countable d) None of these
- 8) A function $f(x)=|x-3|$ on $(-1,1)$ is _____.
a) Continuous as well as differentiable
b) Continuous, but not differentiable
c) Differentiable but not continuous
d) Neither continuous nor differentiable
- 9) Riemann integral is a particular case of _____.
a) Riemann- John integral
b) Riemann-Lebesgue integral
c) Riemann-Stieltje's integral
d) None of the above
- 10) The limit of sequence $S_n = \frac{1}{n^2}, n \in N$ is _____.
a) 0 b) 1
c) 100 d) 2

- B) Fill in the blanks.** **06**
- 1) Every subset of countable set is _____.
 - 2) A finite set with n elements has _____ number of limit points.
 - 3) If for a geometric series, common ratio $r > 1$, then series _____.
 - 4) Greatest lower bound of a set is also called as _____.
 - 5) The function $f(x) = -x^2 + 2x + 3$ has maximum at point $x =$ _____.
 - 6) The union of two closed sets is _____.
- Q.2 Answer the following.** **16**
- a) Construct a bounded set of real numbers with exactly three limit points.
 - b) Prove or disprove:
 - i) A subset of open set is always open.
 - ii) A subset of closed set is always closed.
 - c) Define and illustrate:
 - i) Limit point of a set
 - ii) Interior point of a set.
 - d) What is meant by absolute convergence of a series? Does it imply regular convergence?
- Q.3 Answer the following.**
- a) Prove or disprove: Countable union of countable sets is always Countable **08**
 - b) Let A be an open subset of \mathbb{R} . Then show that every point of A is also its limit point. **08**
- Q.4 Answer the following.**
- a) When do you say a set is countable? Show that set of real numbers \mathbb{R} is uncountable. **08**
 - b) When a series of real numbers is said to be convergent? Discuss ratio test and comparison test of convergence. **08**
- Q.5 Answer the following.**
- a) Define Cauchy sequence. Show that a sequence is convergent if and only if it is a Cauchy sequence. **08**
 - b) Discuss the convergence of the series $\sum \frac{1}{n^p}$. **08**
- Q.6 Answer the following.**
- a) Define continuous function. Show that every continuous function is Riemann integrable. **08**
 - b) Define limit superior and limit inferior of a sequence. Find the same for the following sequences, hence verify their convergence. **08**
 - i) $S_n = 2 + \frac{(-1)^n}{n}, n \in \mathbb{N}$
 - ii) $S_n = \sin \frac{n\pi}{2}, n \in \mathbb{N}$
- Q.7 Answer the following.**
- a) Show that the set of all rationals is countable. **08**
 - b) Discuss the convergence of a geometric series with common ratio r . **08**

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**M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Linear Algebra & Liner Models (MSC16102)

Day & Date: Sunday, 07-01-2024
Time: 3:00 PM To 6:00 PM

Max. Marks: 80

- Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative:

10

- 1) If v_1, v_2, v_3 are three vectors such that $4v_1 + 2v_2 + v_3 = 0$, then
 - a) v_1, v_2, v_3 are linearly dependent vectors
 - b) v_1, v_2, v_3 are linearly independent vectors
 - c) Need to verify other linear combinations to check independence
 - d) None of these
- 2) What is the dimension of the vector space R^2 over the field R ?
 - a) 1
 - b) Infinite
 - c) 2
 - d) 4
- 3) If A is a non-empty subset of B and B is a set of independent vectors, then the vectors in A are _____.
 - a) Independent vectors
 - b) May or may not be independent vectors
 - c) Dependent vectors
 - d) None of these
- 4) If the determinant of a square matrix is zero, then such a matrix is called as
 - a) Zero matrix
 - b) Insignificant matrix
 - c) Non-singular matrix
 - d) None of these
- 5) If for a 3×3 matrix A , the determinant ($|A|$) is zero, then _____.
 - a) Rank (A) = 1
 - b) Rank (A) < 3
 - c) Rank (A) = 3
 - d) None of these
- 6) Let B be any real matrix and A be its inverse then
 - a) $BA = I$
 - b) $AB = I$
 - c) Both (a) and (b)
 - d) None of these
- 7) If A and B are two matrices of order $n \times n$ with ranks r_1 and r_2 , then _____.
 - a) Rank (AB) = $r_1 + r_2$
 - b) Rank (AB) > $r_1 + r_2$
 - c) Rank (AB) $\geq r_1 + r_2 - n$
 - d) Rank (AB) $\leq r_1 + r_2$
- 8) If the transpose of the given matrix is equal to the matrix itself, then it is called _____.
 - a) Orthogonal matrix
 - b) Symmetric matrix
 - c) Scalar matrix
 - d) Identity matrix

- 9) Vectors whose direction remains unchanged even after applying linear transformation with the matrix are called?
 a) Minor of a matrix b) Eigen values
 c) Cofactor matrix d) Eigen vectors
- 10) Which of the following is true for Gauss-Markov model
 a) $Y = X\beta + \varepsilon$ b) $E(\varepsilon) = 0$
 c) $cov(\varepsilon) = \sigma^2 I$ d) All of these

B) Fill in the blanks.

06

- 1) The set of all linear combinations of a finite set of vectors (S) is called as _____.
- 2) If transpose of the given matrix is equal to the identity matrix, then given matrix must be _____.
- 3) Linear combinations of estimable functions are _____.
- 4) The matrix with only one column is called as _____.
- 5) In the system of linear equations $AX = b$ with unique solution, the matrix A is _____.
- 6) The eigen values of a 2×2 matrix A are 3 and x . If $|A| = 12$, the value of x must be _____.

Q.2 Answer the following

16

- a) Write a note of vector space.
- b) Define and illustrate:
 - i) Symmetric matrix
 - ii) Skew-symmetric matrix
- c) Define inverse and g-inverse of a matrix.
- d) Write a note on minor and cofactor of an element of a matrix.

Q.3 Answer the following.

- a) How Gram-Schmidt orthogonalisation is performed to obtain orthogonal vectors? **08**
- b) Let V be a vector space and $S \subset V$, then show that the span of vectors in S is a subspace of V . **08**

Q.4 Answer the following.

- a) When a set of vectors is said to be linearly independent vectors? Check whether following set is a set of linearly independent vectors- **08**

$$A = \left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 0 \\ 5 \end{pmatrix} \right\}$$
- b) Show that every matrix can be written a sum of symmetric and skew-symmetric matrices. **08**

Q.5 Answer the following.

- a) Show that the rank of a matrix is unaltered by multiplication with a non-singular matrix. **08**
- b) State and prove necessary and sufficient condition for estimability of linear parametric functions. **08**

Q.6 Answer the following.

a) Find inverse and g-inverse for the below matrix: 08

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 7 \\ 7 & 1 & 3 \end{bmatrix}$$

b) Show that the below equations are consistent. Also solve them. 08

$$x + y + z = 6$$

$$x + 2y + 3z = 14$$

$$x + 4y + 7z = 30$$

Q.7 Answer the following.

a) Describe echelon form of a matrix. Show that rank of an echelon matrix is equal to the number of non-zero rows in the matrix. 08

b) Define- 08

i) Non-singular matrix

ii) Rank of a matrix

Show that all non-singular matrices of order n have same rank.

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M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Distribution Theory (MSC16103)

Day & Date: Tuesday, 09-01-2024
 Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Question Nos. 1 and 2 are compulsory.
 2) Attempt any three questions from Q. No. 3 to Q. No. 7.
 3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

- 1) If the distribution function of two dimensional random variates X and Y is denoted by $F(x, y)$, then _____.
 - a) $-1 \leq F(x, y) \leq 1$
 - b) $0 \leq F(x, y) \leq 1$
 - c) $-\infty \leq F(x, y) \leq \infty$
 - d) $0 \leq F(x, y) \leq \infty$
- 2) Which of the following is a scale family?
 - a) $U(0, \theta)$
 - b) $N(0, \sigma^2)$
 - c) $Exp(\theta)$
 - d) All the above
- 3) A random variable X is symmetric about point α then _____.
 - a) $f(\alpha + x) = f(\alpha - x)$
 - b) $f(\alpha + x) = f(x - \alpha)$
 - c) $f(\alpha + x) = -f(\alpha + x)$
 - d) none of these
- 4) Let X be distributed as $U(0, \theta)$. Then distribution of $Y = X/\theta$ is _____.
 - a) $U(0, 1/\theta)$
 - b) $U(0, \theta)$
 - c) $U(0, 1)$
 - d) $Exp(\text{Mean } \theta)$
- 5) Suppose X is $N(0, 1)$ and Y is chi-square with n degrees of freedom. Which of the following is always correct?
 - a) $E[X^2 + Y] = 1 + n$
 - b) $X / \sqrt{\frac{Y}{n}}$ is t_n
 - c) $X^2 + Y$ is χ_{n+1}^2
 - d) $Var[X + Y] = 1 + 2n$
- 6) If $\mu_1' = 2, \mu_2' = 8$ and $\mu_3 = 3$ then value of μ_3' is _____.
 - a) 45
 - b) 35
 - c) 25
 - d) 15
- 7) If $X > 0$ then _____.
 - a) $E[\sqrt{X}] = \sqrt{E(X)}$
 - b) $E[\sqrt{X}] \geq \sqrt{E(X)}$
 - c) $E[\sqrt{X}] \leq \sqrt{E(X)}$
 - d) none of these
- 8) The probability generating function (PGF) of geometric distribution with parameter p is $P_X(S) =$ _____.
 - a) $p/(1 - qS)$
 - b) $q/(1 - pS)$
 - c) $p/(1 - q/S)$
 - d) $q/(1 - p/S)$
- 9) Let (X_1, X_2, X_3) have a trinomial distribution with parameters $(15, 0.4, 0.4, 0.2)$. Then the conditional distribution of X_1 given $X_2 = 5$ is _____.
 - a) $B(15, 2/3)$
 - b) $B(10, 2/3)$
 - c) $B(15, 0.4)$
 - d) $B(10, 0.4)$

Q.7 Answer the following.

a) Let (X, Y) has $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain the conditional distribution of Y given $X = x$. **08**

b) Let X and Y are jointly distributed with pdf **08**

$$f(x, y) = \begin{cases} k(x + 2y), & 0 < x < 2, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find marginal distributions of X and Y .

Seat No.	
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M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Estimation Theory (MSC16104)

Day & Date: Thursday, 11-01-2024
 Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos.1 and 2 are compulsory.
 2) Attempt any three questions from Q. No. 3 to Q. No. 7
 3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

- 1) An unbiased estimator of θ based on random sample of size n from a distribution having pdf $f(x, \theta) = 1/\theta, 0 < x < \theta$ is _____.
 - a) sample mean
 - b) sample median
 - c) largest observation
 - d) double of sample mean
- 2) Which of the following is not a member of one-parameter exponential family of distributions?
 - a) Bernoulli $(1, \theta)$
 - b) Cauchy $(1, \theta)$
 - c) Normal $(\theta, 1)$
 - d) Poisson (θ)
- 3) Suppose $f(x, \theta)$ is pdf of random variable X for which differentiation under integral sign is permissible. Then $E\left(\frac{\partial \log f(x, \theta)}{\partial \theta}\right)$ is _____.
 - a) equal to Fisher information
 - b) less than zero
 - c) less than one
 - d) equal to zero
- 4) The denominator of Cramer-Rao inequality gives _____.
 - a) lower bound
 - b) upper bound
 - c) amount of information
 - d) none of the above
- 5) A statistic $T(X)$ for θ is said to be ancillary if _____.
 - a) The distribution of $T(X)$ is independent of θ
 - b) $T(X)$ is independent of θ
 - c) $T(X)$ is dependent of θ
 - d) The distribution of $T(X)$ is depends on θ
- 6) Let X_1, X_2, \dots, X_n are random variables having joint pdf $f_\theta(x_1, x_2, \dots, X_n)$, $\theta \in \theta$, then Fisher information $I(\theta)$ about θ contained in the observations \underline{x} is given by _____.
 - a) $E_\theta \left(\frac{\partial^2 \log_e f_\theta(\underline{x})}{\partial \theta^2} \right)$
 - b) $E_\theta \left(- \frac{\partial^2 \log_e f_\theta(\underline{x})}{\partial \theta^2} \right)^2$
 - c) $E_\theta \left(\frac{\partial \log_e f_\theta(\underline{x})}{\partial \theta} \right)^2$
 - d) None of the above
- 7) Prior distribution is the _____.
 - a) distribution of parameter θ
 - b) distribution of sample X
 - c) conditional distribution of X given θ
 - d) conditional distribution of θ given X

- 8) Bayes estimator of a parameter under absolute error loss function is _____.
 - a) posterior mean
 - b) posterior median
 - c) posterior mode
 - d) posterior variance
- 9) If T_1 is sufficient statistic for θ and T_2 is an unbiased estimator of θ , then an improved estimator of θ in terms of its efficiency is _____.
 - a) $E(T_1 T_2)$
 - b) $E(T_1 + T_2)$
 - c) $E(T_1 / T_2)$
 - d) $E(T_2 / T_1)$
- 10) Regularity conditions of Cramer-Rao inequality are related to _____.
 - a) integrability of functions
 - b) differentiability of functions
 - c) both integrability and differentiability of functions
 - d) neither integrability nor differentiability of functions

B) Fill in the blanks.

06

- 1) Let X_1, X_2 is a random sample from Poisson (λ) distribution. Moment estimator of λ is _____.
- 2) Let X_1, X_2 be a random sample from $U(0, \theta), \theta > 0$. MLE of θ is _____.
- 3) Cramer-Rao inequality with regards to the variance of an unbiased estimator provides _____ bound.
- 4) Suppose T_n sufficient for θ . Then $g(T_n)$ is sufficient for $g(\theta)$ if $g(\cdot)$ is _____ function.
- 5) The MLE of parameter θ is a statistic that _____ the likelihood function L .
- 6) Bhattacharya bound is the generalization of the _____.

Q.2 Answer the following.

16

- a) Define power series distribution. Give an example of the same.
- b) State Basu's theorem. Illustrate the applicability of Basu's theorem with example.
- c) Let random variable X has $B(n, \theta)$ distribution. Show that distribution of X is complete.
- d) Define MLE. Show that an MLE, if exists, is a function of sufficient statistic.

Q.3 Answer the following.

- a) Define one parameter exponential family of distributions. Obtain a minimal sufficient statistic for this family. **08**
- b) Using the definition of sufficient statistic, examine whether $X_1 + X_2$ is sufficient for Poisson parameter λ based on random sample X_1, X_2 on Poisson distribution. **08**

Q.4 Answer the following.

- a) Describe method of moments and method of minimum chi-square. **08**
- b) Let X_1, X_2, \dots, X_n be iid $U(0, \theta), \theta > 0$. **08**
Find
 - 1) Moment estimator θ
 - 2) MLE of θ .

Q.5 Answer the following.

- a) Obtain Bhattacharya bound under regularity conditions to be stated. Obtain C-R lower bound as a special case of Bhattacharya bound. **08**
- b) Let X_1, X_2, \dots, X_n be iid Poisson (λ) random variables. Show that regularity conditions are satisfied. Obtain the C-R lower bound for variance of unbiased estimator of λ . **08**

Q.6 Answer the following.

- a) State and prove Rao Blackwell and Lehmann-Scheffe theorems. **08**
- b) Use Rao-Blackwell theorem to derive *UMVUE* of $P(X_1 = 0)$ based on sample X_1, X_2, \dots, X_n from Poisson (λ), $\lambda > 0$ distribution. **08**

Q.7 Answer the following.

- a) Define prior and posterior distributions. Illustrate with one example for each of them. **08**
- b) Let X_1, X_2, \dots, X_n is a random sample from $B(1, \theta)$ distribution and prior density of θ is $B_1(\alpha, \beta)$. Assuming squared error loss function, find the Bayes estimator of θ . **08**

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M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Statistical Computing (MSC16108)

Day & Date: Friday, 29-12-2023
 Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
 2) Attempt any three questions from Q. No. 3 to Q. No. 7
 3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: 10

- 1) _____ introduced the Jackknife method.
 - a) Newton-Raphson
 - b) Quenouille
 - c) Fisher
 - d) Efron
- 2) In bootstrap resampling _____ method is used.
 - a) SRSWR
 - b) SRSWOR
 - c) Stratified
 - d) Systematic
- 3) When applying Simpson's 3/8th rule the number of sub-intervals should be _____.
 - a) odd
 - b) even
 - c) at least 6
 - d) multiple of 3
- 4) Let $X \sim U(0,1)$ then the cumulative distribution function $F_x(X)$ has _____.
 - a) Gamma
 - b) $U(2, 3)$
 - c) $U(3, 2)$
 - d) $U(0, 1)$
- 5) In EM algorithm 'M' stands for _____.
 - a) Minimax
 - b) Multivariate
 - c) Maximization
 - d) Multinomial
- 6) The two point Gauss - Legendre quadrature formula is _____.
 - a)
$$\int_{-1}^1 f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) + E(I)$$
 - b)
$$\int_{-1}^1 f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) - f\left(\frac{1}{\sqrt{3}}\right) + E(I)$$
 - c)
$$\int_{-1}^1 f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f(0) + f\left(\frac{1}{\sqrt{3}}\right) + E(I)$$
 - d)
$$\int_{-1}^1 f(x)dx \cong \frac{\pi}{2}f\left(\frac{-1}{\sqrt{2}}\right) + \frac{\pi}{2}f\left(\frac{1}{\sqrt{2}}\right) + E(I)$$
- 7) EM is used to obtain _____ estimator.
 - a) Maximum Likelihood
 - b) Unbiased
 - c) Moment
 - d) None of these

- 8) The steepest ascent method is used to find _____ of the given function.
 - a) minimum
 - b) maximum
 - c) nominal level
 - d) mean
- 9) For $f(x) = -x^2$ with $x_0 = 3$ and $\alpha = 0.5$, the maximum value of the given function using steepest ascent method is _____.
 - a) 1
 - b) 3
 - c) 0
 - d) -3
- 10) Latent variable is defined as _____.
 - a) a variable which is not directly observed
 - b) quantitative variable
 - c) qualitative variable
 - d) None of these

B) Fill in the blanks.

06

- 1) In Bootstrap resampling technique, from original sample of size n , we get _____ resample.
- 2) In EM algorithm 'E' stands for _____.
- 3) If $U_i \sim U(0,1)$ then, $Z = \sum_{i=1}^{12} U_i - 6$ Follows _____ distribution.
- 4) To generate single random number from bivariate exponential, it requires _____ independent exponential random numbers.
- 5) Acceptance - Rejection method used to _____.
- 6) Bootstrap is a _____ technique.

Q.2 Answer the following.

16

- a) Describe Monte Carlo integration technique.
- b) Describe the Newton - Raphson method of finding the correct root.
- c) State advantages and disadvantages of bootstrap technique.
- d) What do you mean by gradient search method? Define its types.

Q.3 Answer the following.

- a) Explain theory of importance sampling with application to reduce Monte Carlo error. **08**
- b) What is acceptance rejection ($A - R$) method of random number generation? Derive an algorithm for generating random numbers from $N(\mu, \sigma^2)$. **08**

Q.4 Answer the following.

- a) Describe linear congruential method of random number generation. Illustrate with example. **08**
- b) State and prove the result for generating random numbers from Poisson distribution. **08**

Q.5 Answer the following.

- a) Let X_1, X_2, \dots, X_n be a random sample of size n from the displaced exponential with pdf $e^{-(x-\theta)} I_{[\theta, \infty]}(x)$ then show that, the jackknife estimator is unbiased estimator of θ . **08**
- b) Explain bootstrap technique as bias reduction technique. **08**

Q.6 Answer the following.

- a) What is EM algorithm? When we use EM algorithm? Illustrate with example. **08**
- b) What is convolution of statistical distribution? State and prove the result of convolution for Poisson distribution. **08**

Q.7 Answer the following.

- a) State and prove the result for generating random numbers from discrete uniform distribution. **08**
- b) Let $X \sim U(0,1)$ and $Y \sim U(0,1)$. Define $Z = X + Y$, obtain the distribution of Z using convolution theorem. **08**

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**M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Probability Theory (MSC16201)

Day & Date: Monday, 18-12-2023
Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

- 1) If for a r.v. X , $X(\omega) = c$, a constant for all ω , then r.v. X is called _____.

a) Good r.v.	b) Conjugate r.v.
c) Degenerate r.v.	d) Concave r.v.
- 2) The largest field of subsets of Ω is called as _____.

a) power set	b) Universal set
c) sample class	d) none of these
- 3) If P is a probability measure defined on (Ω, \mathcal{A}) , then $P(\varphi) = \underline{\hspace{2cm}}$
(φ is empty set)

a) Zero	b) One
c) 0.5	d) 0.3325
- 4) Lebesgue measure of a singleton set $\{k\}$ is _____.

a) 0	b) 1
c) k	d) None of these
- 5) A class is a collection of _____.

a) Numbers	b) Alphabets
c) Sets	d) none of these
- 6) The σ - field generated by the intervals of the type $(-\infty, x)$, $x \in R$ is called _____.

a) Standard σ - field	b) Borel σ - field
c) Closed σ - field	d) None of these
- 7) If a random variable X is integrable, then _____.

a) X^+ is integrable	b) X^- is integrable
c) $ X $ is integrable	d) all of these
- 8) If X and Y are independent variables, then $E(X + Y) = \underline{\hspace{2cm}}$.

a) $E(X) \cdot E(Y)$	b) $E(X) + E(Y)$
c) $E(X) - E(Y)$	d) $E(X)/E(Y)$
- 9) The limit of suprema sequence is called as _____.

a) Limit inferior	b) Limit superior
c) Limit	d) None of these
- 10) If a r.v. X is symmetric about zero, then the characteristic function $\varphi_x(t)$ of X is _____.

a) Real	b) doesn't exist
c) Complex	d) None of these

- B) Fill in the blanks.** **06**
- 1) A well-defined collection of sets is called as _____.
 - 2) If A is empty set, then $P(A) =$ _____.
 - 3) If Ω contains 3 elements, then the largest field of subsets of Ω contains _____ sets.
 - 4) A non-empty class contains at least _____ sets.
 - 5) If for events A and $B, A \cup B = \Omega$ then these events are called as _____.
 - 6) If P is a probability measure defined on (Ω, \mathcal{A}) , then $P(\Omega) =$ _____.

- Q.2 Answer the following.** **16**
- a) Prove that inverse mapping preserves all set relations.
 - b) Define conditional probability measure. Show that it is also a probability measure.
 - c) Prove or disprove: Arbitrary union of fields is a field.
 - d) Define mixture of two probability measures. Show that mixture is also a probability measure.

- Q.3 Answer the following.**
- a) Prove that collection of sets whose inverse images belong to a σ - field, is a σ - field. **08**
 - b) Define field and σ - field. Show that there exist classes which are field but not σ - field. **08**

- Q.4 Answer the following.**
- a) Define expectation of simple random variable. If X and Y are simple random variables, prove the following: **08**
 - i) $E(X + Y) = E(X) + E(Y)$
 - ii) $E(cX) = c E(X)$, where c is a real number.
 - iii) If $X > 0$ a. s., then $E(X) > 0$.
 - b) Discuss Borel σ - field. Find the Borel sets. **08**

- Q.5 Answer the following.**
- a) Show that Probability measure is a continuous measure. **08**
 - b) Define convergence in probability and convergence in distribution. Also prove that convergence in probability implies convergence in distribution. **08**

- Q.6 Answer the following.**
- a) Define the characteristic function of a random variable. Also state its inversion theorem and uniqueness property. **08**
 - b) Prove that expectation of a random variable X exists, if and only if $E|X|$ exists. **08**

- Q.7 Answer the following.**
- a) Find the characteristic function for binomial distribution. **08**
 - b) State and prove Yule-Slutsky results. **08**

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M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Stochastic Processes (MSC16202)

Day & Date: Tuesday, 19-12-2023
 Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
 2) Attempt any three questions from Q. No. 3 to Q. No. 7
 3) Figure to right indicate full marks.

Q.1 A) Choose the correct options.

10

- 1) If i and j are communicating states, and state i is persistent, then
 - a) State j is also persistent
 - b) State j is transient
 - c) State j is may or may not be persistent
 - d) None of these
- 2) If period of a state is one, then the state is called as _____.
 - a) Uniperiodic
 - b) Aperiodic
 - c) Periodic
 - d) None of these
- 3) If $\{N(t)\}$ is a counting process, then $N(0) =$ _____.
 - a) 0
 - b) 1
 - c) 10
 - d) 2.71
- 4) If $\{N(t)\}$ is a Poisson process with parameter λ , then $E(N(t)) =$ _____.
 - a) λ
 - b) λt
 - c) t
 - d) λ^2
- 5) The process $\{X(t), t > 0\}$, where $X(t) =$ number of particles in a room at time t , is an example of _____ stochastic process.
 - a) discrete time continuous state space
 - b) discrete time discrete state space
 - c) continuous time continuous state space
 - d) continuous time discrete state space
- 6) For a persistent state 'i' the ultimate first return probability $F_{ii} =$ _____.
 - a) 1
 - b) 0
 - c) 0.5
 - d) 0.33
- 7) In a Branching process if $E(X_1) = m$, then $E(X_n) =$ _____.
 - a) N
 - b) m^n
 - c) n^m
 - d) None of these
- 8) Which of the following are class properties?
 - a) Persistency
 - b) Periodicity
 - c) Transientness
 - d) all of these
- 9) The row sum of every row of a transition probability matrix (TPM) is always _____.
 - a) Two
 - b) Zero
 - c) Non-negative
 - d) One

- 10) A finite Markov chain which contains only one communication class is called as _____.
 a) irreducible Markov chain b) reducible Markov chain
 c) finite Markov chain d) None of these

B) Fill in the blanks **06**

- 1) Recurrent state is also called as _____.
- 2) For non-null recurrent state, the mean recurrent time is _____.
- 3) For a TPM of a reducible Markov chain, the row sum is _____.
- 4) If two states are communicating with each other and one of them is transient, then the other one must be _____.
- 5) For a symmetric random walk, probability 'p' of positive jump is _____.
- 6) The probability of ultimate return for a transient state is _____.

Q.2 Answer the following **16**

- a) Define and illustrate Markov chain.
- b) Give classification of Stochastic processes according to state space and time domain.
- c) Write a short note on Mean recurrent time of a state.
- d) Discuss probability of first return for a state.

Q.3 Answer the following **08**

- a) Explain Gamblers ruin problem. Obtain the probability that starting with i units the Gamblers fortune will reach N before reaching zero. **08**
- b) Verify the states of random walk model for persistency as well as for periodicity **08**

Q.4 Answer the following **08**

- a) Prove that, Markov chain is completely specified by one step t.p.m. and initial distribution **08**
- b) Describe gambler's game. If a gambler starts the game with initial amount 'i', find his winning probability. **08**

Q.5 Answer the following **08**

- a) Define stationary distribution of a Markov chain. Find the same for a Markov chain with state space $\{1,2,3\}$, whose tpm is **08**

$$\begin{bmatrix} 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{5} & \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$

- b) A Markov chain with state space $S = \{1,2,3\}$ has tpm $\begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$ **08**

It is known that the process has started with the state $X_0 = 2$

- i) $P(X_1 = 2)$
- ii) $P(X_2 = 3)$
- iii) $P(X_0 = 1)$
- iv) $P(X_3 = 2/X_1 = 1)$

Q.6 Answer the following

- a) State and prove class property of periodicity. **08**
- b) If $\{N(t)\}$ is a Poisson process, then for $s < t$, obtain the distribution of $N(s)$, **08**
if it is already known that $N(t) = k$.

Q.7 Answer the following

- a) Define pure birth process and obtain its probability distribution. **08**
- b) Discuss stationary distribution of a Markov chain. Also give illustration. **08**

Seat No.	
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**M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Theory of Testing of Hypotheses (MSC16203)

Day & Date: Wednesday, 20-12-2023
Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

- 1) Which of the following is a simple hypothesis for $N(\mu, \sigma^2)$?
 - a) $H_0: \mu = 5, \sigma = 2$
 - b) $H_0: \mu = 10$
 - c) $H_0: \mu = 0, \sigma > 1$
 - d) $H_0: \mu \neq 3, \sigma = 1$
- 2) Let $f_\theta, \theta \in \theta = \{\theta_0, \theta_1\}$. Then MP test is based on _____.
 - a) $H_0: \theta \leq \theta_0$ against $H_1: \theta > \theta_0$
 - b) $H_0: \theta_0 < \theta < \theta_1$ against $H_1: \theta \leq \theta_0$ or $\theta > \theta_1$
 - c) $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1$
 - d) $H_0: \theta \leq \theta_0$ or $\theta > \theta_1$ against $H_1: \theta_0 < \theta < \theta_1$
- 3) An UMP test _____.
 - a) is biased test
 - b) is an unbiased test
 - c) always exist
 - d) none of these
- 4) Let X_1, X_2, \dots, X_n are iid with $N(\theta, 1)$. Let $H_0: \theta = \theta_0$ and $H_1: \theta \neq \theta_0$. For any $\alpha, 0 < \alpha < 1$, _____.
 - a) there does not exist a UMP level α test.
 - b) there exists a UMP level α test.
 - c) there exists a test with one sided.
 - d) none of these.
- 5) If λ is the likelihood ratio test statistic, which one of the following has got its asymptotic distribution as χ^2 distribution?
 - a) $\log_e(\lambda)$
 - b) $\log_e(1/\lambda)$
 - c) $\log_e(\lambda^2)$
 - d) $\log_e(1/\lambda^2)$
- 6) If, for a given $\alpha, 0 \leq \alpha \leq 1$, non-randomized Neyman-Pearson and likelihood ratio test of a simple hypothesis against a simple alternative exists, then which one of the following is correct?
 - a) They are one and the same
 - b) They are equivalent
 - c) They are exactly opposite
 - d) One cannot say anything about it.
- 7) The expected value of the runs in the sequence $XY Y X Y X X$ is _____.
 - a) 3.1
 - b) 4
 - c) 4.4
 - d) 5.2
- 8) In Kruskal-Wallis test of k samples, the appropriate degrees of freedom are _____.
 - a) k
 - b) $k - 1$
 - c) $k + 1$
 - d) $n - k$

- 9) If n_1 and n_2 in Mann-Whitney test are large, the statistic U is distributed with mean _____.
 a) $(n_1 + n_2)/2$ b) $(n_1 - n_2)/2$
 c) $n_1 n_2 / 2$ d) $n_1 n_2$
- 10) If $\phi_1(x)$ and $\phi_2(x)$ are two test functions of size α each then size of $\lambda\phi_1(x) + (1 - \lambda)\phi_2(x)$ is _____.
 a) α b) $\lambda\alpha$
 c) 1 d) Not defined

B) Fill in the blanks. 06

- 1) Level of significance is the probability of _____ type of error.
- 2) The distribution of statistic used in sign test is _____.
- 3) The non-parametric test for goodness of fit of a distribution is _____.
- 4) If all frequencies of classes are same, the value of Chi-square is _____.
- 5) The range of Kendall's rank correlation τ is _____ (-1 to 1)
- 6) UMAU confidence intervals are obtained from _____ tests.

Q.2 Answer the following. 16

- a) Define monotone likelihood ratio (MLR) property.
- b) Distinguish between randomized and non-randomized tests.
- c) Describe signed-rank test in brief.
- d) State two sample U statistic theorem.

Q.3 Answer the following 08

- a) Define most powerful (MP) test. Show that MP test need not be unique using suitable example. 08
- b) Obtain MP test of level α for testing $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1 (> \mu_0)$ based on a random sample of size n from $N(\mu, \sigma^2)$, where σ^2 is known. 08

Q.4 Answer the following 08

- a) When a family of densities is said to have monotone likelihood ratio? Show that the one-parameter exponential family of densities belongs to this class of MLR densities. 08
- b) Let X_1, X_2, \dots, X_n be a random sample from $N(\theta, \sigma^2)$, where θ is known. Obtain UMP level α test for testing $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 > \sigma_0^2$ 08

Q.5 Answer the following 08

- a) Explain the concepts of UMPU tests and show that MP and UMP tests of size α are unbiased. 08
- b) Let X_1, X_2, \dots, X_n be a random sample of size n from $U(0, \theta)$ distribution. Obtain shortest length confidence interval for θ . 08

Q.6 Answer the following. 08

- a) Define confidence set and UMA confidence set of level $(1 - \alpha)$. Derive the relationship between UMA confidence set and UMP test. 08
- b) Let X_1, X_2, \dots, X_n be a random sample from exponential distribution with mean θ . Consider the testing of hypothesis problem $H_0: \theta = \theta_0$ against $H_1: \theta < \theta_0$. Find UMA $(1 - \alpha)$ level family of confidence sets corresponding to size α UMP test. 08

Q.7 Answer the following.

- a) State and prove a necessary and sufficient condition for a similar test to have Neyman structure. **08**
- b) Derive LRT for testing $H_0: \mu = \mu_0$ against $H_1: \mu \neq \mu_0$ based on a sample of size n from $N(\mu, 1)$ distribution. **08**

Seat No.	
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M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS

Sampling Theory (MSC16206)

Day & Date: Thursday, 21-12-2023
Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. 10

- 1) Selection of Indian cricket team for the world cup is _____ sampling.
 - a) random
 - b) systematic
 - c) purposive
 - d) cluster
- 2) In linear systematic sampling of 20 units from a population of 200 units, the probability that units U_{21} and U_{22} are both in a sample is _____.
 - a) 0
 - b) 1/10
 - c) 1/20
 - d) 1/400
- 3) Systematic sampling means _____.
 - a) selecting n continuous units
 - b) selecting n units situated at equal intervals
 - c) selection of n largest units
 - d) selection of any n units
- 4) In sampling with probability proportional to size, the units are selected with probability proportional to _____.
 - a) size of the unit
 - b) size of the sample
 - c) size of the population
 - d) None of these
- 5) In simple random sampling the ratio estimator is _____.
 - a) always unbiased
 - b) always biased
 - c) minimum variance unbiased
 - d) None of these
- 6) A population is divided into clusters and it has been found that all the units within a cluster are same. In this situation, which sampling will be adopted?
 - a) SRSWOR
 - b) Systematic sampling
 - c) Cluster sampling
 - d) Stratified random sampling
- 7) Sampling error can be reduced by _____.
 - a) increasing the population
 - b) decreasing the sample size
 - c) increasing the sample size
 - d) None of the above

- 8) Deming's technique is used to deal with _____.
 a) sampling errors b) non-response errors
 c) non-sampling errors d) None of the above
- 9) Simple regression estimator of population total is given by _____.
 a) $N[\bar{x} + b(\bar{X} - \bar{y})]$ b) $N[\bar{x} + b(\bar{x} - \bar{y})]$
 c) $N[\bar{y} + b(\bar{X} - \bar{x})]$ d) $N[\bar{y} + b(\bar{x} - \bar{y})]$
- 10) Non-response in survey means _____.
 a) non availability of respondent
 b) non return of questionnaire by person
 c) refuse to give information by respondent
 d) All the above

B) Fill in the blanks.

06

- 1) The probability of drawing a unit at each subsequent draw remains same in _____ sampling scheme.
- 2) In the context of sampling the fraction n/N is called _____.
- 3) Variance of optimum allocation is always _____ that of proportional allocation.
- 4) Two stage sampling design is more efficient than single stage sampling if correlation between units in the first stage is _____.
- 5) Under SRSWOR sampling design, the bias of regression estimator of population mean \bar{Y}_{Reg} is given by _____.
- 6) If 100 students are selected out of 500, and 25 students are then selected from the selected 100 students. The procedure adopted is _____.

Q.2 Answer the following

16

- a) Give advantages of sampling method over census method.
- b) Define ratio estimator of population mean and obtain its expected value.
- c) Describe the Lahiri's method of selecting a probability proportional to size sample from a finite population of size N .
- d) What is two stage sampling? Give a practical example where two-stage sampling scheme may be adopted.

Q.3 Answer the following.

- a) Describe simple random sampling. In SRSWOR, show that the probability of drawing a specified unit at every draw is the same. **08**
- b) In SRSWOR, derive an unbiased estimator of a population mean and its sampling variance. **08**

Q.4 Answer the following.

- a) What is proportional allocation? Derive the variance of the estimator of the population mean under this allocation. **08**
- b) Define linear systematic sampling. Derive the sampling variance of the traditional unbiased estimator of a population mean under this scheme. **08**

Q.5 Answer the following.

- a) Define a cluster sampling. Bring out similarities and differences between cluster sampling and stratified sampling. **08**
- b) Define PPSWR sampling design. Obtain an unbiased estimator of population total and its variance when PPSWR sample of size n is drawn from a population of size N . **08**

Q.6 Answer the following.

- a) Define Horvitz-Thompson estimator of population mean and establish its unbiasedness under an arbitrary sampling design. Also derive its sampling variance. **08**
- b) Develop Des Raj's ordered estimator of population mean based sample size 2. Show that it is unbiased. **08**

Q.7 Answer the following.

- a) Define linear regression estimator for a population mean. Derive the approximate expression for bias of the estimator. **08**
- b) Explain the problem of non-response and any one technique to deal with the non-response. **08**

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**M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Asymptotic Inference (MSC16301)

Day & Date: Friday, 05-01-2024
Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative **10**

- 1) If T_n is consistent of θ then _____
 - a) T_n is consistent of θ^2
 - b) T_n is consistent of $\sqrt{\theta}$
 - c) T_n is consistent of e^θ
 - d) None of these
- 2) Consider the following statements:
 - 1) Strong consistency implies weak consistency.
 - 2) Weak consistency implies strong consistency.
 Which of the above statements is / are true?
 - a) only 1
 - b) only 2
 - c) both 1 and 2
 - d) neither 1 nor 2
- 3) Let T_n be an unbiased and consistent estimator of θ then T_n^2 for θ^2 is _____
 - a) unbiased and consistent both
 - b) unbiased only
 - c) consistent only
 - d) neither unbiased nor consistent
- 4) Which one of the following is true for estimation of θ for $U(0, \theta)$ distribution by the MLE _____
 - a) unbiased but not consistent
 - b) consistent but not unbiased
 - c) both consistent and unbiased
 - d) neither consistent nor unbiased
- 5) Based on sample of size n from $N(\theta, 1)$ an estimator \bar{X}_n for θ is _____.
 - a) Unbiased
 - b) Consistent
 - c) CAN
 - d) all the above
- 6) Exponential distribution with location parameter θ is _____.
 - a) one parameter exponential family
 - b) Cramer family
 - c) both (A) and (B)
 - d) neither (A) nor (B)
- 7) For sufficiently large sample size, with probability close to one, the likelihood equation admits _____.
 - a) no consistent solution
 - b) unique consistent solution
 - c) two consistent solutions
 - d) more than two consistent solutions

- 8) Let X_1, X_2, \dots, X_n be iid from Poisson (θ) and \bar{X}_n is CAN for θ . CAN estimator of $P_\theta(X = 1)$ is _____.
 - a) \bar{X}_n
 - b) $e^{-\bar{X}_n}$
 - c) $\bar{X}_n e^{-\bar{X}_n}$
 - d) none of these
- 9) In LRT, under some regularity conditions on $f(x, \theta)$, the random variable $-2 \log \lambda(x)$ [where $\lambda(x)$ is likelihood ratio] is asymptotically distributed as _____.
 - a) chi-square
 - b) exponential
 - c) Normal
 - d) F-distribution
- 10) Mean squared error of an estimator T_n of θ is expressed as _____.
 - a) $Var_\theta(T_n) + Bias$
 - b) $Var_\theta(T_n) + [Bias]^2$
 - c) $[Var_\theta(T_n)]^2 + [Bias]^2$
 - d) $[Var_\theta(T_n) + Bias]^2$

B) Fill in the blanks. 06

- 1) For Cauchy distribution with location parameter θ , consistent estimator of θ is _____.
- 2) The asymptotic distribution of Wald's statistic is _____.
- 3) In testing independence in a 2×3 contingency table, the number of degrees of freedom in χ^2 distribution is _____.
- 4) The variance stabilizing transformation for normal population is _____.
- 5) Exponential family is _____ than Cramer family.
- 6) To investigate the significance difference between variances of several normally distributed populations _____ test is used.

Q.2 Answer the following 16

- a) Define
 - i) Weak consistency
 - ii) Strong consistency
- b) Give an example of consistent estimator which is not CAN.
- c) Describe Rao's score test. State its asymptotic distributions
- d) Let X_1, X_2, \dots, X_n be iid Poisson (λ). Show that sample mean \bar{X}_n is consistent for λ .

Q.3 Answer the following 08

- a) Define consistent estimator. State and prove invariance property of consistent estimator of a real valued parameter θ 08
- b) Let X_1, X_2, \dots, X_n be iid from exponential distribution with mean θ . Obtain consistent estimator for first and third quartiles. 08

Q.4 Answer the following 08

- a) Show that sample distribution function at a given point is CAN for the population distribution function at the same point. 08
- b) Let X_1, X_2, \dots, X_n be iid exponential with location θ . Examine whether $X_{(1)}$ is CAN for θ 08

Q.5 Answer the following 08

- a) Derive asymptotic distribution of Pearson's chi-square statistic. 08
- b) Let X_1, X_2, \dots, X_n be a random sample of size n from $N(\mu, \sigma^2)$. Obtain MLE of (μ, σ^2) . Show that it is CAN. Obtain its asymptotic variance-covariance matrix. 08

Q.6 Answer the following

- a) Explain variance stabilizing transformations and illustrate their use in large sample estimation and tests. **08**
- b) Based on random sample of size n from $N(\theta, \sigma^2)$, find variance stabilizing transformation for S^2 . Using this transformation, obtain $100(1 - \alpha)\%$ confidence interval for σ^2 . **08**

Q.7 Answer the following

- a) Derive Bartlett's test for homogeneity of variances of several normal populations **08**
- b) Let X_1, X_2, \dots, X_n be iid $B(1, \theta)$. Let $\phi(\theta) = \theta(1 - \theta)$. Obtain CAN estimator for $\phi(\theta)$. **08**

Seat No.	
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M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Multivariate Analysis (MSC16302)

Day & Date: Sunday, 07-01-2024
 Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and 2 are compulsory.
 2) Attempt any Three questions from Q. No. 3 to Q. No. 7
 3) Figures to the right indicate full marks.

Q.1 A) Choose Correct Alternative. **10**

- 1) A Wishart distribution has _____ parameter/s.
 a) 1
 b) 2
 c) 3
 d) 4
- 2) Hotelling's T^2 is multivariate extension of _____.
 a) normal distribution
 b) chi-square distribution
 c) t-distribution
 d) F-distribution
- 3) The first principal component have _____ variance.
 a) Least
 b) Largest
 c) Average
 d) none of these
- 4) A divisive hierarchical clustering method employs a _____ strategy.
 a) Top-down
 b) Bottom-up
 c) Random
 d) None of these
- 5) Let X_1, X_2, \dots, X_n be a random sample of size n from p -variate normal distribution with mean vector 0 and covariance matrix Σ . The MLE of Σ is _____.
 a) $\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})(X_i - \bar{X})'$
 b) $\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})(X_i - \bar{X})'$
 c) $\frac{1}{n} \sum_{i=1}^n X_i X_i'$
 d) $\sum_{i=1}^n X_i X_i'$
- 6) Principal components are _____.
 a) orthogonal
 b) uncorrelated
 c) both (a) and (b)
 d) neither (a) nor (b).
- 7) Let random vector $X = (X_1, X_2, X_3)'$ follows $N_3(\mu, \Sigma)$, where $\mu' = [1, 2, 3]$ and $\Sigma = \begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $X_1 - X_2 + X_3$ follows _____ distribution.
 a) $N(2, 5/2)$
 b) $N(0, 2)$
 c) $N(2, 2)$
 d) $N(2, 3)$
- 8) As the distance between two populations increases, misclassification error _____.
 a) Decreases
 b) Increases
 c) remains constant
 d) none of these

- 9) Let X be a random vector with covariance matrix Σ . A decrease in variances of p variables in X will lead to _____.
 a) increase $\text{trace}(\Sigma)$
 b) decrease $\text{trace}(\Sigma)$
 c) does not affect $\text{trace}(\Sigma)$
 d) nothing can be said
- 10) Total variation explained by all principal components is _____ that by the original variables.
 a) equal to
 b) greater than
 c) less than
 d) none of these

B) Fill in the blanks.

06

- 1) Let X_1, X_2, \dots, X_n be a random sample of size n from p -variate normal distribution with mean vector μ and covariance matrix Σ . The distribution of mean vector \bar{X} is _____.
- 2) The mean vector of $(X_1 + X_2, X_1 - X_2)$ is $(10, 0)$ then mean vector of $(X_1, 2X_1 - X_2)$ is _____.
- 3) While applying _____ clustering algorithm, the distance between two clusters is taken to be the smallest distance between observations from two clusters.
- 4) A _____ is a graphical device for displaying clustering results.
- 5) If X has $N_p(\mu, \Sigma)$ distribution then moment generating function of vector X is _____.
- 6) Let vector Y has $N_p(\mu, \Sigma)$ distribution. For a constant matrix $A_{q \times p}$ and vector $b_{q \times 1}$ the distribution of $X = AY + b$ is _____.

Q.2 Answer the following.

16

- a) Describe singular and non-singular normal distribution.
- b) Find maximum likelihood estimator for $\underline{\mu}$ based on a random sample from multivariate normal distribution $N_p(\underline{\mu}, \Sigma)$.
- c) Show that two p -variate normal vectors \underline{X}_1 and \underline{X}_2 are independent if and only if $\text{cov}(\underline{X}_1, \underline{X}_2) = 0$.
- d) Define variance-covariance matrix. State its properties.

Q.3 Answer the following.

- a) With usual notations, find the mean and variance-covariance matrix of multivariate normal distribution. **08**
- b) Obtain the characteristic function of multivariate normal distribution. **08**

Q.4 Answer the following.

- a) Discuss the concept of discriminant analysis in detail. **08**
- b) Find maximum likelihood estimator of Σ based on a random sample from multivariate normal distribution $N_p(\mu, \Sigma)$. **08**

Q.5 Answer the following.

- a) Discuss hierarchical and non-hierarchical clustering. Discuss agglomerative clustering in detail. **08**
- b) Explain the concept of clustering in brief. Discuss k-means clustering. **08**

Q.6 Answer the following.

- a) Describe- **08**
1) Single linkage
2) Complete linkage
Illustrate with the help of an example.
- b) Derive expressions for principle components. Show that total variation explained by principal components is same as total variation in original variables. **08**

Q.7 Answer the following.

- a) If $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$, then find the distribution of the following: **08**
1) $\underline{a}'\underline{X}$, where \underline{a} is a p-dimensional vector of constants
2) $A\underline{X}$, where A is matrix of order $m \times p$
- b) Obtain Fisher's discriminant function for two populations. **08**

Seat No.	
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**M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Planning and Analysis of Industrial Experiments (MSC16303)

Day & Date: Tuesday, 09-01-2024
Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

- 1) In two replications of 2^4 experiment total number of observations are _____.
a) 32 b) 16
c) 64 d) 31
- 2) In one-fourth fraction of 2^7 experiment with defining relation $I = ABCD = CDEFG$. The aliases of AB are _____.
a) $CD, ABCDEFG, EFG$ b) $AB, CDEFG, CD$
c) $CD, ABCDEFG, CDEFG$ d) $ACDEFG, ABEFG, ABCD$
- 3) A connected block design is _____ orthogonal.
a) always b) never
c) may or may not d) none of these
- 4) When the interaction effect ABC is confounded in 2^3 factorial design:
a) the block effect and main effect A are identical
b) the block effect and interaction AB are identical
c) the block effect and interaction ABC are identical
d) the block effect and interaction AC are identical
- 5) In a BIBD (v, b, r, k, λ) , which is not a parametric relationship?
a) $rv = bk$ b) $N'N = (r - \lambda)I_v + \lambda E_{vv}$
c) $r(k - 1) = \lambda(v - 1)$ d) $b \geq v$
- 6) In principle block of a 2^4 experiment in two blocks with $ABCD$ as generator which of the following treatment is present?
a) $(1), abcd, abc, abd, bcd, acd, bc, cd$
b) $(1), ab, ac, ad, bc, bd, cd, abcd$
c) $a, b, c, d, abc, abd, bcd, acd$
d) $ab, ac, ad, bc, bd, cd, abc, abd$
- 7) The two-way ANOVA model without interaction can be written as _____.
a) $Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ijk}$ $i = 1, 2, \dots, v; j = 1, 2, \dots, b$
b) $Y_{ij} = \alpha_i \times \beta_i + \epsilon_{ijk}$ $i = 1, 2, \dots, v; j = 1, 2, \dots, b$
c) $Y_{ij} = \mu - \alpha_i + \beta_j + \epsilon_{ijk}$ $i = 1, 2, \dots, v; j = 1, 2, \dots, b$
d) $Y_{ij} = \mu + \alpha_i - \beta_j + \epsilon_{ijk}$ $i = 1, 2, \dots, v; j = 1, 2, \dots, b$
- 8) In a BIBD, if number of treatments is equal to the number of plots in a block, then BIBD is _____.
a) reduces to CRD b) reduces to RBD
c) reduces to LSD d) none of these

- 9) In one-way ANOVA model with v treatments, which of the following is not assumption of errors?
 a) errors are uncorrelated b) errors have constant variance
 c) errors have mean zero d) errors have binomial distribution
- 10) Smaller the experimental error _____ efficient the design.
 a) Less b) more
 c) equally d) none of these

B) Fill in the blanks.

06

- 1) In partial confounding _____ effects are confounded in _____ replications.
- 2) In a RBD with 5 blocks and 4 treatments, number of plots in each block is _____.
- 3) ANOVA is statistical method of comparing _____ of several populations.
- 4) A design in which main effects are confounded with 2-way interactions is Resolution _____ design.
- 5) The rank of estimation space in one-way ANOVA with v treatment is _____.
- 6) The degrees of freedom corresponding to error in four replicate of 2^4 design are _____.

Q.2 Answer the following.

16

- a) Define one-way classification model. Derive least square estimates of parameters in one-way classification model.
- b) Write a short note on effects in factorial experiments.
- c) Define Total and Partial confounding. Illustrate any one type of confounding.
- d) Write lay out of 3^3 factorial experiment in single replicate.

Q.3 Answer the following.

- a) Derive the test for testing treatments in one-way classification. **08**
- b) Obtain half-fraction of 2^6 experiments. Write its complete alias structure. **08**

Q.4 Answer the following.

- a) Define connected block design. State and prove necessary and sufficient condition for orthogonality of a connected block design. **08**
- b) Describe analysis of 2^n factorial experiment. **08**

Q.5 Answer the following.

- a) Obtain half fraction of 2^5 experiments. Write its consequences. **08**
- b) Define resolution of design and minimum aberration design. Illustrate both. **08**

Q.6 Answer the following.

- a) Write 2^5 experiments in four blocks. Explain analysis of confounded experiments. **08**
- b) Define one-way ANCOVA model. Derive test for testing hypothesis of treatment in one-way ANCOVA. **08**

Q.7 Answer the following.

- a) Define **08**
 - 1) Orthogonal block design
 - 2) Balanced block design
 - 3) BIBD
- b) State and prove properties of $Q = T - NK^{-\delta}B$. **08**

Seat No.	
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**M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Regression Analysis (MSC16306)

Day & Date: Thursday, 11-01-2024
Time: 11:00 AM To 02:00 PM

Max. Marks: 80

- Instructions:** 1) Question Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions.

10

- 1) The estimate of β in the regression model $Y = \alpha + \beta X + \varepsilon$ by method of least squares is _____.
 - a) biased
 - b) unbiased
 - c) inconsistent
 - d) none of these
- 2) In a multiple linear regression model, the hat matrix is _____.
 - a) Symmetric but not idempotent
 - b) idempotent but not Symmetric
 - c) Symmetric and idempotent
 - d) Skew-symmetric but not idempotent
- 3) In a multiple linear regression model with $\varepsilon \sim N(0, \sigma^2 I)$, the distribution of LSE $\hat{\beta}$ is _____.
 - a) $N(\beta, H\sigma^2)$
 - b) $N(\beta, (I - H)\sigma^2)$
 - c) $N(\beta, \sigma^2 I)$
 - d) $N(\beta, (X'X)^{-1}\sigma^2)$
- 4) The difference between the observed value Y_i and corresponding fitted value \hat{Y}_i is called _____.
 - a) intercept
 - b) error
 - c) residual
 - d) none of these
- 5) Normal probability plots show _____.
 - a) residuals plotted versus cumulative probability
 - b) residuals plotted versus observation numbers
 - c) residuals plotted versus predicted values
 - d) Y-values plotted versus X-values
- 6) The coefficient of determination (R^2) is the square of correlation coefficient between (where Y is response) _____.
 - a) Y and hat matrix
 - b) Y and its predicted value
 - c) regressors
 - d) none of these
- 7) The multicollinearity problem in regression concerns the _____.
 - a) error terms
 - b) regressors
 - c) response variable values
 - d) regression coefficients
- 8) The variance stabilizing transformation \sqrt{Y} is used when distribution of Y is _____.
 - a) Poisson
 - b) Binomial
 - c) Normal
 - d) none of these

- 9) Orthogonal polynomials are used to fit a polynomial model of _____.
 - a) first order in one variable
 - b) second order in two variables
 - c) any order in one variable
 - d) any order in two variables
- 10) Logistic regression model is an appropriate model when response variable is distributed as _____.
 - a) Poisson
 - b) Binomial
 - c) Normal
 - d) Gamma

B) Fill in the blanks:

06

- 1) In a simple linear regression model, the distribution of error term is assumed to be _____.
- 2) Cochran-Orkut method of parameter estimation is used in the presence of _____.
- 3) The model $y = \beta_0 x^{\beta_1}$ can be linearized by using _____ transformation.
- 4) The non-linear model transformed to an equivalent linear form is called _____ linear.
- 5) The joint points of pieces in polynomial fitting are usually called _____.
- 6) In a multiple linear regression model with $\varepsilon \sim N(0, \sigma^2 I)$, the variance of residual vector e is _____.

Q.2 Answer the following

16

- a) State assumptions of error vector in multiple regression model.
- b) In a multiple linear regression model, show that the hat matrix is symmetric and idempotent.
- c) Describe eigen value analysis of matrix $X'X$ method for detection of multicollinearity.
- d) Describe polynomial models in one variable and two variables.

Q.3 Answer the following.

- a) Describe multiple linear regression model. Stating the assumptions, obtain mean and variance of least squares estimators of β .
- b) Discuss confidence interval for regression coefficient and prediction interval for future observation in the context of multiple linear regression.

08

08

Q.4 Answer the following.

- a) Discuss the concept of multicollinearity with suitable example. Discuss the examination of correlation matrix method for detection of multicollinearity.
- b) Describe forward selection method for variable selection and state its limitations.

08

08

Q.5 Answer the following.

- a) State the autocorrelation problem. Explain Durbin-Watson test for detecting autocorrelation. What are its limitations?
- b) Explain the residual plots. Outline the procedure of construction of normal probability plot and procedure for checking normality assumption.

08

08

Q.6 Answer the following.

- a) Define k^{th} order polynomial regression model in one variable. Describe orthogonal polynomial to fit the polynomial model in one variable.
- b) Describe the least squares method for parameter estimation in non-linear regression. Discuss the same for $y = \theta_1 e^{\theta_2 x} + \varepsilon$.

08

08

Q.7 Answer the following.

- a) Define generalized linear model. Derive the maximum likelihood estimates of the parameters in generalized linear model. **08**
- b) Define logistic regression model. Derive the maximum likelihood estimates of parameters involved in the single covariate logistic regression model. **08**

Seat No.	
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M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Data Mining (MSC16401)

Day & Date: Monday, 18-12-2023
 Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos.1 and 2 are compulsory.
 2) Attempt any three questions from Q. No. 3 to Q. No. 7
 3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

- 1) The class label of training tuples is not known and the number or set of classes to be learned is also not known in advance. Then it is known as:
 - a) Self learning
 - b) Unsupervised learning
 - c) Supervised learning
 - d) None of these
- 2) _____ maps data into predefined groups.
 - a) Regression
 - b) Time series analysis
 - c) Prediction
 - d) Classification
- 3) An agglomerative hierarchical clustering method uses a _____ strategy.
 - a) Top-down
 - b) Bottom-up
 - c) Random
 - d) None of these
- 4) In k-nearest neighbor algorithm, k stands for _____.
 - a) Number of neighbors that are investigated
 - b) Number of iterations
 - c) Number of total records
 - d) Random number
- 5) In data mining, ANN stands for _____.
 - a) Artificial Neural Network
 - b) A-nearest neighbor
 - c) Adjacent Neural Network
 - d) None of these
- 6) Data used to verify performance of the built model is called _____.
 - a) training data
 - b) trained data
 - c) testing data
 - d) pre-analysis data
- 7) k-nearest neighbor method can be used _____.
 - a) only when class labels are qualitative
 - b) only when class labels are quantitative
 - c) Both (a) and (b)
 - d) None of these

- 8) Unlike in regression problem, the class label in classification problem is _____.
a) numeric (ratio scale) b) Categorical
c) Integer only d) Rational only
- 9) In a feed- forward network, the connections between layers are _____ from input to output.
a) Bidirectional b) Unidirectional
c) Multidirectional d) None of these
- 10) Data by itself is not useful unless _____.
a) It is massive
b) It is properly stated
c) It is collected from diverse sources
d) It is processed to obtain information

B) Fill in the blanks.**06**

- 1) Student learns things in the presence of a teacher. This is considered as _____ learning.
- 2) In _____ machine learning method, patterns are inferred from the unlabeled input data.
- 3) The part of the entire data, which is used for building the model is called as _____.
- 4) The problem of finding hidden structure in unlabeled data is called _____.
- 5) Classification of new species to one of the earlier known families of species is _____.
- 6) In data mining, SVM stands for _____.

Q.2 Answer the following.**16**

- a) What are the advantages of unsupervised learning?
b) Discuss, with illustration, the concept of unsupervised learning.
c) Why Bayes' classifier is called Naive classifier?
d) Discuss accuracy and precision of a classifier.

Q.3 Answer the following.

- a) Discuss information gain in decision tree.
b) Describe decision tree classifier in detail.

08**08****Q.4 Answer the following.**

- a) Discuss k-nearest neighbor classifier in detail.
b) Write down the algorithm for Bayesian classifier.

08**08****Q.5 Answer the following.**

- a) Discuss the different metrics for Evaluating Classifier Performance.
b) Discuss density based methods for unsupervised learning.

08**08**

Q.6 Answer the following.

- a) Describe unsupervised learning. Also explain in detail, association rules and prediction. **08**
- b) Describe - **08**
 - i) Sensitivity of a model
 - ii) Specificity of a modelIllustrate with the help of example.

Q.7 Answer the following.

- a) Explain in detail, market basket analysis. **08**
- b) Discuss characteristics of logistic regression. **08**

Seat No.	
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**M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Noc-2023
STATISTICS**

Industrial Statistics (MSC16402)

Day & Date: Tuesday, 19-12-2023
Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

- 1) Quality is inversely proportional to _____.
 - a) cost
 - b) variability
 - c) method
 - d) time
- 2) Which of the following is useful in data collection activity?
 - a) check sheet
 - b) control chart
 - c) histogram
 - d) Pareto chart
- 3) The S chart is preferred over R chart for _____ sample sizes.
 - a) small
 - b) large
 - c) moderate
 - d) moderate to large
- 4) The use of warning limits used in control charts increases _____.
 - a) proportion of defectives
 - b) process capability
 - c) risk of false alarms
 - d) process variability
- 5) The capability index C_p involves _____ parameter(s) to be estimated.
 - a) only μ
 - b) only σ
 - c) both μ and σ
 - d) none of the above
- 6) In double sampling plan _____.
 - a) only two units are checked
 - b) only first and last lot is checked
 - c) only two samples of respective n_1 and n_2 units are checked necessarily
 - d) only two samples of respective n_1 and n_2 units are checked conditionally
- 7) The curve showing the probability of acceptance of a lot of quality p is known as _____.
 - a) AOQ curve
 - b) ASN curve
 - c) OC curve
 - d) ARL curve
- 8) In a demerit system, the unit will cause personal injury or property damage is classified as _____ defect.
 - a) class A
 - b) class B
 - c) class C
 - d) class D

- 9) If sample size inspected at each stage is one, sequential procedure is called _____
- lot-by-lot sequential sampling
 - item-by-item sequential sampling
 - group sequential sampling
 - none of the above
- 10) An appropriate distribution of run length is _____
- normal
 - Bernoulli
 - geometric
 - Poisson

B) Fill in the blanks

06

- The control limits of the p chart are based on the assumption that the number of nonconforming items follows _____ distribution.
- Usually 2σ limits are called as _____
- An out-of-control signal given by a control chart when the process is actually in-control, is called _____
- V-mask method is used to implement _____ chart.
- To determine location of a defect _____ SPC tool is used.
- In 'DMAIC', M stands for _____

Q.2 Answer the following

16

- Describe process control and product control.
- Explain the use of Pareto chart with suitable example.
- Describe a single sampling plan for attributes.
- Explain producer's risk and consumer's risk.

Q.3 Answer the following

- List seven SPC tools and explain in detail any two of them.
- Outline the steps involved in the construction of \bar{X} and S charts.

08

08

Q.4 Answer the following

- What is an EWMA control chart? Explain the procedure of obtaining the control limits of the same.
- Explain in detail demerit control chart.

08

08

Q.5 Answer the following

- Define process capability index C_p . Obtain $(1 - \alpha)$ level confidence interval for the same.
- Define process capability index C_{pk} with necessary underlying assumptions if any. State and prove its relationship with probability of nonconformance.

08

08

Q.6 Answer the following

- Explain the assumptions, construction and operation of Hotelling's T^2 control chart.
- Explain the variable sampling plan when upper specification is given with known standard deviation.

08

08

Q.7 Answer the following

- Describe six-sigma methodology and DMAIC cycle in detail.
- Discuss nonparametric sign control chart to monitor location of a process.

08

08

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**M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS**

Reliability and Survival Analysis (MSC16403)

Day & Date: Wednesday, 20-12-2023
Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

- 1) In a parallel system, the system reliability is _____ than the reliability of single component in the system.
 - a) same as
 - b) smaller than
 - c) larger than
 - d) none of these
- 2) The i^{th} component X_i of state vector is _____ random variable.
 - a) Geometric
 - b) Bernoulli
 - c) Poisson
 - d) Normal
- 3) A series system is a special case of $k - out - of - n$ system when _____.
 - a) $k = 1$
 - b) $k = 2$
 - c) $k = n - 1$
 - d) $k = n$
- 4) If a distribution function $F(t)$ is IFRA if and only if its survival function $\bar{F}(t)$ satisfies that _____.
 - a) $\bar{F}(\alpha t) \geq \alpha \bar{F}(t)$
 - b) $\bar{F}(\alpha t) \geq [\bar{F}(t)]^\alpha$
 - c) $\bar{F}(\alpha t) \leq \alpha \bar{F}(t)$
 - d) $\bar{F}(\alpha t) \leq [\bar{F}(t)]^\alpha$
- 5) For which of the following family, each member has non-monotonic failure rate?
 - a) Exponential
 - b) Weibull
 - c) Lognormal
 - d) Gamma
- 6) Which of the following rate function corresponds to DFR distribution?
 - a) $h(t) = t$
 - b) $h(t) = e^t$
 - c) $h(t) = e^{-t}$
 - d) $h(t) = t e^t$
- 7) In survival analysis, the outcome variable is _____.
 - a) continuous
 - b) discrete
 - c) dichotomous
 - d) none of the above
- 8) Actuarial method of estimation of survival function is used when data consists of _____.
 - a) only censored observations
 - b) only uncensored observations
 - c) complete data
 - d) all the above

- 9) The scaled TTT transform for exponential distribution with mean λ is _____.
 a) λt b) λ
 c) $\frac{1}{\lambda}$ d) t
- 10) Which of the following distribution has no ageing property?
 a) lognormal b) exponential
 c) gamma d) none of these

B) Fill in the blanks.

06

- 1) Series system of n components has _____ minimal path sets.
 2) IFRA property is preserved under _____.
 3) The distribution of i^{th} component X_i of state vector is _____.
 4) Study period is fixed in _____ censoring.
 5) For a distribution with finite mean, the degree of estimability of mean is _____.
 6) The survival function ranges between _____ and _____.

Q.2 Answer the following.

16

- a) Define reliability of component. Obtain the reliability of series system of n independent components.
 b) Define dual of a structure function. Show that dual of dual is primal.
 c) Describe Type-I censoring with one illustration.
 d) Define:
 i) survival function
 ii) cumulative hazard function

Q.3 Answer the following.

08

- a) Define coherent system. Show that k-out-of-n system is coherent system.
 b) For a coherent system with n components prove that:

08

i) $\phi(0) = 0$ and $\phi(1) = 1$
 ii) $\prod_{i=1}^n X_i \leq \phi(X) \leq \prod_{i=1}^n X_i$

Q.4 Answer the following.

08

- a) Define IFR and IFRA class of distributions. If $F \in$ IFR then show that $F \in$ IFRA.

08

- b) If failure time of item has Weibull distribution with distribution function

$$F(t) = \begin{cases} 1 - e^{-(\lambda t)^\alpha}, & t > 0 \\ 0, & otherwise \end{cases}$$

Examine whether it belongs to IFR or DFR.

Q.5 Answer the following.

08

- a) Define Poly function of order 2 (PF₂). Prove that if $f \in$ PF₂ then $F \in$ IFR.

08

- b) Discuss maximum likelihood estimation of parameters of a gamma distribution under complete data.

Q.6 Answer the following.

08

- a) Describe type II censoring. Obtain maximum likelihood estimate of mean of the exponential distribution under type II censoring.

08

- b) Describe Kaplan-Meier estimator and derive an expression for the same.

Q.7 Answer the following.

- a) Describe Gehan's test for two sample testing problem in presence of censoring. **08**
- b) Obtain the actuarial estimator of the survival function. Clearly state the assumption that you need to make. State Greenwood's formula for the variance of the estimator. **08**

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M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Optimization Techniques (MSC16404)

Day & Date: Thursday, 21-12-2023

Max. Marks: 80

Time: 03:00 PM To 06:00 PM

- Instructions:** 1) Q. Nos.1 and 2 are compulsory.
 2) Attempt any Three questions from Q.No.3 to Q.No.7.
 3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative.**10**

- 1) For a given *LPP*, if Z is objective function, then _____.
 - a) $Max Z = -Min Z$
 - b) $Max Z = Min (-Z)$
 - c) $Max Z = -Max Z$
 - d) None of these
- 2) An optimal solution to an *LPP* _____.
 - a) Always corresponds to an extreme point of feasible region
 - b) Always lies on the boundary of feasible region
 - c) Always exists
 - d) None of these
- 3) The dual has an infeasible solution, primal has _____.
 - a) an unbounded solution
 - b) an infeasible solution
 - c) a feasible
 - d) None of these
- 4) The size of the payoff matrix of a game can be reduced by using the principle of _____.
 - a) game inversion
 - b) dominance
 - c) game transpose
 - d) rotation
- 5) Dual simplex method is applicable to those *LPPs* that starts with _____.
 - a) an infeasible solution
 - b) a feasible solution
 - c) an infeasible but optimum
 - d) a feasible but optimum
- 6) In an integer linear programming problem _____.
 - a) all decision variables are integers
 - b) all decision variables are real numbers
 - c) all decision variables are complex numbers
 - d) all decision variables are non-negative
- 7) Which of the following is not associated with *LPP*?
 - a) Proportionality
 - b) Uncertainty
 - c) Additivity
 - d) Divisibility

SLR-ER-26

- 8) The right-hand side of a constraint in a primal problem appears in the corresponding dual as _____.
a) a coefficient in the objective function
b) a right-hand side of a constraint
c) an input-output coefficient
d) None of these
- 9) In quadratic programming problem constraints are _____.
a) non-linear equation form
b) non-linear inequality form
c) linear inequality form
d) none of these
- 10) If two constraints do not intersect in the positive quadrant of the graph, then _____.
a) a solution is infeasible b) a solution is unbounded
c) a solution is feasible d) a solution is degenerate

Q.1 B) Write true or false.

06

- 1) Feasible region may or may not be bounded.
- 2) In two phase simplex method the value of the objective function of phase-I cannot exceed zero.
- 3) For an LPP having n decision variables, there must be equal number of constraints.
- 4) Two phase simplex method is an alternative method to Big M method.
- 5) If only proper subset of the decision variables in a LPP are restricted to integer values the problem is known as mixed integer programming.
- 6) In a two-person zero sum game the optimal gain of two players is zero.

Q.2 Answer the following.

16

- a) Write a note on Dominance property.
- b) Write a note on Sensitivity analysis.
- c) Show that dual of dual is primal.
- d) Write a short note on Big-M method.

Q.3 Answer the following.

- a) Write down simplex algorithm to solve linear programming problem. **08**
- b) Develop necessary Kuhn Tucker conditions for an optimal solution to a quadratic programming problem. **08**

Q.4 Answer the following.

- a) Show that: i^{th} constraint in the primal is an equality if i^{th} dual variable is unrestricted sign. **08**
- b) Describe effect of change in coefficients of objective function c_j' s in sensitivity analysis. **08**

Q.5 Answer the following.

- a) Write down dual simplex algorithm. **08**
- b) Solve following game **08**

$$\begin{array}{c}
 \text{Player B} \\
 \text{Player A} \begin{pmatrix} 10 & 5 & -2 \\ 13 & 12 & 15 \\ 16 & 14 & 10 \end{pmatrix}
 \end{array}$$

Q.6 Answer the following.

- a) Solve the following LPP **08**
 Maximize $Z = -x_1 + 2x_2 - x_3$
 sub to
 $3x_1 + x_2 - x_3 \leq 10$
 $-x_1 + 4x_2 + x_3 \leq 6$
 $x_2 + x_3 \leq 4$
 $x_1, x_2, x_3 \geq 0$
- b) Explain Gomory's cutting plane method to solve integer programming problem. **08**

Q.7 Answer the following.

- a) Use Branch and Bound method to solve following integer programming problem **08**
 Maximize $Z = 7x_1 + 9x_2$, subject to constraints
 $-x_1 + 3x_2 < 6$, $7x_1 + x_2 \leq 35$, $x_2 \leq 7$, $x_1, x_2 \geq 0$ and integers
- b) i) State and prove weak duality theorem. **08**
 ii) State and prove strong duality theorem.

Seat No.	
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M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
STATISTICS
Time Series Analysis (MSC16407)

Day & Date: Friday, 22-12-2023
 Time: 03:00 PM To 06:00 PM

Max. Marks: 80

- Instructions:** 1) Question no. 1 and 2 are compulsory.
 2) Attempt any three questions from Q. No. 3 to Q. No. 7.
 3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

- 1) The time series $X_t - 0.5X_{t-1} = Z_{t-1}$, where $\{Z_t\} \sim WN(0, \sigma^2)$ is _____.
 - a) causal only
 - b) invertible only
 - c) both causal and invertible both
 - d) none of these
- 2) The mean of a stationary process $X_t = \mu + \phi_1 X_{t-1} + Z_t$ where $\{Z_t\} \sim WN(0, \sigma^2)$ is _____.
 - a) 0
 - b) $\mu + \phi$
 - c) μ
 - d) $\mu^2 + \phi$
- 3) The variance of the process $X_t - 0.5X_{t-1} = Z_{t-1}$ where $\{Z_t\} \sim WN(0, 1)$ is _____.
 - a) $4/3$
 - b) $3/4$
 - c) $1/4$
 - d) $1/2$
- 4) Autoregressive process of order two can be represented as _____.
 - a) $X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} * Z_t$
 - b) $X_t = \mu + \phi_1 X_{t-1} + \phi_2 X_{t-2} + Z_t$
 - c) $X_t = \mu \times \phi_1 X_{t-1} + \phi_1 X_{t-1} \times Z_t$
 - d) $X_t = \mu + \phi_1 X_{t-1} + Z_t$
- 5) The two-sided moving average method is defined as _____.
 - a) $X_t = \frac{1}{2q} \sum_{j=0}^q X_{t-j}$
 - b) $X_t = \frac{1}{2q} \sum_{j=-q}^q X_{t-j}$
 - c) $X_t = \frac{1}{(2q+1)} \sum_{j=-q}^q X_{t-j}$
 - d) $X_t = \frac{1}{(2q+1)} \sum_{j=0}^q X_{t-j}$
- 6) The single exponential smoothing equation is _____.
 - a) $S_t = \alpha Y_{t-1} + (1 - \alpha) S_{t-1} \quad t \geq 2$
 - b) $S_t = \alpha^2 Y_{t-1} + (1 - \alpha) S_{t-1} \quad t \geq 2$
 - c) $S_t = \alpha Y_{t-1} + (1 - \alpha)^2 S_{t-1} \quad t \geq 2$
 - d) $S_t = \alpha^2 Y_{t-1} + 2(1 - \alpha) S_{t-1} \quad t \geq 2$

- 7) If there is trend and seasonal component present in the given series, then _____ method can be used.
 - a) Single exponential smoothing
 - b) Double exponential smoothing
 - c) Triple exponential smoothing
 - d) Quadratic smoothing
- 8) The parameters of $AR(p)$ model can be estimated using _____ equations.
 - a) Least square
 - b) Yule walker
 - c) Likelihood
 - d) Geometric
- 9) The sample autocorrelation follows _____ distribution.
 - a) Student's t
 - b) Asymptotic Normal
 - c) Chi-square
 - d) F distribution
- 10) The process $X_t = \phi_1 X_{t-1} + Z_t$ where $\{Z_t\} \sim WN(0, \sigma^2)$ is causal process if _____.
 - a) $|\phi_1| < 1$
 - b) $|\phi_1| > 1$
 - c) $|\phi_1| = 1$
 - d) $|\phi_1| < 1.5$

B) Fill in the blanks.

06

- 1) The additive model of time series is given by _____.
- 2) The ACF of $AR(p)$ process is _____.
- 3) The smoothing parameter in single exponential method is selected such that, _____ is minimum.
- 4) _____ method can be used to eliminate both trend and seasonality.
- 5) Causality is the property of _____ in the process $\phi(B)X_t = \theta(B)Z_t$.
- 6) The causal representation of $ARMA(p, q)$ process $\{X_t\}$ is _____.

Q.2 Answer the following.

16

- a) Define weak stationary process and strong stationary process. Give one example each.
- b) Explain moving average as a method of elimination of trend only.
- c) Write a short note on single exponential smoothing.
- d) Write a short note on conditional heteroscedastic models.

Q.3 Answer the following.

- a) Define $AR(1)$ process and hence obtain its partial autocorrelation function. **08**
- b) Define $ARMA(1,1)$ process. Obtain causal representation of the same process. **08**

Q.4 Answer the following.

- a) Describe the diagnostic checking methods in time series analysis. **08**
- b) Verify whether the process $X_t + 0.6X_{t-1} = Z_t + 0.04Z_{t-1}$, is causal or not. Hence derive its autocovariance function. **08**

Q.5 Answer the following.

- a) Derive the expression for autocorrelation function $ARMA(1,1)$ process. **08**
- b) Derive the Yule - Walker equations for parameter estimation in $AR(p)$ process. **08**

Q.6 Answer the following.

- a) Explain Turning point test and Difference sign - test for testing trend. **08**
- b) Describe analysis of $ARIMA(p, d, q)$ process. **08**

Q.7 Answer the following.

- a) Explain moving average as a method of estimation and elimination of trend. **08**
- b) Discuss in detail residual analysis in time series analysis. **08**