Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Group and Ring Theory (2317101)

Day & Date: Friday, 05-01-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) Consider the following statements
P: Every field is Euclidean domain.
Q: R is integral domain iff $R[x]$ is integral domain.
   a) P is true and Q is false b) P is false and Q is true
   c) Both P and Q are true d) Both P and Q are false

2) Which of the following polynomial is irreducible over Q?
   a) $x^2 + 1$ b) $x^3 + x^2 - 2x - 1$
   c) $1 + x + x^2 + x^3 + x^4$ d) All of the above

3) If $G = \{i, -i, 1, -1\}$ is group with respect to multiplication then $O(-1) =$ ___.
   a) 1 b) 2
   c) 3 d) 4

4) $(2Z, +, .)$ is ______.
   a) Commutative ring
   b) Commutative ring with identity
   c) Commutative ring with multiplicative inverse
   d) Field

5) Which one of the following is correct?
   a) Every integral domain is a field
   b) An infinite integral domain is a field
   c) A finite integral domain is a field
   d) Integral domain is not a field

6) Which of the following is class equation of abelian group of order 10?
   a) $1 + 2 + 2 + 5$ b) $1 + 1 + 3 + 5$
   c) $1 + 1 + 2 + 6$ d) $1 + 1 + 1 + \ldots + 1$ (10 times)

7) A group G is said to be solvable iff there exists some positive integer k $s.t$ $G^k =$ ______.
   a) $\{e\}$ b) G
   c) $\emptyset$ d) None of these

8) Which of the following is cyclic group?
   a) S_3 b) Z_5
   c) D_4 d) K_4

B) Fill in the blanks. **04**

1) Units are those elements in R which possess _____ inverse.
2) Every normal series is _____ series.
3) A Ring R in which multiplication is commutative is called _____.
4) If G is abelian $\Leftrightarrow Z(G) =$ _____.

Q.2 Answer the following. (Any Six) **12**

a) Explain group action on a set with one example.
b) Show that $x + 1$ is factor of $x^4 + 3\,x^3 + 2x + 4$ in $Z_5[x]$.
c) Define:
1) Derived subgroup of group G
2) Normalizer of H

d) Define:
1) Simple group
2) Principal series

e) If $|G| = 24$ then how many Sylow 2-subgroups exist?
f) Explain concept primitive polynomial.
g) Find all zeros of the polynomial $f(x) = x^5 + 3x^3 + x^2 + 2x$ in $Z_5[x]$.
h) Prove that: Every Nilpotent group is solvable.

Q.3 Answer the following. (Any three) **12**

a) If G be a finite group then prove that G is a p- group iff $|G|$ is power of prime p.
b) If D is Unique Factorization domain then show that the finite product of primitive polynomials is again a primitive polynomial.
c) If G' be the commutator subgroup of a group G then prove that G is abelian iff $G = \{e\}$ where e is identity element of G.
d) Prove that: F be a field, an element $a \in F$ is a zero of $f(x) \in F[x]$ iff $(x - a)$ is a factor of $f(x)$ in $F[x]$.

Q.4 Answer the following. (Any two) **12**

a) Show that: No group of order 36 is simple.
b) If F is a field then prove that the ideal generated by $p(x) \neq 0$ of $f(x)$ is maximal iff $p(x)$ is irreducible over F
c) State and prove Burnside theorem.

Q.5 Answer the following. (Any two) **12**

a) State and prove Gauss lemma.
b) Prove that: Any two composition series of a group G are isomorphic.
c) If G be a finite group with $|G| = p^n m$ where p is a prime number and $p \nmid m$ then prove that
i) G contains a subgroup of order p^i for each $i, 1 \leq i \leq n$
ii) Every subgroup of order p^i is normal subgroup of order p^{i+1} for each $i, 1 \leq i \leq\ n - 1$.

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Real Analysis (2317102)

Day & Date: Sunday, 07-01-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) A necessary and sufficient condition for integrability of a bounded function is _____.
   a) $\lim_{\mu(P)\to\infty}(U(P,f)-L(P,f))=0$
   b) $\lim_{\mu(P)\to\infty}(U(P,f)+L(P,f))=0$
   c) $\lim_{\mu(P)\to 0}(U(P,f)+L(P,f))=0$
   d) $\lim_{\mu(P)\to 0}(U(P,f)-L(P,f))=0$

2) If $f(x)=x$ on $[0,1], n=2$ by dividing the interval into two equal sub intervals then $U(P,f)=$ _____.
   a) 0.75 b) 0.25
   c) 0 d) 7.5

3) If we plot p points in between a and b of $[a,b]$ then number of sub intervals created are _____.
   a) p b) $p+1$
   c) $2p$ d) None of thes

4) A bounded function f is integrable on $[a,b]$ if the set of points of discontinuity has _____ limit points.
   a) unique b) no
   c) finite d) infinite

5) A function $f=(f_1,f_2,\dots f_n)$ has continuous partial derivative on an open set S in R^n and the Jacobian determinant is non zero at some point a in S then there is an n-ball $B(a)$ on which f is _____.
   a) onto b) one one
   c) continuous d) open mapping

6) If S is convex set then _____ for all $x,y\in S$
   a) $L(x,y)\subseteq S$ b) $L(x,y)\supseteq S$
   c) $L(x,y)=S$ d) None of these

7) If $f:R\to R$ then Total derivative is _____.
   a) Real number b) Gradient vector
   c) Real matrix d) None of these

8) A function can have finite directional derivative $f'(C:u)$ but may fail to be _____ at C.
   a) derivable b) finite
   c) integrable d) continuous

B) Fill in the blanks **04**

1) The directional derivative of $f(x,y) = x^2y$ at point $(1,2)$ in the direction $(1,1)$ is _____.
2) For any partition P, the norm of partition is defined as $\mu(p) =$ ______.
3) The partial derivatives of a function describes the rate of change of a function in the direction of _____.
4) The condition of _____ is necessary for a function to assume its mean value ξ in given interval by first mean value theorem.

Q.2 Answer the following (Any Six) **12**

a) Define:
   i) Upper Sum
   ii) Lower Sum
b) Find the integration of $f(x) = x$ on $[-1, 1]$ by Riemann Sum method.
c) Find the directional derivative of $f(x,y) = x^2 + y^2$ at point $(1,2)$ in the direction $(2,3)$
d) State first fundamental theorem of calculus.
e) Write second definition of integrability (Using Riemann sum).
f) Define: Total Derivative
g) Write short note on Jacobian Matrix.
h) If $\int_{-1}^{2} x^2 dx = 3$ then find its mean value.

Q.3 Answer the following (Any Three) **12**

a) Solve $\int_0^3 (2x + 5)dx$
b) If f_1 and f_2 are two bounded and integrable functions on $[a, b]$ then prove that $f_1 + f_2$ is also integrable on $[a, b]$ and also prove that $\int_a^b (f_1 + f_2)dx = \int_a^b f_1 dx + \int_a^b f_2 dx$
c) Examine whether the function $f(x) = x^2 + 4x + 3$ on $[-10,10]$ have local extrema or not.
d) If f is differentiable function at c with total derivative T_c then prove that the directional derivative $f'(c; u)$ exists for every u in R^n and also prove that $T_c(u) = f'(c; u)$.

Q.4 Answer the following (Any Two) **12**

a) Prove that: A necessary and sufficient condition for the integrability of a bounded function f is that for every $\epsilon > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta, U(P.f) - L(P,f) < \epsilon$
b) If P^* is a refinement of a partition P then for a bounded function f prove that
   i) $L(P^*, f, \alpha) \geq L(P, f, \alpha)$
   ii) $U(P^*, f, \alpha) \leq U(P, f, \alpha)$
c) Prove that: A function f is bounded and integrable on $[a, b]$ and there exists a function F such that such that $F' = f$ on $[a, b]$ then prove that $\int_a^b f(x)dx = F(b) - F(a)$

Q.5 Answer the following (Any Two) **12**

a) Find directional derivative of

$$f(x) = \begin{cases} \dfrac{x^2.y}{x^4 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

b) If a function $f = (f_1, f_2, \dots f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n and the Jacobian determinant $J_f(a) \neq 0$ for some point a in S then prove that there is an $n - ball\ B(a)$ on which f is one to one.

c) Prove that: Every continuous function is integrable.

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Number Theory (2317107)

Day & Date: Tuesday, 09-01-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) When the sum $1^5 + 2^5 + 3^5 + \cdots + 100^5$ is divided by 4 then the remainder is _____.
   a) 0 b) 1
   c) 2 d) 3

2) If $a^n \equiv a\ (mod\ n)$ fails to hold for some choice of a then n is necessarily _____.
   a) prime b) square free integer
   c) composite d) perfect number

3) If 'a' has order k (mod n) then a^h has order k (mod n) iff _____.
   a) gcd $(k, h) = 2$ b) gcd $(a, h) = 1$
   c) gcd $(a, k) = 2$ d) gcd $(k, h) = 1$

4) If $F(n) = \sum_{d|n} f(d)$ then _____.
   a) $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$ b) $f(n) = \sum_{d|n} \mu(d) F\ (d)$
   c) $f(n) = \sum_{d|n} \mu\left(\frac{n}{d}\right) F\ (d)$ d) Both a and c

5) If p is an odd prime then there exists a primitive root r of p such that _____.
   a) $r^{p-1} \equiv 1(mod\ p^2)$ b) $r^{p-1} \not\equiv 1(mod\ p)$
   c) $r^{p-1} \not\equiv 1(mod\ p^2)$ d) $r^{p-1} \equiv 1(mod\ p)$

6) For any positive integer $n, \varphi(n) =$_____.
   a) $n \sum_{d|n} \frac{\mu(d)}{d}$ b) $n \sum_{d|n} \mu(d)$
   c) $\sum_{d|n} \frac{\mu(d)}{d}$ d) $d \sum_{d|n} \frac{\mu(d)}{n}$

7) If $\gcd(a, b) = 1$, then for any integer $c, \gcd(ac, b) =$ _____.
   a) 1 b) $\gcd(a, c)$
   c) $\gcd(ab, c)$ d) $\gcd(b, c)$

8) If $p(x) = \sum_{k=0}^{m} c_k x^k$ be a polynomial function of x with integral coefficients c_k. If $a \equiv b(mod\ n)$ then _____.
   a) $p(b) \equiv 0(mod\ n)$ b) $p(a) \equiv 1(mod\ n)$
   c) $p(b) \equiv 1(mod\ n)$ d) $p(a) \equiv p(b)(mod\ n)$

B) Fill in the blanks. **04**

1) The linear congruence $ax \equiv b(mod\ n)$ has a solution iff _____.
2) The remainder when $3^{24}.5^{13}$ is divided by 17 is _____.
3) The order of 3 modulo 8 is _____.
4) A function whose domain of definition is set of positive integers is called ___.

Q.2 Answer the following. (Any Six) **12**

a) If $ac \equiv bc\ (mod\ n)$ then show that $a \equiv b\ \left(mod\ \frac{n}{d}\right)$, where $d = \gcd(c, n)$.

b) Show that one of every three consecutive integer is divisible by 3.

c) Find the highest power of 13 contained in $20000!$.

d) Show that 3 is primitive root of 17.

e) Find $\tau(10000)$ and $\sigma(10000)$.

f) Define the following terms:
- i) Square free integers
- ii) Linear Congruence

g) If $a = bq + r$ then show that $\gcd(a, b) = gcd\ (b, r)$.

h) If $f(n) = n^2 + 2$ and $n = 6$ then show that $\sum_{d|6} f(d) = \sum_{a|6} F\left(\frac{6}{d}\right)$.

Q.3 Answer the following. (Any Three) **12**

a) If a has order $k\ mod\ n$ then show that a^h has order $\frac{k}{d} mod\ n$ where $d = \gcd(k, h)$.

b) If f and F be two number theoretic functions related by the formula $F(n) = \sum_{d|n} f(d)$ then show that, $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$.

c) Find the general solution of the linear Diophantine equation $11x + 5y = 79$.

d) Find all the primes less than 130.

Q.4 Answer the following. (Any Two) **12**

a) Find an integer which leaves remainder 5 when divided by 11 and 2 when divided by 19.

b) Write a note on Fermat factorization method and factorize 340663.

c) If a is a primitive root modulo n and b, c and k are any integers, then prove that
- i) $b \equiv c(mod\ n) \Rightarrow ind\ b \equiv ind\ c\ \big(mod\ \varphi(n)\big)$
- ii) $ind.(bc) \equiv ind\ b + ind\ c\ (mod\ \varphi(n))$
- iii) $ind\ b^k \equiv k\ ind\ b\ \big(mod\ \varphi(n)\big)$

Q.5 Answer the following. (Any Two) **12**

a) State and prove Chinese Reminder Theorem.

b) If $n = {p_1}^{k_1} {p_2}^{k_2} \ldots {p_r}^{r}$ is a prime factorization of n then prove that,
- i) $\tau(n) = (k_1 + 1)(k_2 + 1) \ldots (k_r + 1)$
- ii) $\sigma(n) = \left(\frac{{p_1}^{k_1+1} - 1}{p_1 - 1}\right)\left(\frac{{p_2}^{k_2+1} - 1}{p_2 - 1}\right) \ldots \left(\frac{{p_r}^{k_r+1} - 1}{p_r - 1}\right)$

c) Show that if one of the two integers $2a + 3b$ or $9a + 5b$ is divisible by 17 then so can the other.

SLR-EO-7

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Research Methodology in Mathematics (2317103)

Day & Date: Thursday, 11-01-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.8

Q.1 A) Choose correct alternative. 08

1) Research can be classified as: _____
 a) Basic, Applied Research
 b) Philosophical, Historical, Survey and Experimental Research
 c) Quantitative and Qualitative Research
 d) All the above

2) Bibliography given in a research report: _____
 a) shows vast knowledge of the researcher
 b) helps those interested in further research
 c) has no relevance to research
 d) all the above

3) Who defined “Research as systematized effort to gain new knowledge”?
 a) C. R. Kothari b) Redman and Mory
 c) Clifford Woody d) Ross Taylor

4) A hypothesis is a _____.
 a) Tentative statement whose validity is still to be tested
 b) Supposition which is based on the past experiences
 c) Statement of fact
 d) All of the above

5) The i-10 index indicates the number of academic publications an author has written that have been cited by _____ sources.
 a) exactly 10 b) more than 10
 c) at least 10 d) less than 10

6) UGC CARE list is maintained by _____.
 a) Savitribai Phule Pune University, Pune
 b) Punyashlok Ahilyadevi Holkar Solapur University, Solapur
 c) University Grants Commission
 d) Maharashtra Government

7) The sampling in which each and every item in the population has equal chance of inclusion in the sample is known as _____.
   a) Systematic sampling b) Stratified sampling
   c) Simple random sampling d) Sequential Sampling

8) The Data of research is, generally ______.
   a) Qualitative only b) Quantitative only
   c) Both 'a' and 'b' d) Neither 'a' nor 'b'

B) State True/False. (one mark each) **04**

1) Research is an original contribution to the existing stock of knowledge making for its advancement.
2) UGC CARE is a quality mandate for all academicians over the world.
3) The quality of research journal is indicated by impact factor.
4) ISI stands for Institute for Scientific Information.

Q.2 Answer the following. (Any Six) **12**

a) Define: Research (Write at least two definitions)
b) Write different types of sampling.
c) Explain the terms: Lemma, theorem, corollary and preposition.
d) Define: h-index, i10 index
e) Give the longform of UGC CARE.
f) Write short note on Abstract of research article.
g) Write short note on motivation in research.
h) Write basic postulates of Scientific method.

Q.3 Answer the following. (Any Three) **12**

a) Give the difference between Research methods and Research Methodology.
b) Explain the term: Preparing the research design.
c) Write the problems encountered by researchers in India.
d) Write short note on citation index.

Q.4 Answer the following. (Any Two) **12**

a) Write short note on collecting the data.
b) Explain Do's and Don'ts of Mathematical writing.
c) Write the text file format of Research article.

Q.5 Answer the following. (Any Two) **12**

a) Write an expository note on UGC CARE list, journal including objective, need and scope of UGC CARE.
b) Give details about "Words versus symbols".
c) Write an expository note on Keywords and Subject classification.

Seat No.	

Set P

M.Sc. (Semester-I) (Old) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Number Theory (MSC15108)

Day & Date: Friday, 05-01-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) If $ca \equiv cb \ (mod\ n)$ and $gcd(c, n) = d$ then _____.
   a) $a \equiv b \ (mod\ n)$ b) $a \equiv b \ (mod\ d)$
   c) $a \equiv b \ (mod\ nd)$ d) $a \equiv b \ (mod\ \frac{n}{d})$

2) For positive integers a and b, $lcm(a, b) = a.b$ iff ______.
   a) $a \nmid b$ b) $b \nmid a$
   c) $\gcd(a, b) = 1$ d) $\gcd(a, b) = ab$

3) The exponent of the highest power of prime p that divides is $\frac{(2n)!}{(n!)^2}$ is ____.
   a) $\sum_{k=1}^{\infty}\left(\left[\frac{2n}{p^k}\right]+\left[\frac{n}{p^k}\right]\right)$ b) $\sum_{k=1}^{\infty}\left(\left[\frac{2n}{p^k}\right]-2\left[\frac{n}{p^k}\right]\right)$
   c) $\sum_{k=1}^{\infty}\left(\left[\frac{(2n)!}{p^k}\right]-2\left[\frac{n!}{p^k}\right]\right)$ d) $\sum_{k=1}^{\infty}\left(\left[\frac{(2n)}{p^k}\right]+3\left[\frac{n}{p^k}\right]\right)$

4) Consider the statements: _____.
   I) If p is a prime number then $(p - 1)! \equiv 1 (mod\ p)$
   II) If $a^{m-1} \equiv 1 \ (mod\ m)$ then m is a prime number.
   a) only I is true b) only II is true
   c) both I and II are true d) both I and II are false

5) If $p(x) = \sum_{k=0}^{m} c_k x^k$ be a polynomial function of x with integral coefficients c_k and $a \equiv b (mod\ n)$ then _____.
   a) $p(b) \equiv 0 (mod\ n)$ b) $p(a) \equiv 1 (mod\ n)$
   c) $p(b) \equiv 1 (mod\ n)$ d) $p(a) \equiv p(b) (mod\ n)$

6) If $a > 1$ and m, n are positive integers then $gcd(a^m - 1, a^n - 1) =$____.
   a) $a^{\gcd(m,n)} - 1$ b) $\gcd(m, n) - 1$
   c) $a^{\gcd(m,n)}$ d) $\gcd(m, n)$

7) If the integer a has order k modulo n, then _____.
   a) $a^i \equiv a^j (mod\ n)\ if\ i \equiv j (mod\ n)$
   b) $a^i \equiv a^j (mod\ n)\ iff\ i \equiv j (mod\ n)$
   c) $a^i \equiv a^j (mod\ n)\ iff\ i \equiv j (mod\ k)$
   d) $i \equiv j (mod\ n)\ if a^i \equiv a^j (mod\ n)$

8) The last two digits in the decimal representation of $3^{100}$are ______.
   a) 31 b) 11
   c) 21 d) 01

9) The solution of the linear congruence $17x \equiv 9(mod\ 276)$ is _____.
   a) 297 b) 23
   c) 33 d) 243

10) If m and n are relatively prime positive integers then _____.
   a) $m^{\varphi(n)} + n^{\varphi(m)} \equiv 1(mod(m+n))$
   b) $m^{\tau(n)} + n^{\varphi(m)} \equiv 1(mod\ mn)$
   c) $m^{\sigma(n)} + n^{\tau(m)} \equiv 1(mod\ mn)$
   d) $m^{\varphi(n)} + n^{\varphi(m)} \equiv 1(mod\ mn)$

B) Fill in the blanks. **06**

1) The system of linear congruences $ax + by \equiv r\ (mod\ n)$ *and* $cx + dy \equiv s\ (mod\ n)$ has a unique solution $(mod\ n)$, Whenever ____.
2) The highest power of 12 contained in $500!$ is _____.
3) The largest integer value of $[\pi]$ is _____.
4) The simultaneous solution of the system of linear congruences, $x \equiv 3\ (mod\ 6), x \equiv 5\ (mod\ 7), x \equiv 2\ (mod\ 11)$ is ____.
5) The factors of 340663 are _____.
6) If n has primitive root then it has exactly _____ primitive roots.

Q.2 Answer the following **16**

a) If a is an odd integer then show that $\frac{a^4+4a^2+11}{16}$ is an integer.

b) Show that 1729 is an absolute pseudo prime.

c) If f is multiplicative function and $S(n) = \sum_{d|n} f(d)$ then prove that $S(n)$ is also multiplicative function.

d) Construct the index table for 17 with primitive root 5.

Q.3 Answer the following.

a) If $n = {p_1}^{k1}{p_2}^{k2}$____ ${p_r}^{kr}$ is a prime factorization of n then prove that. **08**

i) $\tau(n) = (k_1+1)(k_2+1)$ _____ (k_r+1)

ii) $\sigma(n) = \left(\frac{{p_1}^{k1+1}-1}{p_1-1}\right)\left(\frac{{p_2}^{k2+1}-1}{p_2-1}\right)$ _____ $\left(\frac{{p_r}^{kr+1}-1}{p_r-1}\right)$

b) Find an integer which leaves the remainder 5 when divided by 11 and 2 when divided by 19. **08**

Q.4 Answer the following

a) State and prove Eulers theorem and show that the sum of positive integers less than n and relatively prime to n is equal to $\frac{1}{2}u\varphi(n)$. **10**

b) If a has order $k\ mod\ n$ then show that a^h has order $\frac{k}{d} mod\ n$ where $d = \gcd(k, h)$. **06**

Q.5 Answer the following.

a) If $gcd(a, b) = d$ then the equation $ax + by = c$ has a solution iff $d|c$, further if $(x_{0,}y_0)$ is a solution of $ax + by = c$ then show that all the other solutions are in the form $x_1 = x_0 - \frac{b}{a}t, y_1 = y_0 + \frac{a}{d}t$ for any integer t. **10**

b) Find the primes not exceeding 150 by using the method Sieve of Eratosthenes. **06**

Q.6 Answer the following.

a) State and prove Fermat's theorem also find the remainder when 72^{1001} is divided by 31. **10**

b) Show that if one of the two integers $2a + 3b \; or \; 9a + 5b$ is divisible by 17 then so can the other. **06**

Q.7 Answer the following.

a) Show that the integer 2^n has no primitive root for $n \geq 3$. **08**

b) If p is a prime and $f(x) = a_n x^n + a_{n-1} x^{n-1} + - - - + a_1 x + a_{0,}$ $a_n \not\equiv 0 (mod \; p)$ is a polynomial of degree $n \geq 1$ with integral coefficients then show that $f(x) = 0 (mod \; p)$ has at least n incongruent solutions $mod \; p$. **08**

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Object Oriented Programming using C++ (MSC15109)

Day & Date: Friday, 05-01-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) _____ means the ability to take more than one form.
   a) Inheritance b) Abstraction
   c) Polymorphism d) None of these

2) A _____ is a collection of objects of similar type.
   a) Object b) Class
   c) Polymorphism d) Inheritance

3) _____ is used to declare integer data type.
   a) int b) integer
   c) Integer d) INT

4) Which feature of OOP indicates code reusability?
   a) Abstraction b) Polymorphism
   c) Encapsulation d) Inheritance

5) _____ refers to the variable name.
   a) keywords b) identifiers
   c) string d) operators

6) _____ are operators that are used to format data display.
   a) string b) identifiers
   c) keyboards d) manipulators

7) An _____ function is a function that is expanded in line when it is invoked.
   a) inline b) multiline
   c) pointer d) undefined

8) Wrapping data and its related functionality into a single entity is known as _____.
   a) Abstraction b) Encapsulation
   c) Polymorphism d) Modularity

9) C++ is_____.
   a) procedural programming language
   b) object oriented programming language
   c) functional programming language
   d) both procedural and object oriented programming language

10) Identify the incorrect constructor type.
   a) Friend constructor b) Default constructor
   c) Parameterized constructor d) Copy constructor

B) State whether True or False. **06**

1) The smallest individual unit in a program is called Token.
2) Class is a basic run time entity.
3) The use of same function name to create functions that perform a variety of different tasks is known as function overloading.
4) A derived class with only one base class is called as multiple inheritance.
5) Constructors should declared in the public section.
6) By default, members of the class are public.

Q.2 Answer the following. **16**

a) Explain the basic Data types used in C++.
b) What is Token? Explain different types of Tokens.
c) What is Object? Explain with example.
d) What is function prototyping? Explain with example.

Q.3 Answer the following. **16**

a) What is an algorithm? Explain the characteristics of algorithm.
b) Explain the basic concepts of OOP.

Q.4 Answer the following. **16**

a) What is Inheritance? Explain Single Inheritance with suitable example.
b) Explain the use of scope resolution operator with example.

Q.5 Answer the following. **16**

a) What is array? Explain One dimensional array with example.
b) What is constructor? Explain the use of Parameterized constructor.

Q.6 Answer the following. **16**

a) Explain the use of new and delete operators used in C++.
b) Write a C++ program to implement function overloading (Assume your own data)

Q.7 Answer the following. **16**

a) Explain the use of call by value with suitable example.
b) Write a C++ program to implement multilevel inheritance. (Assume your own data)

SLR-EO-10

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Algebra - I (MSC15101)

Day & Date: Sunday, 07-01-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If D is Euclidean domain, then D is _____.
   a) Principal ideal domain b) Unique factorization domain
   c) Integral domain d) All of these

2) Any group of order p^n where p is prime then G is _____.
   a) Abelian b) Non abelian
   c) Nilpotent d) None of these

3) In $Z\,[x]$, content of $3x^2 + 6x - 9$ is _____.
   a) 1 b) -1
   c) -3 d) 3

4) If a group G is finite cyclic group of order n, then number of generators of G is _____.
   a) At least 2 b) 2
   c) n d) $n + 1$

5) If G is a group then which of the following necessarily imply that $G' = \{e\}$ _____.
   a) G is non abelian b) G is abelian
   c) G is cyclic d) None of these

6) If $\mathbb{F}$ is a field, then _____.
   a) F is Integral domain b) F is Principal ideal domain
   c) F is Euclidean domain d) All of these

7) Class equation of S_3 is _______.
   a) $2 + 2 + 2$ b) $1 + 1 + 4$
   c) $1 + 2 + 3$ d) $1 + 1 + 1 + 1 + 1 + 1$

8) Which of the following is an integral domain?
   a) Z b) 2Z
   c) 3Z d) 5Z

9) If G is a cyclic group then which of the following is always true?
   a) $G' = G$ b) $G' \neq \{e\}$
   c) $G' = \{\,\}$ d) $G' = \{e\}$

10) For every field F there exist at most _____ ideals.
   a) 1 b) 2
   c) 3 d) 4

B) Fill in the blanks. **06**

1) A non-zero element in an integral domain D having improper divisors are called ____.
2) There exist at least _____ composition series for every finite group G.
3) If a, b, c be any element in Euclidean domain R & $gcd\ (a, b) = 1$ if $a|bc$ then _______.
4) Class equation of $Q_8 = \{1, -1, i, -i, j, -j, k, -k\}$ is _______.
5) Units in ring of Gaussian integer i.e $\{a + ib/a, b \in Z\}$ is/are _____.
6) Two subnormal series of a group G are have ______ refinement.

Q.2 Answer the following **16**

a) Define Cyclotomic polynomial and show that it is irreducible over Q.
b) Show that the ring of integer is Euclidean domain.
c) Define i) Centre of group ii) Nilpotent group.
d) Find the homomorphism from Z_6 to Z_8

Q.3 Answer the following.

a) State and prove 2nd Sylow theorem. **08**
b) Prove that: Two subnormal series of a group G are having isomorphic refinement. **08**

Q.4 Answer the following.

a) Prove that: A group G is solvable iff the n^{th} derived subgroup of G is $\{e\}$. **08**
b) If G be a finite group with $O(G) + p^n$ where p is a prime number then prove that $Z(G)$is a non-trivial i.e. $Z(G) \neq \{e\}$. **08**

Q.5 Answer the following.

a) Prove that: No group of order 36 is simple. **08**
b) Prove that: A polynomial ring $F[x]$ over the field F is principal ideal domain. **08**

Q.6 Answer the following.

a) If $f(x) = x^6 + 3x^5 + 4x^2 - 3x + 2$ and $g(x) = x^2 + 2x - 3$ be in $Z_7[x]$ then find $q(x)$ and $r(x)$ such that $f(x) = q(x)g(x) + r(x)$ and degree of $r(x) < 2$ **08**
b) Prove that: Every Euclidean domain is principal ideal domain. **08**

Q.7 Answer the following.

a) State and prove Eisenstein criteria of irreducibility over Q. **08**
b) Prove that $f(x) = x^3 + x^2 - 2x - 1$ in $Z[x]$ is irreducible over Q. **08**

SLR-EO-11

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Real Analysis - I (MSC15102)

Day & Date: Tuesday, 09-01-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) The lower integral of a function f on $[a, b]$ is_____.
   a) infimum of set of upper sums
   b) infimum of set of lower sums
   c) supremum of set of upper sums
   d) supremum of set of lower sums

2) If $f: R \to R$ then Total derivative is _____.
   a) Real number b) Gradient vector
   c) Real matrix d) None of these

3) If f and $|f|$ are bounded and integrable on $[a, b]$ then, $|\int_a^b f(x)\,dx|$ _____.
   a) $\geq \int_a^b |f|\,dx$ b) $\leq \int_a^b |f|\,dx$
   c) $= \int_a^b |f|\,dx$ d) None of these

4) Consider the following statements:
   I) Every monotonic increasing function on $[a, b]$ is bounded.
   II) Every monotonic increasing function on $[a, b]$ is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

5) A function can have finite directional derivative $f'(C: u)$ but may fail to ______ at C.
   a) derivable b) finite
   c) integrable d) continuous

6) If f and g are integrable functions then ____ is also integrable.
   a) $f + g$ b) $f - g$
   c) $f.g$ d) all of the above

7) With usual notations, the condition of integrability for a function f over $[a, b]$ is ______.
   a) $U(P, f) - L(P, f) < \epsilon$ b) $U(P, f) + L(P, f) < \epsilon$
   c) $L(P, f) - U(P, f) < \epsilon$ d) $U(P, f) - L(P, f) > \epsilon$

8) By first mean value theorem, if a function f is continuous on $[a, b]$ then there exist a number ξ in $[a, b]$ such that $\int_a^b f(x)\,dx =$ ______.
   a) $f(\xi)(a - b)$ b) $f(\xi)(b - a)$
   c) $f(\xi)(a + b)$ d) $f'(\xi)(a - b)$

9) If P_1 and P_2 are two partitions of $[a, b]$ then their common refinement is given by $P^* =$ _____.
   a) $P_1 \cap P_2$ b) $P_1 + P_2$
   c) $P_1 - P_2$ d) $P_1 \cup P_2$

10) The statement $\int_a^b f(x)\,dx$ exists implies that the function f is _____ and ______.
   a) continuous, integrable b) bounded, integrable
   c) bounded, continuous d) finite, continuous

B) Fill in the blanks. **06**

1) A bounded function f is integrable on $[a, b]$ if the set of points of discontinuity has _____ limit points.
2) The directional derivative of $f(x, y) = x^2y$ at point $(1,2)$ in the direction $(1,1)$ is _____.
3) The lower sum of a function is defined as $L(P, f) =$ _____.
4) The partial derivatives of a function describe the rate of change of a function in the direction of ______.
5) If $f(x) = x$ on $[0,1], n = 2$ by dividing the interval into two equal sub intervals then $U(P, f) =$ _____.
6) The condition of _____ is necessary for a function to assume its mean value ξ in given interval by first mean value theorem.

Q.2 Answer the following. **16**

a) Define: Upper sum, Lower sum, Upper Integral, Lower Integral.

b) If a function f is continuous on $[a, b]$ then prove that there exists a number ξ in $[a, b]$ such that $\int_a^b f(x)\,dx = f(\xi)(b - a)$

c) Examine whether the function $f(x) = x^2 + 4x + 3$ on $[-10,10]$ have local extrema or not.

d) If a function f is continuous on $[a, b]$ then prove that there exists a number ξ in $[a, b]$ such that $\int_a^b f(x)\,dx = f(\xi)(b - a)$

Q.3 Answer the following.

a) If f is differentiable function at c with total derivative T_c then prove that the directional derivative $f'(c;\, u)$ exists for every u in R^n and also prove that $T_c(u) = f'(c; u)$ **08**

b) If f_1 and f_2 are two bounded and integrable functions on $[a, b]$ then prove that $f_1 + f_2$ is also integrable on $[a, b]$ and also prove that $\int_a^b (f_1 + f_2)\,dx = \int_a^b f_1\,dx + \int_a^b f_2\,dx$ **08**

Q.4 Answer the following.

a) If P^* is a refinement of a partition P then for a bounded function f prove that **08**
   1) $L(P^*, f) \geq L(P, f)$
   2) $U(P^*, f) \leq U(P, f)$

b) Solve $\int_1^2 (x^2 + 3)\,dx$ by Riemann sum method. **08**

Q.5 Answer the following.

a) If a function f is bounded and integrable on $[a, b]$ then prove that the function F defined as, $F(x) = \int_a^x f(t)dt; a \leq x \leq b$ is continuous on $[a, b]$. Furthermore if f is continuous at a point c of $[a, b]$ then prove that F is derivable at c and $F'(c) = f(c)$ **08**

b) Prove that: A function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$ then prove that $\int_a^b f(x)dx = F(b) - F(a)$ **08**

Q.6 Answer the following.

a) If a function f is monotonic on $[a, b]$ then prove that it is integrable on $[a, b]$ **08**

b) Prove that: A necessary and sufficient condition for the integrability of a bounded function f is that for every $\epsilon > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a, b]$ with norm $\mu(P) < \delta, U(P, f) - L(P, f) < \epsilon$ **08**

Q.7 Answer the following.

a) Check whether directional derivative exists or not for following function. **08**

$$f(x, y) = \frac{xy}{x + y}, x \neq 0, y \neq 0$$

$$f(x, y) = 0, x = 0, y = 0$$

b) If S is an open set connected subset of R^n and $f: \rightarrow R^m$ is differentiable at each point of S and if $f'(c) = 0$ for each $c \in S$ then prove that f is constant on S. **08**

SLR-EO-12

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Differential Equations (MSC15103)

Day & Date: Thursday, 11-01-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The order of differential equation whose solutions are $\sin x, \cos x$ is _____.
   a) 1 b) 2
   c) 3 d) 4

2) A linear differential equation $L(y) = b(x)$ is said to be non-homogeneous if $b(x) =$ _____.
   a) Non-zero b) Two
   c) Three d) Zero

3) If r_1, r_2 are distinct roots of characteristic polynomial p where $p(r) = r^2 + a_1 r + a_{2,}$ then the functions ϕ_1, ϕ_2 are defined as _____.
   a) $\phi_1(x) = e^{-r_1 x}$ and $\phi_2(x) = e^{-r_2 x}$
   b) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{r_2 x}$
   c) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = x e^{r_2 x}$
   d) $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{-r_2 x}$

4) If differential operator L involves differentiation with respect to x then
   i) $\frac{\partial}{\partial r} L(e^{rx}) = L(\frac{\partial}{\partial r} e^{rx})$
   ii) $L(xe^{rx}) = [p'(r) + xp(r)]e^{rx}$
   a) Both true b) both false
   c) i) true and ii) false d) i) false and ii) true

5) In $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = b(x)$, points where $a_0(x) = 0$ are called _____.
   a) singular points b) ordinary point
   c) regular singular point d) none of these

6) With usual notation $\frac{d}{dx}[x^n J_n(x)] =$ _____.
   a) $x^n J_{n+1}(x)$ b) $x^n J_{n-1}(x)$
   c) $x^{n-1} J_n(x)$ d) $x^{n+1} J_n(x)$

7) Indicial polynomial for Euler equation of order 2 is _____.
   a) $r(r+1) + ar + b$ b) $r(r-1) + ar + b$
   c) $r(r-1) - ar + b$ d) None of these

8) The two solutions of $y'' - 25y = 0$ are $\emptyset_1(x) =$ _____ and $\emptyset_2(x) =$ _____.

a) e^x, e^{2x} b) e^{5x}, e^{-5x}
c) e^{5x}, xe^{5x} d) None

9) The solution of $y'' + 4y = 0$ are _____.

a) x, x^2 b) $\sin x, \cos x$
c) $2x, -2x$ d) $\sin 2x, \cos 2x$

10) The differential equation $2xydx + (1 + x^2)dy$ is _____.

a) exact b) not exact
c) can not say d) none of these

B) Fill in the blanks. **06**

1) On an interval I containing x_0 there exists _____ solution of the initial value problem.

2) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1y' + a_2y = 0$ then $W(\phi_1, \phi_2)(x) =$ _____ $W(\phi_1, \phi_2)(x_0)$

3) The function g is analytic at x_0 if g can be expressed in power series about x_0 which has _____ radius of convergence.

4) Two functions $x, |x|$ are linearly _____.

5) The Legendre equation is _____.

6) If p is a polynomial such that $\deg(p) = n$ and $p(z) = (z - a)q(z)$ then q has _____ root.

Q.2 Answer the following. **16**

a) Find solution of $y^{(2)} - 2y^{(1)} - 3y = 0$ satisfying $y(0) = 0, y'(0) = 1$.

b) Prove that if ϕ_1, ϕ_2 are two solution of $L(y) = 0$ then $c_1\phi_1 + c_2\phi_2$ is also solution of $L(y) = 0$, where c_1, c_2 are any two constants.

c) ϕ_1, ϕ_2 be the basis of $y'' + \propto (x)y = 0$ satisfying $\emptyset_1(0) = 1, \emptyset_2(0) = 0, \emptyset_1'(0) = 0, \emptyset_2'(0) = 1$. Compute $W(\emptyset_1, \emptyset_2)(0)$

d) Show that $f(x, y) = x^2 \cos^2 y + y \sin^2 x$ satisfies Lipschitz condition on set $S: |x| \leq 1, |y| < \infty$

Q.3 Answer the following.

a) Prove that two solutions ϕ_1, ϕ_2 of $L(y) = 0$ are linearly independent on an interval I if $W(\emptyset_1, \emptyset_2)(x) \neq 0$. **08**

b) Solve $x^2y^{(2)} + xy^{(1)} - 4y = x$ for positive values of x. **08**

Q.4 Answer the following.

a) Compute the Wronskian of solutions of $y''' - 3r_1y'' + 3r_1^2y' - r_1^3y = 0$. **08**

b) Prove that $W(\emptyset_1, \emptyset_2)(x) = e^{-a_1(x - x_0)} W(\emptyset_1, \emptyset_2)(x_0)$ if $\emptyset_1, \emptyset_2$ are two solutions of $L(y) = 0$ on an interval I containing point x_0. **08**

Q.5 Answer the following.

a) Prove that a function $\emptyset$ is solution of the IVP $y' = f(x, y), y(x_0) = y_0$ on an interval I iff it is a solution of $y = y_0 + \int_{x_0}^{x} f(t, \emptyset(t))dt$ on I. **08**

b) Solve $y^{(3)} - y^{(1)} = x$ **08**

Q.6 Answer the following.

a) Prove that $W(\emptyset_1, \emptyset_2, \cdot\cdot \emptyset_n)(x) = \exp\{-\int_{x_0}^{x} a_1(t)dt\} W(\emptyset_1, \cdot\cdot \emptyset_n)(x_0)$ **08**

b) Solve $y' = xy, y(0) = 1$ using the method of successive approximation. **08**

Q.7 Answer the following. **08**

a) Derive Bessel function of zero order of the first kind.

b) Let α, β be any two constants and let x_0 be any real number on any interval I containing x_0 . Prove that there exist at most one solution $\emptyset$ of IVP $L(y) = 0, y(x_0) = \alpha, y'(x_0) = \beta$ **08**

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Classical Mechanics (MSC15104)

Day & Date: Friday, 29-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Determinant value of an orthogonal matrix is _____.
   a) 1 b) -1
   c) either 1 or -1 d) neither 1 nor -1

2) Lagrangian is defined as _____.
   a) $L = T - V$ b) $L = T + V$
   c) $2T + V$ d) $L = 2T - V$

3) Newton's equation of motion can be derived from Lagrange's equation.
   a) True b) False
   c) can't say d) may be

4) Conservative force is only depends on _____.
   a) Time b) Velocity
   c) Co-ordinates d) Both (a) and (b)

5) Routhian is a function which usually replaces _____.
   a) Lagrangian b) Hamiltonian
   c) Both a and b d) None of a and b

6) The rotation matrix in 3-dimensions has _____ degrees of freedom.
   a) 9 b) 6
   c) 3 d) 1

7) Hamiltonian H is independent of _____.
   a) Generalized coordinates b) generalized velocity
   c) Generalize momentum d) Time

8) Rheonomic constraint depends on _____.
   a) co-ordinates b) time
   c) momentum d) both a and b

9) Geodesic on the surface of sphere is _____.
   a) parabola b) cycloid
   c) hyperbola d) arc of great circle

10) Which of the following does not represents a rotation?
   a) orthogonal matrix with determinant -1
   b) orthogonal matrix with determinant $+1$
   c) Eulerian angles
   d) Both b and c

B) Fill in the blanks. **06**

1) Euler - Lagrange's differential equations are _____ conditions for extremum of a functional.
2) Brachistochrone problem deals with _____.
3) If two particles in the 3 D-space are constrained to maintain a fixed distance from each other then degrees of freedom are _____.
4) The curve is _____ for which area of surface of revolution is minimum when revolved about y-axis.
5) Shortest distance between any two points is a _____.
6) Scleronomic constraint are not depending on _____.

Q.2 Answer the following. **16**

a) If q is cyclic in L then show that it is cyclic in H.
b) State modified Hamilton's principle.
c) Show that: The generalised momentum corresponding to cyclic co-ordinates is conserved.
d) Show that frictional force is not conservative.

Q.3 Answer the following.

a) Show that: The path followed by a particle in sliding from one point to another under the influence of gravity is a cycloid. **08**
b) Derive Newton's equation of motion from Lagrange's equation of motion. **08**

Q.4 Answer the following.

a) Find Euler-Lagrange's differential equation satisfied by $y(x)$ for which the in tergal
$I = \int_{x_1}^{x_2} f(y, y', x)dx$ has extremum value, where $y(x)$ is twice differentiable function satisfying $y(x_1) = y_1$ and $y(x_2) = y2$. **08**

b) Derive Lagranges equation of motion from Hamilton's principle. **08**

Q.5 Answer the following.

a) Prove that: The product of two linear orthogonal transformations is again a linear orthogonal transformation and hence show that finite rotations of a rigid body about the fixed point of body are not commutative. **08**
b) Find the extremal for an isoperimetric problem $I[Y(x)] = \int_0^1 (y'^2 + x^2)dx$ subject to condition $\int_0^1 (y^2)dx = 2, \mathcal{Y}(0) = 0, y(1) = 0$ **08**

Q.6 Answer the following.

a) Derive the equation of motion of Atwood's machine. **08**
b) Show that: The shortest distance between two points in a plane is a straight line. **08**

Q.7 Answer the following.

a) State and prove Hamilton's principle by using Lagranges's equation. **08**
b) Establish the relation between $\delta - variation\ and \bigtriangleup - variation.$ **08**

SLR-EO-14

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Algebra - II (MSC15201)

Day & Date: Monday, 18-12-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If F is of field rational number K is field of real number then the dimension of $K(F)$ is, ______.
   a) 1 b) 2
   c) 3 d) None of these

2) The degree of extension of $Q(\sqrt{2}, \sqrt{3}, \sqrt{11})$ over Q is _____.
   a) 2 b) 4
   c) 5 d) 6

3) The extension K of a field F is called simple extension of F if _____ for some a in K.
   a) $K = F(a)$ b) $F = K(a)$
   c) $F(a) = F$ d) None of these

4) Which of the following is not algebraic over Q _____.
   a) $\sqrt{5}$ b) $\sqrt{7}$
   c) e d) None of these

5) The Splitting field of $x^2 + 2 \in R[x]$ over R is _____.
   a) Q b) R
   c) C d) None of these

6) If K is finite extension of a field F and $\mathrm{G(K, F)}$ is finite group then which of the following is true,
   a) $\mathrm{O}(\mathrm{G}(\mathrm{K}, \mathrm{F})) = [\mathrm{K}: \mathrm{F}]$ b) $\mathrm{O}(\mathrm{G}(\mathrm{K}, \mathrm{F})) < [\mathrm{K}: \mathrm{F}]$
   c) $\mathrm{O}(\mathrm{G}(\mathrm{K}, \mathrm{F})) > [\mathrm{K}: \mathrm{F}]$ d) $\mathrm{O}(\mathrm{G}(\mathrm{K}, \mathrm{F})) \leq [\mathrm{K}: \mathrm{F}]$

7) The number of automorphism of field on complex number is/are _____.
   a) 1 b) 2
   c) 3 d) 0

8) If $[K: F] = n$ then each element in K is algebraic over F of degree _____.
   a) Equal to n b) less than n
   c) greater than n d) at most n

9) For every prime number p and every integer m there exists a field having _____ elements.
   a) m b) p
   c) p^m d) pm

10) The Splitting field of $x^2 - 1 \in R[x]$ over Q is _____.
   a) Q b) R
   c) C d) None of these

B) Fill in the blanks **06**

1) Any two-field having _____ numbers of element are isomorphic.
2) The field R of real number is a _____ extension of the field of real number Q.
3) The number of automorphism of field of real number is/are _____.
4) Any finite extension of a field F of characteristic _____ is simple extension.
5) If F is field then the dimension of $F(F)$ is _____.
6) If $[Q(\sqrt{3}):Q] = 2$ then each element in $Q(\sqrt{3})$ is algebraic over Q of degree _____.

Q.2 Answer the following. **16**

a) Prove that: If L is a finite extension of F and if K is a subfield of L which contains F then $[K:F]$ is a divisor of $[L:F]$.
b) Define Algebraic element and check whether $\sqrt{2}$ and π are algebraic over Q or not.
c) Define the following the terms:
   1) Degree of field extension
   2) Finite field extension
   3) Simple field extension
   4) Minimal polynomial of an algebraic element
d) If α is constructible element then show that $\sqrt{\alpha}$ is constructible element.

Q.3 Answer the following.

a) If a, b in K are algebraic over F then prove that $a \pm b, ab, \frac{a}{b} (b \neq 0)$ are all algebraic over F, where K is extension of F. **08**
b) If the complex number z is a root of $p(x)$ having real coefficients then prove that $\bar{z}$ is also root of $p(x)$. **08**

Q.4 Answer the following.

a) If F be a field of rational numbers then determine the degree of spitting field of the polynomial $x^3 - 1$ over F. **08**
b) If K be an extension of a field F then prove that the element $a \in K$ is algebraic over F iff $F(a)$ is finite extension of F. **08**

Q.5 Answer the following.

a) Define Derivative of a polynomial and show that if F be a field and let $f(x) \in F[x]$ be a polynomial such that $f'(x) = 0$ then prove that, **10**
   i) If characteristic of $F = 0$ then $f(x) = a \in f(x)$ is a constant polynomial
   ii) If the characteristic of $F = p \neq 0$ then $f(x) = g(x^p)$ *for* some polynomial $g(x) \in F[x]$.
b) If $f(x) \in F[x]$ is irreducible and characteristic of F is 0 then prove that $f(x)$ has no multiple roots. **06**

Q.6 Answer the following.

a) Prove that a field of characteristic 0 is perfect field. **08**
b) Find the Galois group of $x^2 - 2$ over the field of rational number. **08**

Q.7 Answer the following.

a) Show that $\sqrt{2}$ and $\sqrt{3}$ are algebraic over Q.Exhibit the polynomial over Q of degree 4 satisfied by $\sqrt{2}+\sqrt{3}$. **08**

b) Show that if α and β is constructible then prove that $\alpha\beta$ and $\frac{\alpha}{\beta}(\beta \neq 0)$ is constructible. **08**

SLR-EO-15

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Real Analysis – II (MSC15202)

Day & Date: Tuesday, 19-12-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

1) If ϕ is an empty set then $m^*(\phi) =$ _____.
   a) zero b) finite
   c) non-zero d) infinite

2) If f and g are two real valued measurable functions defined on the same domain D then _____.
   a) $f + g$ is measurable
   b) $f - g$ is measurable
   c) cf is mesurable for some c in R
   d) All the above

3) The negative part f^- of a function f is given by $f^-(x) =$ _____.
   a) $\max(f(x), 0)$ b) $\min(-f(x), 0)$
   c) $\max(-f(x), 0)$ d) $f(x)$

4) Let f be a non-negative measurable function on $[a, b]$ such that $\int_a^b f(x)dx = 0$ then ______.
   a) $f(x) = 0$ almost everyehere on $[a, b]$
   b) $f(x) \neq 0 \ \forall x \in [a, b]$
   c) $f(x) \geq 0 \ \forall x \in [a, b]$
   d) None of the above

5) A property is said to be hold almost everywhere if there exists a set of points where it fails to hold is of measure _____.
   a) zero b) > 0
   c) < 0 d) finite

6) A countable intersection of open set is called _____.
   a) F_σ set b) G_δ set
   c) $F_{\sigma\delta}$ d) $G_{\sigma\delta}$

7) Let m^* be a outer measure and $m^*(E) = 0$ then _____.
   a) E is measurable b) E is countable
   c) E is uncountable d) None of these

8) Outer measure is defined on _____.
   a) R b) $P(R)$
   c) measurable sets d) open sets

9) If Z is a set of integers then outer measure of $Z, m^*(Z)$ is _____.
   a) one   b) finite
   c) non zero   d) zero

10) The outer measure of an interval is its _____.
   a) cardinality   b) supremum value
   c) infimum value   d) length

B) Fill in the blanks. **06**

1) A set $E \subseteq R$ is called measurable if for any subset A of $R, m^*(A) =$ ____.
2) A set which is countable union of closed set is called _____.
3) If A and B are disjoint sets then $\chi_{A \cup B} =$ _____.
4) A simple function ϕ is written as a linear combinations of _____ functions.
5) A continuous function defined on a measurable set is _____.
6) If f is a non negative measurable function defined over a measurable set E then $\int_E f =$ _____, where h is a bounded measurable function.

Q.2 Answer the following. **16**

a) If A is countable set then prove that $m^*(A) = 0$

b) Define:-
   i) Outer Measure
   ii) Lebesgue Measure
   iii) Measurable set
   iv) Measurable function

c) If f be a non negative measurable function and $\{E_i\}$ be a disjoint sequence of measurable sets and $E = \cup E_i$ then prove that $\int_E f = \sum_i \int_{E_i} f$

d) If f is function of bounded variations on $[a, b]$ then with usual notations prove that

$$T_a^b = P_a^b + N_a^b$$

Q.3 Answer the following.

a) If ϕ and ψ be the simple function which vanishes outside a set of finite measure $E,$ then prove the following results: **08**
   i) $f a\phi + b\psi = a\int \phi + b\int \psi$
   ii) $\phi \geq \psi \; a.e \Rightarrow \int \phi \geq \int \psi$

b) Prove that collection $\mathcal{M}$ of all measurable sets is σ - algebra. **08**

Q.4 Answer the following.

a) If $\{E_n\}_{n=1}^{\infty}$ be an infinite increasing sequence of measurable sets then prove that **08**

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} m\,(E_n)$$

b) State and prove Fatou's Lemma. **08**

Q.5 Answer the following.

a) Prove That: A function f is of bounded variations on $[a, b]$ if and only if f is difference of two monotone real valued functions on $[a, b]$. **08**

b) If f and g are two non negative measurable function and f is integrable over E such that $g(x) < f(x)$ on E then prove that g is integrable and $\int_E f - g = \int_E f - \int_E g$ **08**

Q.6 Answer the following.

a) If f and g are two measurable functions on the same domain then prove that functions $f + c, cf, f + g, f - g$ and $f.g$ are also measurable where c is constant. **08**

b) If $E_1$and E_2 are measurable sets then prove that, **08**

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$$

Q.7 Answer the following.

a) If E be a measurable set then prove that translation $E + y$ is a measurable set and $m(E + y) = m(y)$. **08**

b) Prove that Cantor's set C is an uncountable set with outer measure zero. **08**

SLR-EO-16

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
General Topology (MSC15203)

Day & Date: Wednesday, 20-12-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative (MCQ). **10**

1) In a discrete topological space $< X, \Im >$, every subset of X is _____.
   a) closed b) open
   c) both open and closed d) None of these

2) If X is a finite set, then the co-finite topology on X coincides with _____.
   a) co-countable topology
   b) discrete topology
   c) indiscrete topology
   d) p-inclusion topology

3) In indiscrete topological space $< X, \Im >$, if $A \subset X$ with $|A| > 1$, then $d(A) =$
   a) A b) φ
   c) X d) A^c

4) If $X = \{a, b, c\}, \Im = \{\varphi, \{a\}, \{b, c\}, X\}$ and $A = \{a, c\}$, then $i(A) =$ _____.
   a) $\{a\}$ b) $\{a, c\}$
   c) $\{c\}$ d) X

5) If $X = \{a, b, c\}, \Im = \{\varphi, \{a\}, \{b, c\}, X\}$ and $A = \{b\}$, then $c(A) =$ _____.
   a) $\{b\}$ b) $\{b, c\}$
   c) $\{a, c\}$ d) $\{a, b\}$

6) Every T_1 space is _____.
   a) T_0 space b) T_2 space
   c) T_3 d) None of the above

7) Subspace $< Y, \Im^* >$ of a Lindelof space $< X, \Im >$ is again Lindelof if ____.
   a) Y is closed subspace of X b) Y is open subset of X
   c) Y is infinite d) Y is uncountable

8) A topological space $< X, \Im >$ is said to be separable if _____.
   a) there exists a dense in itself subset in X
   b) There exists a dense set in X
   c) There exists a countable dense subset in X
   d) There exists an uncountable dense subset in X

9) If a topological space $< X, \Im >$ is closed if _____.
   a) $d(A) = X$ b) $d(A) \subset A$
   c) $A \subset d(A)$ d) $X \subset d(A)$

10) In any topological space $< X, \Im >$, a set A is open iff _____.
   a) $i(A) \subset A$ b) $i(A) = A$
   c) $i(A) = \varphi$ d) All of the above

B) True or False. **06**

1) Every T_2 space is T_1 space.
2) Every Lindelof space is compact space.
3) To prove that a set A in a topological space $< X, \Im >$ open, it enough to prove that $A \subset i(A)$.
4) The usual topological space $< \mathbb{R}, \Im_u >$ is compact.
5) Every co-finite topology on X is compact
6) Every T_1 space is T_3.

Q.2 Answer the following. **16**

a) Define first axiom space, second axiom space, separable space and Lindelof space.

b) Prove that being a T_0 space is a hereditary property.

c) Prove that being a T_2 space is a topological property

d) Prove that continuous image of every connected space is a connected space.

Q.3 Answer the following. **16**

a) For any set A in a topological space $< X, \Im >$, prove that $\bar{A} = A \cup d(A)$.

b) Define continuous function between two topological spaces. If $< X, \Im >, < Y, \Im^* >$ are two topological spaces and if $f: < X, \Im > \to < Y, \Im^* >$ is a function, then prove that f is continuous on X iff $f^{-1}(G^*)$ is open in X for every open set G^* in X^*.

Q.4 Answer the following. **16**

a) If $< X, \Im >$ is any topological space, then prove that $< X, \Im >$ is compact iff every family of closed sets in $< X, \Im >$ having finite intersection property has a non-empty intersection.

b) Prove that being a T_3 space is a topological property.

Q.5 Answer the following. **16**

a) Prove that a topological space X is normal iff for any closed set F and an open set G containing F, there exists an open set H such that $F \subset H \subset \bar{H} \subset G$.

b) If $X = \{a, b, c, d\}, \Im = \{\varphi, \{a\}, \{c\}, \{b, d\}, \{a, c\}, \{a, b, d\}, \{b, c, d\}, X\}$ and $A = \{a, b, c\}$ then find $d(A)$.

Q.6 Answer the following **16**

a) Define completely regular space. Prove that being a completely regular space is a hereditary property.

b) Let X be an infinite set. Define $\Im = \{\varphi\} \cup \{A \subset X | X - A \text{ is finite}\}$. Then prove that $\Im$ is a topology on X.

Q.7 Answer the following. **16**

a) Prove that a topological space $< X, \Im >$ is a T_2 space iff any two disjoint compact subsets of X are separated by disjoint open sets.

b) Let $< X, \beth >$ and $< X^*, \beth^* >$ be two topological spaces. Let $f: X \to X^*$ be one-one, onto mapping. Then prove that f is a homeomorphism iff $f[i(E)] = i^*[f(E)]$, for any $E \subseteq X$.

SLR-EO-17

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Complex Analysis (MSC15206)

Day & Date: Thursday, 21-12-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) The value of $\int_C \frac{1}{z^2+4}\,dz$, where C is the circle $|z-2i| = 1$ is _____.
   a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$
   c) $2\pi i$ d) 0

2) The function $f(z)$ = $secz$ is _____.
   a) analytic for all z b) not analytic at $z = \frac{\pi}{2}$
   c) not analytic at $z = \pi$ d) nowhere analytic

3) If z is any complex number then $|z+5|^2 + |z-5|^2 = 75$ represents _____.
   a) a circle b) an ellipse
   c) a triangle d) straight line

4) Which of the following mapping does not change the shape of the figure but it changes size of the figure?
   a) Rotation b) Translation
   c) Magnification d) Bilinear Transformation

5) The residue of the function $f(z) = \frac{\sin z}{z^8}$ at $z = 0$ is _____.
   a) $\frac{1}{7!}$ b) $\frac{-1}{7!}$
   c) 1 d) 0

6) If pole of the bilinear transformation lies on the boundary then the image is _____.
   a) Circle b) Triangle
   c) Straight line d) Parabola

7) If f have an isolated singularity at $z = a$ and $f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n$ is its Laurent expansion about $z = a$ then the residue of f at $z = a$ is _____.
   a) a_{-1} b) a_0
   c) a_{-2} d) a_1

8) In Laurent's expression, singularities of different types are distinguished by _____.
   a) Analytic part b) Real part
   c) Imaginary part d) Principal part

9) If image of an open set is not open under an analytic function then the function is _____.
   a) Not analytic b) Constant
   c) Non-constant d) Not differentiable

10) The transformation $w = \frac{1}{z}$ maps $|z| < 1$ into ______.
   a) $|w| < 1$ b) $|w| = 1$
   c) $|w| \neq 1$ d) $|w| > 1$

B) Fill in the blanks. **06**

1) The function $f(z) = \frac{\sin z}{(z-\pi)^2}$ have the pole of order_____ at $z = \pi$.
2) If $z = a$ is a singularity of $f(z)$ such that $f(z)$ is analytic at each point in its neighbourhood then $z = a$ is called as _____.
3) If $T_1(z) = \frac{z+2}{z+3}$ and $T_2(z) = \frac{z}{z+1}$, then $T_2T_1(z)$ is _____.
4) If $f(z)$ has a pole of order n at $z = a$ then residue of function $f(z)$ st a is ___
5) A non-constant analytic function maps open set to a _____.
6) If $f: C \to C$ defined by $f(z) = z^2 + 1$ is an analytic function then the set of zeros of the function f is _____.

Q.2 Answer the following **16**

a) Evaluate: $\int_\gamma \frac{z-3\cos z}{\left(z-\frac{\pi}{2}\right)^5}\, dz$ over $\gamma : |z| = 5$

b) State and prove Cauchy estimate theorem.

c) If S is a Mobius transformation then prove that S is the composition of Translation, Dilation and Inversion.

d) Find $Res(f;-1)$, $Res(f;2)$ for $f(z) = \frac{z^2}{(z+1)^2(z-2)}$

Q.3 Answer the following.

a) If f has an essential singularity at $z = a$ then show that $f(ann(a;0,\delta))$ is dense in C for all $\delta > 0$. **08**

b) Show that $\int_0^\pi \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$ **08**

Q.4 Answer the following.

a) If z_1, z_2, z_3, z_4 be the four distinct points in C_∞ then show that the cross ratio (z_1, z_2, z_3, z_4) is real iff all four points lie on a circle or straight line. **08**

b) If γ is a rectifiable curve and suppose φ be a function defined and continuous on $\{\gamma\}$. For each $m \geq 1$, let $F_m(z) = \int_\gamma \frac{\varphi(w)}{(w-z)^m} dw\, ; z \notin \{\gamma\}$. The show that each F_m is analytic on $C - \{\gamma\}$ and $F_m'(z) = mF_{m+1}(z)$. **08**

Q.5 Answer the following.

a) State and prove Taylor's theorem. **10**

b) If $|z| < 1$ then show that, **06**

$$\int_0^{2\pi} \frac{e^{is}}{e^{is}-z} ds = 2\pi$$

Q.6 Answer the following.

a) If G be a region and $f: G \to C$ be an analytic function such that there is a point 'a' in G with $|f(z)| \leq |f(a)|\ \forall z \in G$ then show that f is a constant. **08**

b) If f has an isolated singularity at $z = a$ then prove that the point $z = a$ is removable singularity iff $\lim_{z \to a}(z - a)f(z) = 0$. **08**

Q.7 Answer the following.

a) Define the following terms with one example: **10**

1) Isolated Singularity
2) Non-Isolated Singularity
3) Removable Singularity
4) Pole
5) Essential Singularity

b) Find the Mobius transformation which maps the given points $z_1 = 0, z_2 = 1$ and $z_3 = \infty$ onto the points $w_1 = -1, w_2 = -i$ and $w_3 = 1$. **06**

SLR-EO-19

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Functional Analysis (MSC15301)

Day & Date: Friday, 05-01-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) An idempotent linear transformation on a linear space N is called _____.
   a) operator b) norm
   c) projection d) metric

2) Consider the statements.
   I) Every finite dimensional normed linear space is a Banach space.
   II) Every Banach space is finite dimensional linear space.
   a) only I is true b) only II is true
   c) both are true d) both are false

3) Pick the INCORRECT statement:
   a) Every Hilbert space is a normed linear space
   b) Every Banach space is a topological space
   c) Every normed space is a metric space
   d) Every Banach space is a Hilbert space

4) If M and N are subspaces of a Hilbert space and $M \perp N$ then _____.
   a) $M \cup N = \{0\}$ b) $M \cap N = \{0\}$
   c) $M + N = \{0\}$ d) $M + N = H$

5) A continuous linear transformation $T: N \rightarrow N'$ is said to be open mapping if for every open set G in $N, T(G)$ is _____ in N'.
   a) closed b) bounded
   c) open d) Finite

6) By closed graph theorem, if B and B' are Banach spaces and T is a linear transformation of B into B' then T is continuous mapping iff ______.
   a) its graph is open set b) its graph is closed set
   c) its graph is finite set d) its graph is countable set

7) In a Hilbert space, for any $x, y \in H$ the vectors x, y are said to be orthogonal if _____.
   a) $\langle x, y\rangle \neq 0$ b) $\langle x, y\rangle = 0$
   c) $\langle x, y\rangle \leq 0$ d) $\langle x, y\rangle \geq 0$

8) In a linear space, a vector is called unit vector if $\|x\| =$ _____.
   a) 1 b) 0
   c) finite d) lion-negative

9) Consider the following statements:
   I) Every cauchy sequence in normed linear space is convergent.
   II) Every convergent sequence in normed linear space is cauchy.
   a) only I is true b) only II is true
   c) both are true d) both are false

10) As self adjoint operator T is said to be positive if _____.
   a) $T \leq 0$ b) $T \geq 0$
   c) $I + T = 0$ d) $\langle T(x), x\rangle$ is real

B) State whether following statements are true or false. **06**
1) If H is a Hilbert space then its conjugate space H^* is also Hilbert space.
2) Every closed subspace of normed linear space is complete.
3) The inner product in Hilbert space is jointly continuous.
4) The mapping $\phi: H \to H^*$is linear.
5) Any two finite dimensional normed linear spaces over same scalar field are topologically isomorphic.
6) There exist a Hilbert space in which parallelogram law is not true.

Q.2 Answer the following. **16**
a) State and prove Pythagorean theorem.
b) Define: Inner Product and Norm.
c) If $\|.\|_1, \|.\|_2$ are equivalent norms defined on the linear space X then show that $< X, \|.\|_1 >$ is a Banach space iff $< X, \|.\|_2 >$ is a Banach space.
d) If $S(x, r)$ is an open sphere in B with centre at x and radius r, S_r is the open with centre at origin and radius r then prove that $S(x, r) = x + S(0, r)$.

Q.3 Answer the following.
a) Show that the real linear space and complex linear space are Banach spaces under the norm, $\|x\| = |x|, x \in \mathbb{R}$ or $\mathbb{C}$. **08**
b) If M is a linear subspace of normed linear space N and f is a functional defined on M then prove that f can be extended to a functional F defined on whole space N such that $\|f\| = \|F\|$. **08**

Q.4 Answer the following.
a) State and Prove Schwarz's inequality. **08**
b) If N and N' are two normed linear spaces and D a subspace of N then prove that a linear transformation $T: D \to N'$ is closed if and only if its graph T_G is closed. **08**

Q.5 Answer the following.
a) If X is a complex IPS then Prove that: **08**
   1) $< ax - by, z >= a < x, z > -b < y, z >$
   2) $< x, ay + bz >= \bar{a} < x, y > +\bar{b} < x, z >$
   3) $< x, ay - bz >= \bar{a} < x, y > -\bar{b} < x, z >$
   4) $< x, 0 >= 0 \; and < 0, x >= 0, \forall x \in X$

b) If H is a Hilbert space then prove that H^* is also Hilbert space with the inner product defined by $< f_x, f_y >=< y, x >$. **08**

Q.6 Answer the following.

a) If M is a closed linear subspace of a Hilbert space H, x be a vector not in M and $d = d(x, M)$ then prove that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$. **06**

b) Prove that: Any two n-dimensional normed spaces over the same sacalar field are topologically isomorphic **10**

Q.7 Answer the following.

a) If N is a normed linear space and two norms $\|.\|_1$and $\|.\|_2$ are defined on N then prove that these two norms are equivalent if and only if there exists a positive real numbers m and M such that $m\|x\|_1 \leq \|x\|_2 \leq M\|x\|_1, \forall x \in N$. **08**

b) If Y is complete then prove that $B(X, Y)$ is complete. **08**

SLR-EO-20

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: oct/Nov-2023
MATHEMATCIS
Advanced Discrete Mathematics (MSC15302)

Day & Date: Sunday, 07-01-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The relation $\{(1,2),(1,3),(3,1),(1,1),(3,3),(3,2),(1,4),(4,2),(3,4)\}$ is _____.
   a) Reflexive b) Symmetric
   c) Transitive d) None of these

2) In how many ways can 5 balls be chosen so that 2 are red and 3 are black?
   a) 910 b) 990
   c) 970 d) 124

3) If B is a Boolean Algebra, then which of the following is true?
   a) B is a finite but not complemented lattice
   b) B is a finite, complemented and distributive lattice
   c) B is a finite, distributive but not complemented lattice
   d) B is not distributive lattice.

4) How many different words can be formed out of the letters of the word VARANASI?
   a) 64 b) 120
   c) 40320 d) 720s

5) The complete graph with four vertices has k edges where k is ____.
   a) 3 b) 4
   c) 5 d) 6

6) A graph with n vertices will definitely have a parallel edge or self-loop if the total number of edges are _____.
   a) more than n b) more than $n+1$
   c) more than $(n+1)/2$ d) more than $n(n-1)/2$

7) A tree contains an ____.
   a) pedant vertex b) loop
   c) isolated vertex d) parallel edges

8) What is the recurrence relation for the sequence $1,3,7,15,31,63,..?$
   a) $a_n = 3a_{n-1} - 2a_{n+2}$ b) $a_n = 3a_{n-1} - 2a_{n-2}$
   c) $a_n = 3a_{n-1} - 2a_{n-1}$ d) $a_n = 3a_{n-1} - 2a_{n-3}$

9) The connectivity of a connected graph G is one if and only if _____.
   a) $G = K_1$ b) $G = K_2$
   c) G has cut vertex d) both b and c

10) For any connected graph G, _____.
   a) $rad\ (G) \leq 2\ rad\ (G)$ b) $rad\ (G) \leq diam\ (G)$
   c) $diam\ (G) \leq 2\ rad\ (G)$ d) All of these

B) Fill in the blanks. **06**

1) The coefficient of $\chi^{10} in(\chi^3 + \chi^4 + \chi^5 + - - -)^3$ is _____
2) The edges of a graph G which are not in spanning tree are called as _____
3) The characteristic equation of $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$ is _____
4) If $(S, \lesssim)$be a $POSET$ and every two elements of S are comparable, then S is called _____.
5) If $(n+1)$ objects are put into n boxes then at least one box contains _____
6) The generating function for the sequence $1, 6, 36, 216, \ldots.$ is _____

Q.2 Answer the following. **16**

a) Prove that in any graph G there is an even number of odd vertices.
b) Prove that $n_{c_r} + n_{c_r - 1} = n + 1_{c_r} (0 \leq r \leq n)$
c) Show that an acyclic graph with n vertices is tree iff it contains precisely $(n-1)$ edges.
d) Draw the Hasse diagram of the poset $(P(S), \subseteq)$ where $P(S)$ is the power set on $S = \{a, b, c\}$.

Q.3 Answer the following. **08**

a) Find the primes less than 100 by using the principle of inclusion-exclusion?
b) Show that a graph G is connected iff given any pair u and v of vertices there is a path from u to v. **08**

Q.4 Answer the following.

a) If G be a graph with n vertices and q edges,$w\ (G)$denotes the number of connected component in G then prove that G has at least $n - w(G)$ edges. **08**
b) Among the integers $1\ to\ 1000$.Find how many of them are not divisible by 3, nor by 5, nor by 7. **08**

Q.5 Answer the following.

a) Solve, **10**
   i) $y_{n+2} + y_{n\_+1} - 2y_n = n^2$
   ii) $y_{n+2} - 4y_{n+1} + 4y_n = 2^n$
b) If L be any lattice and $a, b, c, \in L$ then prove that **06**
   i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$
   ii) $a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

Q.6 Answer the following.

a) If G be a graph with n vertices $v_1, v_2, v_3, \ldots v_n$ & A denote the adjacency matrix of G with respect to this listing of vertices. Let $B = [b_{i,j}]$ be the matrix $B = A + A^2 + A^3 + \ldots + A^{n-1}$.Then show that G is connected graph iff for every pair of distinct indices i, j we have $b_{i,j} \neq 0$. **10**
b) Show that a graph G is connected if and only if it has a spanning tree. **06**

Q.7 Answer the following.

a) Show that in a complemented, distributive lattice, the following are equivalent **08**

i) $a \lesssim b$

ii) $a \wedge b' = 0$

iii) $a' \vee b = 1$

iv) $b' \lesssim a'$

b) Write a short note on the matrix representation of graph with two examples. **08**

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Linear Algebra (MSC15303)

Day & Date: Tuesday, 09-01-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) Characteristic values of $\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ are _____.
   a) $1,1$ b) $1, i$
   c) i, i d) $-1, i$

2) If V is n-dimensional vector space over the field F then the dimension of dual space V^* of V is _____.
   a) n b) $\frac{n}{2}$
   c) n^2 d) $n+1$

3) Which of the following mapping $T: R^3 \rightarrow R^3$ is not a linear transformation?
   a) $\mathrm{T(x, y, z) = (x - y, y - z, z - x)}$
   b) $\mathrm{T(x, y, z) = (x - y, 3z, 0)}$
   c) $\mathrm{T(x, y, z) = (x + 2y, y + z, x - z)}$
   d) $\mathrm{T(x, y, z) = (x + y, x - y, z + 1)}$

4) If W be a subspace of a vector space V and T be a linear operator on V then W is said to be invariant under T if _____.
   a) $T(W) \subseteq W$ b) $T(W) \supseteq W$
   c) $T(W) = 0$ d) $T(W) = V$

5) A linear operator T on Inner product space V is said to be _____ if $T = T^*$
   a) Self adjoint b) Unitary
   c) Normal d) Identity

6) The monic polynomial of lowest degree over the field F that annihilates a linear operator T is called ______.
   a) Minimal polynomial b) Characteristic polynomial
   c) Annihilating polynomial d) Constant polynomial

7) If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ then the characteristic polynomial of A is _____.
   a) $\mathrm{x}^2 + 1$ b) $\mathrm{x}^2 - 1$
   c) $\mathrm{x}^2 + 2$ d) $\mathrm{x}^2 - 2$

8) It x and y be twovectors in an inner product space V then x is said to be orthogonal to y if _____.
   a) $< x, y >= 1$ b) $< x, y >= 0$
   c) $< x, y >= -1$ d) $< x, y >= \sqrt{5}$

9) If V be finite dimensional vector space V over the field F and W be subspace of V then _____.
   a) $\mathrm{dimW} + \mathrm{dimW}^0 < \mathrm{dimV}$
   b) $\mathrm{dimW} + \mathrm{dimW}^0 > \mathrm{dimV}$
   c) $\mathrm{dimW} + \mathrm{dimW}^0 = \mathrm{dimV}$
   d) None of these

10) If ⋋ is characteristic value of a linear operator T then the _____ multiplicity of ⋋ is defined to be the multiplicity of ⋋ as a root of the characteristic polynomial of T.
   a) Minimal
   b) Geometric
   c) Algebraic
   d) unique

B) Fill in the blanks. **06**

1) The value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ is _____.

2) If V be an inner product space and $S = \{0\}$ is a subspace of V then $S^{\perp} =$_____.

3) The solution of the system of equations $3x + 2y - 6z = 1, 2x - 3y + 3z = -1, x - 4y + z = -6$ is _____.

4) If V be a vector space over the field F then a linear transformation $T: V \to V$ is called _____ on V.

5) If V and W be inner product space over the same field F and T be a linear transformation from V into W then T preserves norm if $\|T(\propto)\| =$ _____ $\forall \propto \in V$.

6) If $\lambda_1, \lambda_2,$ _____, λ_n are the eigenvalues of A then the eigenvalues of kA are _____.

Q.2 Answer the following. **16**

a) Prove that the minimal polynomial of a matrix or of a linear operator T is unique.

b) If V be finite dimensional inner product space and W is subspace of V which is invariant under T then prove that the orthogonal complement of W is invariant under T^*

c) Find all characteristic values and characteristic vectors of a matrix $\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

d) Define the following terms:
   i) Unitary operator
   ii) Self adjoint operator
   iii) Normal operator
   iv) Hermitian form

Q.3 Answer the following.

a) Slate and prove Cayley Hamilton's theorem. **08**

b) If V be an finite dimensionl vector space over the field F and T be a linear operator on V then prove that T is triangulable if and only if the minimal polynomial for T is product of linear polynomial over F. **08**

Q.4 Answer the following.

a) If V and W be inner product spaces over the same field F and T be linear transformation form V into W then prove that T preserves inner product iff T preserves norm. **08**

b) If $V = W_1 \oplus W_2 \oplus W_3 \oplus \ldots \oplus W_k$ then prove that there exists k linear operators $E_1, E_2 \ldots., E_k$ on V such that **08**

i) Each E_1 is a projection on V

ii) $E_i E_j = 0$ if $i \neq j$

iii) $I = E_1 + E_2 + E_3 + \cdots \cdot\cdot + E_k$

iv) The range of E_i is W_i

Q.5 Answer the following.

a) If $B = \{(-1,1,1), (1,-1,1), (1,1,-1)\}$ is a basis of $V_3(R)$ then find the dual basis of B. **08**

b) If V be an inner product space and T be self-adjoint operator on V then prove that each characteristic value is real and characteristic vector associated with distinct characteristic values are orthogonal. **08**

Q.6 Answer the following.

a) If S and T are linear operators on an inner product space V and c is any scalar then prove that. **08**

i) $(S+T)^* = S^* + T^*$

ii) $(cT)^* = cT^*$

iii) $(ST)^* = T^* S^*$

iv) $(T^*)^* = T$

b) If $\beta_1 = (3,0,4), \beta_2 = (-1,0,7)$ and $\beta_3 = (2,9,11)$ then find the orthogonal and orthonormal basis for R_3 with the standard inner product by using Gram Schmidt orthogonalization process. **08**

Q.7 Answer the following.

a) Show that the matrix $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ is diagonalizable. **08**

b) Obtain the Jordan canonical forms of the $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$. **08**

SLR-EO-22

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Differential Geometry (MSC15306)

Day & Date: Thursday, 11-01-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Select the correct alternative. **10**

1) If U_1, U_2, U_3 are natural frame fields at p, then $U_i[f] =$ _____.
   a) $\frac{df}{dx}$ b) $\frac{\partial f}{\partial_{x_i}}$
   c) $\frac{\partial f_i}{\partial_x}$ d) None of these

2) If v_p is tangent vector of $T_p(E^3)$ at a point p then $df(v_p) =$ _____.
   a) 0 b) 1
   c) $v_p[f]$ d) does not exist

3) A curve $\alpha: I \to E^3$ is said to be a regular curve if _____.
   a) $\alpha'(t) \neq 0, \forall t \in I$ b) $\alpha''(t) \neq 0, \forall t \in I$
   c) $\alpha'(t) = 0, \forall t \in I$ d) $\alpha'(t) \neq 0,$ for same $t \in I$

4) If T, N, B are frenet frame fields, then which of the following is true.
   a) $T.B = 0$ b) $B.N = 0$
   c) $N.T = 0$ d) all of the above

5) For minimal surfaces, Gaussian curvature K is _____.
   a) always positive b) always negative
   c) always zero d) non-negative

6) If $\overline{T}: E^3 \to E^3$ is an isometry, then it preserves _____.
   a) Norm b) metric
   c) dot product d) all of the above

7) For a patch $X: D \to E^3, F =$ _____.
   a) $F = X_u \cdot X_u$ b) $F = X_u \cdot X_v$
   c) $F = X_v \cdot X_v$ d) $F = ||X_u||$

8) Cylinders are surfaces obtained by translating a _____.
   a) A line along the curve b) A circle along the curve
   c) An ellipse along the line d) helix along the line

9) If $\overline{T}: E^3 \to E^3$ is a translation by $\bar{a}$, then $\overline{T}^{-1}$ is a translation by _____.
   a) $||\bar{a}||$ b) $\bar{0}$
   c) $\bar{p}$ d) $-\bar{a}$

10) In case of torus, profile curve is a _____.
   a) Straight line b) ellipse
   c) helix d) circle

B) State whether true or false. **06**

1) Torus is a surface.
2) Directional derivative of a function f at the point p in the direction of vector v_p is a vector quantity.
3) For a plane curve, torsion is zero.
4) Gaussian curvature for a surface is product of principle curvatures.
5) $T' = -\kappa N$.
6) For circle, curvature k is constant.

Q.2 Answer the following. **16**

a) If $\bar{W} = x^2U_1 + yzU_3, \bar{v} = (-1,0,2), \bar{p} = (2,1,0)$, then find $\nabla_{\bar{v}}\bar{W}$ at $\bar{p}$.

b) Define coordinate patch and Proper coordinate patch.

c) Show that the shape operator describes the cylindrical surface as half flat and half round.

d) Find the unit speed reparameterization of a circle of radius r and hence compute the tangent vector field of the curve.

Q.3 Answer the following.

a) If $\alpha: I \to E^3$ is a regular curve in E^3 then show that **10**

$T = \frac{\dot{\alpha}}{\|\dot{\alpha}\|}, B = \frac{\dot{\alpha}\times\ddot{\alpha}}{\|\dot{\alpha}\times\ddot{\alpha}\|},\ N = B \times T, \kappa = \frac{\|\dot{\alpha}\times\ddot{\alpha}\|}{\|\dot{\alpha}\|^3}, \quad \tau = \frac{\dot{\alpha}\cdot(\ddot{\alpha} \times \dddot{\alpha})}{\|\dot{\alpha} \times \ddot{\alpha}\|^3}$

b) Define directional derivative of a function along a vector field. Further, if $\bar{V},\bar{W}$ are vector fields on E^3 and f, g, h are real valued functions, then show that **06**

1) $(f\bar{V} + g\bar{W})[h] = f\bar{V}[h] + g\bar{W}[h]$
2) $\bar{V}[af + bg](p) = a\bar{V}[f] + b\bar{V}[g]$
3) $\bar{V}[fg] = \bar{V}[f]g + f\bar{V}[g]$

Q.4 Answer the following.

a) Prove that every isometry of E^3 can be uniquely described as orthogonal transformation followed by translation. **08**

b) Define a regular mapping. Prove that a mapping $X: D \to E^3$ is regular iff $X_u \times X_v \neq 0, \forall(u, v) \in D$ **08**

Q.5 Answer the following.

a) Define 1-form. Prove that $df = \sum_i \frac{\partial f}{dx_i} dx_i$, where $f_i = f(\bar{U}_t)$ **08**

b) Show that $M: z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is a surface and $X(u, v) = (aucosv, businv, u^2)$ is a parametrization of M. **08**

Q.6 Answer the following.

a) Compute the Frenet apparatus for $\alpha(t) = \left(2t, t^2, \frac{t^3}{3}\right)$ at $t = 0$. **10**

b) For a non-unit speed regular curve in E^3, prove that. **06**

$$\begin{bmatrix} \dot{T} \\ \dot{N} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} 0 & \kappa v & 0 \\ -\kappa v & 0 & \tau v \\ 0 & -\tau v & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Q.7 Answer the following.

a) Let $X: E^2 \to E^3$ be the mapping defined by $X(u, v) = (u + v, u - v, uv)$. Show that X is a proper patch and that the image of X is the surface $z = \frac{x^2-y^2}{4}$ **08**

b) Show that $\bar{F}$ defined by $\bar{F}(\bar{p}) = -\bar{p}$ is an isometry of E^3. If so, find its translation and orthogonal part. **08**

SLR-EO-24

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Measure & Integration (MSC15401)

Day & Date: Monday, 18-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) The characteristic function $\chi(A)$ of A is measurable then A is _____.
   a) measurable b) not measurable
   c) need not be measurable d) None of these

2) If $(X, \mathcal{B}, \mu)$ be a measure space, $E \subseteq X$ then E is called finite measure if _____.
   a) $\mu(X) < \infty$ b) $\mu(E) < \infty$
   c) $\mu(\mathcal{B}) < \infty$ d) All of the above

3) If $(X, \mathcal{B}, \mu)$ be a measure space, a subset $E \subseteq X$ is said to be _____ if $E \cap B \in \mathcal{B}$ for each $B \in \mathcal{B}$.
   a) finite b) saturated
   c) locally measurable d) complete

4) Consider the following statements.
   i) Every σ - finite measure is saturated.
   ii) Every measurable set is locally measurable.
   a) only i) is true b) only ii) is true
   c) both are true d) both are false

5) If $E \in \mathcal{B}$ with $\mu(E) < \infty$ then $\int_E 1 \, d\mu$= _____.
   a) zero b) one
   c) $\mu(\mathcal{B})$ d) $\mu(E)$

6) A set with positive measure _____.
   a) is a positive set b) need not be a positive set
   c) is a negative set d) need not be a negative set

7) Two measures v_1 and v_2 on a measurable space $(X, \mathcal{B})$ are said to be mutually singular if there exist sets A and B with $X = A \cup B$ such that _____.
   a) $v_1(A) = 0, v_1(B) = 0$ b) $v_1(B) = 0, v_1(A) = 0$
   c) both a and b d) none of the above

8) If v is a signed measure and μ is measure such that $v \perp \mu$ and $v << \mu$ then _____.
   a) $v = 0$ b) $v \neq 0$
   c) $v < 0$ d) $v > 0$

9) For any set $A \in \mathcal{A}$ (Algebra), following relation holds
   a) $\mu_*(A) \leq \mu^*(A)$ b) $\mu_*(A) \geq \mu^*(A)$
   c) $\mu_*(A) < \mu^*(A)$ d) $\mu_*(A) = \mu^*(A)$

10) If f be a non-negative measurable function and $\int f = 0$ then _____.
    a) $f = 0$
    b) $f = 0$ almost everywhere
    c) $f \geq 0$
    d) $f \geq 0$ almost everywhere

B) Fill in the blanks. **06**

1) If A and B are two disjoint sets then the characteristic function $\chi_{A \cup B} =$ _____.
2) The measure μ defined on a measure space $(X, \mathcal{B}, \mu)$ is called σ finite measure if _____.
3) The measure μ defined on a measure space $(X, \mathcal{B}, \mu)$ is called saturated if every locally measurable set is _____.
4) The integration of a simple function $\phi = \sum_{i=1}^{n} C_i \cdot \chi E_i$ is given as $\int_E \phi =$ _____.
5) If f_n is a sequence of non-negative measurable functions such that $f_n \to f$ almost everywhere then Fatou's lemma implies _____.
6) If f and g are non negative measurable functions and a, b are non-negative constants then $\int af + bg =$ _____.

Q.2 Answer the following. **16**

a) If f and g are measurable functions then prove that $f + g$ is also measurable function.

b) If f_n is a sequence of non-negative measurable functions which converges almost everwhere to f and $f_n \leq f$ for all n then prove that $\int f = \lim \int f_n$

c) If E is a positive set then prove that $v^{-}(E) = 0$.

d) If $A \in \mathcal{A}$ (Algebra) then with usual notations prove that $\mu^{*}(A) = \mu(A)$

Q.3 Answer the following.

a) If $(X, \mathcal{B}, \mu)$ is a measure space and $\mathcal{C}$ be the σ-algebra of locally measurable sets, for any $E \in \mathcal{C}$ define $\bar{\mu}(E) = \mu(E)$ if $E \in \mathcal{B}$ and $\bar{\mu}(E) = \infty$ if $E \notin \mathcal{B}$ then prove that $(X, \mathcal{C}, \bar{\mu})$ is a measure space. **08**

b) State and prove Lebesgue convergence theorem. **08**

Q.4 Answer the following.

a) If v is a signed measure on a measurable space then prove that there is a positive set A and negative set B such that $X = A \cup B, A \cap B = \phi$ **08**

b) If $A \in \mathcal{A}$ (Algebra) and $\{A_i\}$ is a sequence of sets in $\mathcal{A}$ such that $A \subseteq \cup_{i=1}^{\infty} A_i$ then prove that $\mu(A) \leq \sum_{i=1}^{\infty} \mu(A_i)$ **08**

Q.5 Answer the following.

a) Prove that: The collection $\mathcal{R}$ of measurable rectangles forms semi algebra. **08**

b) Define product measure and prove that if E is measurable subset $X \times Y$ then **08**
  i) $(E^c)_x = E_x^C$
  ii) $(\cup_{i=1}^{\infty} E_i)_x = \cup_{i=1}^{\infty} (E_i)_x$

Q.6 Answer the following.

a) If $E_i \in \mathcal{B}, \mu(E_1) < \infty$ and $E_i \supseteq E_{i+1}, \forall i$ then prove that $\mu(\cap_{i=1}^{\infty} E_i) = \lim_{n \to \infty} \mu(E_n)$ **08**

b) Prove that: Every σ - finite measure is saturated. **08**

Q.7 Answer the following.

a) If $E \subseteq F$ then with usual notations prove that $\mu_*(E) \leq \mu_*(F)$ **08**

b) If $\mathcal{R}$ is a measurable rectangle and $x \in X$ is any element then for $E \in R_{\sigma\delta}$ prove that E_x is measurable subset of Y. **08**

SLR-EO-25

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Partial Differential Equations (MSC15402)

Day & Date: Tuesday, 19-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Second order partial differential equations are classified in to_____.
   a) Hyperbolic type b) Parabolic type
   c) Elliptic type d) All of these

2) The solution of $\frac{\partial^3 z}{\partial x^3} = 0$ is _____.
   a) $z = f_1(y) + xf_2(y) + x^2 f_3(y)$
   b) $z = (1 + y + y^2)f(x)$
   c) $z = (1 + x + x^2)f(y)$
   d) None of these

3) Integral of $yzdx + xzdy + xydz = 0$ is_____.
   a) $xyz = 0$ b) $xyz = c$
   c) $x + y + z = c$ d) None of these

4) The two solutions of Neumann problem differ by_____.
   a) function of x and y b) function of x
   c) function of y d) constant

5) Eliminating a, b from $z = (x + a)(y + b)$ gives_____.
   a) $pq = z$ b) $\frac{p}{q} = z$
   c) $p + q = z$ d) None of these

6) The equation $(2x + 3y)p + 4xq - 8pq = x + y$ is_____.
   a) linear b) non-linear
   c) quasi-linear d) semi-linear

7) A function $f(x, y)$ is said to be a homogeneous function of x and y of degree n if it satisfies_____.
   a) $f(\lambda x, \lambda y) = \lambda^n f(x, y)$ b) $xf_x + yf_y = nf$
   c) Both (a) and (b) d) None of these

8) The complete integral of the pde $z = px + qy + \log pq$
   a) $z = x + y$ b) $z = ax + by + \log ab$
   c) $z = ax + by$ d) None of these

9) The general solution of $P_p + Q_q = R$ is_____.
   a) $\phi(u, v) = 1$ b) $\phi(u, v) = -1$
   c) $\phi(u, v) = c$ d) $\phi(u, v) = 0$

10) The general integral of $yzp + xzp = xy$ is ______.
a) $F(x^2 - y^2, z^2 - y^2) = 0$ b) $z^2 = y^2 + G(x^2 - y^2)$
c) Both (a) and (b) d) None

B) State true or false **06**

1) There always exists an integrating factor for Pfaffian differential equation in two variables.
2) Parametric equations of curve are not unique.
3) Complete integral of $z^2(1 + p^2 + q^2) = 1$ is $(x - a)^2 + (y - b)^2 + z^2 = 1$
4) The $p.d.e. pq = z$ is linear equation.
5) A two parameter family of solutions $z = F(x, y, a, b)$ is called complete integral if the rank of the matrix $\begin{pmatrix} F_a & F_{xa} & F_{ya} \\ F_b & F_{xb} & F_{yb} \end{pmatrix}$ is two.
6) $f(x, y, z, p, q) = 0$ and $g(x, y, z, p, q) = 0$ are compatiable on D if $\frac{\partial(f,g)}{\partial(p,q)} \neq 0$, $dz = pdx + qdy$ is integrable.

Q.2 Answer the following. **16**

a) Find complete integral of $p + q - pq = 0$
b) $\bar{X}\ curl\ \bar{X} = 0$ where $X = P\bar{\imath} + Q\bar{\jmath} + R\bar{k}$ and μ is an arbitrary differentiable function of x, y and z then prove that $\mu\bar{X} \cdot curl(\mu\bar{X}) = 0$
c) Define complete integral and general integral.
d) Show that the solution of the Dirichlet problem if it exists is unique.

Q.3 Answer the following.

a) Show that the surfaces $f(x, y, z) = x^2 + y^2 + z^2 = c, c > 0$ can form an equipotential family of surfaces. **08**
b) Let $u(x, y)$ and $v(x, y)$ be two functions of x and y such that $\frac{\partial v}{\partial y} \neq 0$. If further $\frac{\partial(u,v)}{\partial(x,y)} = 0$, then prove that there exist a relation between u and v not involving x and y explicity. **08**

Q.4 Answer the following.

a) Find complete integral of $p^2x + q^2y = z$ by using Charpits method. **08**
b) Prove that the necessary and sufficient condition for the integrability of $dz = \phi(x, y, z)dx + \Psi(x, y, z)dy$ is $[f, g] = 0$ where $f(x, y, z, p, q) = 0, g(x, y, z, p, q) = 0$ **08**

Q.5 Answer the following.

a) Prove that if $u(x, y)$ is harmonic in a bounded domain D and continuous in $\bar{D} = D \cup B$, then u attains its maximum on the boundary B of D **08**
b) Find an expression of d'Alembert's solution which describes the vibrations of an infinite string. **08**

Q.6 Answer the following.

a) Derive canonical form for hyperbolic type of equations. **08**
b) Solve $xu_x + yu_y = u_z^2$ by using Jacobi's method. **08**

Q.7 Answer the following.

a) As $h_1 = 0$ and $h_2 = 0$ are compatible with $f(x, y, z, u_x, u_y, u_z) = 0$ then prove that $\frac{\partial(f,h)}{\partial(x,u_x)} + \frac{\partial(f,h)}{\partial(y,u_y)} + \frac{\partial(f,h)}{\partial(z,u_2)} = 0$, where $h = h_i (i = 1,2)$ **08**
b) Solve $yzp + xzq = x + y$ **08**

SLR-EO-26

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Integral Equations (MSC15403)

Day & Date: Wednesday, 20-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) An integral equation $g(x)u(x) = f(x) + \int_a^b K(x,t)\, u(t)dt$ is said to be homogeneous if _____.
   a) $g(x) = 2$ b) $g(x) = 1$
   c) $f(x) = 0$ d) $f(x) \neq 0$

2) Which of the following is not a degenerate kernel?
   a) $K(x,t) = 2xt$ b) $K(x,t) = xt^2 - x^2t$
   c) $K(x,t) = cos(x+t)$ d) $K(x,t) = e^{\frac{x}{t}}$

3) Which of the following type of integral equation may have eigenvalues?
   a) homogeneous Fredholm integral equation
   b) Volterra integral equation
   c) both Fredholm and Volterra integral equation
   d) Neither Fredholm nor Volterra equation

4) Which of the following is not a symmetric kernel?
   a) $K(x,t) = x + t$ b) $K(x,t) = \cos(x^2 - t)$
   c) $K(x,t) = e^{x^2+t^2}$ d) $K(x,t) = \log(2x + 2t)$

5) A Volterra integral equation can be solved using Laplace transform if the kernel is _____.
   a) symmetric b) separable
   c) convolution type d) positive

6) The second iterated kernel for $K(x,t) = \frac{1+x}{1+t}$ of a Volterra integral equation is _____.
   a) $K_2(x,t) = \left(\frac{1+x}{1+t}\right)(x-t)$ b) $K_2(x,t) = \left(\frac{1+x}{1+t}\right)$
   c) $K_2(x,t) = \left(\frac{1+x}{1+t}\right)(x+t)$ d) $K_2(x,t) = \left(\frac{1+x}{1+t}\right)(xt)$

7) Solution of $y(x) = 1 - x + \int_0^x y(t)dt$ is _____.
   a) 1 b) x
   c) e^x d) None of these

8) Solution of $y(x) = 2 - \int_0^1 y(t)dt$ is _____.
   a) x b) 1
   c) -1 d) 0

9) Which of the following is a formula to find n-th iterated kernel of a Fredholm Volterra integral equation $u(x) = f(x) + \int_a^b K\,(x,t)u(t)dt$?
   a) $K_n(x,t) = \int_0^x K(x,z)K_{n-1}(z,t)dz$
   b) $K_n(x,t) = \int_a^b K_{n-1}(x,z)K(z,t)dz$
   c) $K_n(x,t) = \int_t^x K(x,z)K_{n-1}(z,t)dz$
   d) All of the above

10) $\int_0^x \int_0^x y(t)dt^2$ _____.
   a) $\int_0^x y(t)dt$
   b) $\int_0^x \frac{(x-t)^2}{2} y(t)dt$
   c) $\int_0^x \frac{(x-t)^3}{3} y(t)dt$
   d) $\int_0^x (x-t)\, y(t)dt$

B) State whether True or False. **06**

1) Eigenvalues of Fredholm integral equation are always real.
2) $y(x) = 1$ is a solution of $y(x) = \int_0^1 y(t)dt$
3) The kernel $K(x,t) = \log(xt)$ is separable.
4) If $y_n(x)$ is nth order approximation to the solution of $y(x) = f(x) + \lambda \int_a^b K(x,t)\, y(t)dt$, then its solution is given by $y(x) = f(x) + \lambda \int_a^b K(x,t)y_n(t)dt$,
5) If a BVP of order 7 has Green's function, then its 5th order derivative has jump discontinuity at $x = t$.
6) An Volterra integral equation gets converted into a boundary value problem.

Q.2 Answer the following. **16**

a) Solve: $\int_0^x F(x)\cos px\, dx = \begin{cases} 1, 0, \le p < 1 \\ 2, 1 \le p < 2 \\ 0, \quad p \ge 2 \end{cases}$

b) Convert the following differential equation into an integral equation without using substitution method.

$$y'' - \sin x\, y' + e^x y = x, y(0) = 1, y'(0) = -1$$

c) Solve: $y(x) = \lambda \int_0^{2\pi} \sin x \sin t\, y(t)dt$

d) Find the n^{th} iterated kernel for the kernel $K(x,t) = e^x \cos t; a = 0, b = \pi.$

Q.3 Answer the following.

a) Show that $y(x) = \cos 2x$ is a solution of **08**
$y(x) = \cos x + 3\int_0^\pi K(x,t)y(t)dt$
where $K(x,t) = \begin{cases} \sin x \cos t\ , & 0 \le x \le t \\ \cos x \sin t\ , & t \le x \le \pi \end{cases}$

b) Solve: $y(x) = 1 + \int_0^1 (1 + e^{x+t})y(t)dt$ **08**

Q.4 Answer the following.

a) Solve $y(x) = \cos x - x - 2 + \int_0^x (t-x)y(t)\, dt$ by iterative method. **08**

b) Find the eigenvalues and eigen functions of **08**
$y(x) = \lambda \int_0^1 K(x,t)y(t)dt$ where
$K(x,t) = \begin{cases} x(t-1), & 0 \le x \le t \\ t(x-1), & t \le x \le 1 \end{cases}$

Q.5 Answer the following.

a) Find the Green's function for $y'' = 0, y(0) = y(m) = 0$ **08**

b) Solve by the method of successive approximations: **08**

$$y(x) = 1 + \int_0^x (x-t)y(t)dt \,, y_o(x) = 1$$

Q.6 Answer the following.

a) Convert the boundary value problem $y'' + y = 0, y(0) = 1, y'(1) = 0$ into an integral equation. Also recover the boundary value problem from the integral equation obtained. **10**

b) Solve using Laplace transform; $Y(t) = t^2 + \int_0^t Y(x)\sin(t-x)dx$ **06**

Q.7 Answer the followings.

a) Find the solution of $y(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^{1}(xt + x^2t^2)y(t)dt$ using Hilbert-Schmidt theorem. **08**

b) Define Symmetric kernel. Prove that if a kernel is symmetric, then all its iterated kernel are also symmetric. **08**

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Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Operations Research (MSC15404)

Day & Date: Thursday, 21-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 10

1) Any solution to a Linear Programming Problem which also satisfies the non- negative restriction of the problem has _____.
   a) Solution b) basic solution
   c) basic feasible solution d) feasible solution

2) The right hand side constant of a constraint in a primal problem appears in the corresponding dual as _____.
   a) A coefficient in the objective function
   b) a right hand side constant of a function
   c) An input output coefficient of a left hand side constraint
   d) Coefficient variable

3) A set of feasible solution to a Linear Programming Problem is _____.
   a) Triangle b) Polygon
   c) Convex d) Square

4) If any value in X_B column of final simplex table is negative, then the solution is _____.
   a) Feasible b) Infeasible
   c) Bounded d) No solution

5) When the sum of gains of one player is equal to the sum of losses to another player in a game, this situation is known as _____.
   a) two-person game b) two-person zero-sum game
   c) non-zero-sum game d) None of these

6) If the set of feasible solutions of the system $AX = B,\ X \geq 0$, is a convex polyhedron, then at least one of the extreme points gives a/an:
   a) Unbounded solution b) Bounded but not optimal
   c) Optimal solution d) Infeasible solution

7) If at least one Δ_j is negative then the solution of linear programming problem is _____.
   a) Not optimal b) Not feasible
   c) Not bounded d) Not basic

8) A quadratic form $Q(x)$ is said to be positive semi definite if _____.
   a) $Q(x) \geq 0\ for\ all\ x\ \neq 0\ \in\ R^n$
   b) $Q(x) > 0\ for\ all\ x\ \neq 0\ \in\ R^n$
   c) $Q(x) < 0\ for\ all\ x\ \neq 0\ \in\ R^n$
   d) $Q(x) \leq 0\ for\ all\ x\ \neq 0\ \in\ R^n$

9) For a maximization problem, the objective function co-efficient for an artificial variable is _____.
   a) $+M$ b) $-M$
   c) Zero d) None of these

10) According to simplex method the slack variable assigned zero coefficients because _____.
   a) No contribution in objective function
   b) High contribution in objective function
   c) Divisors contribution in objective function
   d) Base contribution in objective function

B) Fill in the blanks. **06**

1) The method used to solve Linear Programming Problem without use of the artificial variable is called _____.
2) The coefficient of slack\surplus variables in the objective function are always assumed to be _____.
3) In a Linear Programming Problem functions to be maximized or minimized are called _____.
4) Beal's method is used to solve _____ programming problem.
5) The convex hull of X is the _____ convex set containing X.
6) The dual of dual of a given primal problem is _____.

Q.2 Answer the following **16**

a) Show that closed half space is a convex set.
b) Define the following terms:
   i) Basic feasible solution
   ii) Optimum basic feasible solution
c) Write the general rules for converting any primal into its dual.
d) Describe the algorithm of Two-phase method.

Q.3 Answer the following.

a) Solve the linear programming problem by simplex method. **08**

$$Max. Z = 7x_1 + 5x_2$$

Subject to condition,

$$x_1 + 2x_2 \leq 6$$
$$4x_1 + 3x_2 \leq 12$$
$$and\ x_1, x_2 \geq 0$$

b) State and prove that fundamental theorem of linear programming problem. **08**

Q.4 Answer the following.

a) Solve the linear programming problem by Big-M method. **08**

$$Min\ Z = 2x_1 + x_2\ \ subject\ to\ condition$$
$$3x_1 + x_2 = 3, \quad 4x_1 + 3x_2 \geq 6, \quad x_1 + 2x_2 \leq 4\ \ and\ \ x_1, x_2 \geq 0$$

b) If the k^{th} constraint of the primal is an equality then prove that the dual variable w_k is unrestricted in sign. **08**

Q.5 Answer the following.

a) Show that: The dual of dual of a given primal is the primal. **08**
b) Write the algorithm of Beale's method for solving a quadratic programming problem. **08**

Q.6 Answer the following.

a) Solve the following integer programming problem. **10**

$$Max\ Z = 3x_2 \qquad subject\ to\ condition$$
$$3x_1 + 2x_2 \leq 7,\ \ x_1 - x_2 \geq -2,\ \ x_1, x_2 \geq 0\ and\ are\ integer$$

b) Define the following quadratic form **06**

i) Positive definite

ii) Negative definite

iii) Indefinite

Q.7 Answer the following.

a) Explain the construction of Kuhn-Tucker condition for solving the quadratic programming problem. **08**

b) Solve the 3*3 game by simplex method of linear programming problem whose payoff matrix is given by, **08**

$$\begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix}$$

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Numerical Analysis (MSC15408)

Day & Date: Friday, 22-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The root of the equation $f(x) = 0$ lies in interval (a, b) if _____.
   a) $f(a)f(b) = 0$ b) $f(a)f(b) > 0$
   c) $f(a)f(b) < 0$ d) $f(a)f(b) = 1$

2) If $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ then the eigen value of A are _____.
   a) 0,0 b) 0,1
   c) 1,1 d) 1,5

3) Newton's _____ difference interpolation formula is useful for interpolation near the end of tabular values.
   a) Forward b) Backward
   c) Central d) None of these

4) Euler's method is used to solve _____.
   a) Numerical Integration b) Transcendental Equation
   c) Numerical Differentiation d) Linear Equations

5) The method of false position is also known as _____.
   a) Secant Method b) Newton-Raphson Method
   c) LU-decomposition d) Regula Falsi Method

6) Shifting operator is also called as _____ operator.
   a) Translation b) Averaging
   c) Differential d) Unit

7) Taking $x = 0, x = 1$ (initial guesses) the value of x alter first step for the equation $x = e$ using Regula-falsi method is _____.
   a) 0.613 b) 0.143
   c) 0 d) 1.234

8) What is a root correct to three decimal places of the equation $x^3 - 3x - 5 = 0$ by Using Newton-Raphson method?
   a) 2.279 b) 2.222
   c) 2.345 d) 2.275

9) If approximate solution of the set of equations, $2x + 2y - z = 6$, $x + y + 2z = 8$ and $-x + 3y + 2z = 4$, is given by $x = 2.8$ $y = 1$ and $z = 1.8$. Then, what is the exact solution?
   a) $x = 1, y = 3, z = 2$ b) $x = 2, y = 3, z = 1$
   c) $x = 3, y = 1, z = 2$ d) $x = 1, y = 2, z = 2$

10) The positive root of the equation $x^3 - 4x - 9 = 0$ using Regula Falsi method and correct to 4 decimal places is _____.
   a) 2.7065 b) 2.7123
   c) 2.7214 d) 2.0602

B) Fill in the blanks. **06**

1) The approximate value of $y(0.1)$ from $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$ is _____.
2) Rounded off value of 0.859378 to four significant figures is _____.
3) The relation between percentage error and relative error is _____.
4) The Newton Raphson method fails if $f'(x)$ is _____.
5) If A is upper triangular matrix then A^{-1} is _____.
6) An approximate value of $\frac{1}{3}$ is 0.30, then the absolute error E_A is _____.

Q.2 Answer the following **16**

a) Evaluate the sum $S = \sqrt{5} + \sqrt{7} + \sqrt{11}$ correct to three significant figures and find absolute and relative error.

b) Define the following terms with examples:
   i) Tridiagonal Matrix
   ii) Upper Triangular Matrix

c) Write a note on absolute error, relative error and percentage error.

d) Construct a formula for Newton-Raphson method.

Q.3 Answer the following.

a) Find a real root of the equation $xe^x = 1$ by Bisection method, correct upto three decimal places. **08**

b) Solve the following system of equations **08**
$5x - 2y + z = 4, 7x + y - 5z = 8, 3x + 7y + 4z = 10$ by using Gauss elimination method.

Q.4 Answer the following.

a) Solve the following system of equations **08**
$2x + 3y + z = 9, x + 2y + 3z = 6, 3x + y + 2z = 8$ by using LU decomposition.

b) Solve the following system of equations **08**
$6x + y + z = 20, x + 4y - z = 6, x - y + 5z = 7$
by using Gauss-Seidal method.

Q.5 Answer the following.

a) Find all the eigen values and eigen vectors of the matrix. $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$. **10**

b) If $y'' - xy' - y = 0$ be a differential equation with initial conditions $y(0) = 1$ and $y'(0) = 0$ then find the value of $y(0.1)$ using Taylors series. **06**

Q.6 Answer the following.

a) Solve $10\frac{dy}{dx} = x^2 + y^2, y(0) = 1$ for the interval $0 \le x \le 0.4$ $with$ $h = 0.1$ by using Runge-Kutta method. **10**

b) Write a note on Euler's method. **06**

Q.7 Answer the following.

a) Reduce the matrix $A = \begin{bmatrix} 1 & 3 & 4 \\ 3 & 1 & 2 \\ 4 & 2 & 1 \end{bmatrix}$ to the tridiagonal form. **08**

b) Explain the convergence of Secant method. **08**

SLR-EO-30

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2023
MATHEMATICS
Probability Theory (MSC15410)

Day & Date: Friday, 22-12-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) If $\{A_n\}$ is decreasing sequence of sets, then it converges to ____.
   a) $lim\ inf\ A_n$ b) $lim\ sup\ A_n$
   c) both (a) and (b) d) None of the above

2) If for two independent events A and B, $P(A) = 0.3$, $P(B) = 0.1$, then $P(AUB) =$ _____.
   a) 0.68 b) 0.37
   c) 0.40 d) None of these

3) Which of the following is the weakest mode of convergence?
   a) convergence in r^{th} mean
   b) convergence in probability
   c) convergence in distribution
   d) convergence in almost sure

4) If events A and B are independent events, then which of the following is correct?
   a) $P(A \cap B) = P(A) + P(B)$
   b) $P(A \cup B) = P(A) + P(B) - P(A) * P(B)$
   c) $P(A \cup B) = P(A) * P(B)$
   d) $P(A \cap B) = P(A) - P(B)$

5) If F_1 and F_2 are two fields defined on subsets of Ω, then which of the following is/are always a field?
   a) $F_1 \cup F_2$ b) $F_1 \cap F_2$
   c) both (a) and (b) d) neither (a) nor (b)

6) A class Ƒ is said to be closed under finite intersection, if $A, B \in$ Ƒ implies ______.
   a) $A \cap B \in$ Ƒ, for all $A, B \in$ Ƒ
   b) $A^c \in$ Ƒ , $B^C \in$ Ƒ
   c) both (a) and (b)
   d) None of these

7) Lebesgue measure of a singleton set $\{k\}$ is _____.
   a) 0 b) 1
   c) k d) None of these

8) The sequence of sets $\{A_n\}$, where $A_n = (0,2 + \frac{1}{n})$ converges to _____.
   a) $(0,2)$ b) $(0,2]$
   c) $[0,3)$ d) $[0,2]$

9) The $\sigma -$ field generated by the intervals of the type $(-\infty, x), x \in R$ is called _____.
   a) Standard $\sigma -$ field
   b) Borel $\sigma -$ field
   c) Closed $\sigma -$ field
   d) None of these

10) Indicator function is a ______.
   a) Simple function
   b) Elementary function
   c) Arbitrary function
   d) All of these

B) Fill in the blanks. **06**

1) A well-defined collection of sets is called as _____,
2) If $F(.)$ is a distribution function for some random variable, then $\lim_{x \to \infty} F(x) =$ _____.
3) If P is a probability measure defined on ($\Omega, \mathcal{A}$), then $P(\Omega)$ = _____.
4) If $A \subset B$, then $P(A)$ _____ $P(B)$.
5) The convergence in _____ is also called as a weak convergence.
6) Expectation of a random variable X exists, if and only if ____ exists.

Q.2 Answer the following **16**

a) Prove that inverse mapping preserves all set relations.
b) Write a note on Lebesgue measure.
c) Prove or disprove: Arbitrary union of fields is a field.
d) Write a note on characteristic function of a random variable.

Q.3 Answer the following.

a) State and prove monotone convergence theorem. **08**
b) Prove that probability measure is a continuous measure. **08**

Q.4 Answer the following.

a) Prove that collection of sets whose inverse images belong to a $\sigma -$ field, is a also a $\sigma -$ field. **08**
b) Prove that an arbitrary random variable can be expressed as a limit of sequence of simple random variables. **08**

Q.5 Answer the following.

a) Define, explain and illustrate the concept of limit superior and limit inferior of a sequence of sets. **08**
b) Prove that inverse image of $\sigma -$ field is also a $\sigma -$ field. **08**

Q.6 Answer the following.

a) Prove or disprove: **08**
   i) Convergence in distribution implies convergence in probability
   ii) Convergence in probability implies convergence in distribution
b) Define expectation of simple random variable. If X and Y are simple random variables, prove the following: **08**
   i) $E(X+Y) = E(X) + E(Y)$
   ii) $E(cX) = c\,E(X)$, where c is a real number
   iii) $If\ X > 0$ a.s., then $E(X) > 0$.

Q.7 Answer the following.

a) Prove that expectation of a random variable X exists, if and only if $E|X|$ exists. **08**
b) State and prove Borel-Cantelli lemma. **08**