SLR-MM – 418

Seat No.	

M.Sc. (Part – I) (Sem. – I) Examination, 2015
MATHEMATICS (New – CBCS)
Paper – I : Object Oriented Programming Using C++

Day and Date : Monday, 16-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternatives : **10**

1) How we can access data members using objects ?

A) Object@datamember B) Object*datamember

C) Object.datamember D) Object->datamember

2) Which of the following is not a type of constructor ?

A) Copy constructor B) Friend constructor

C) Default constructor D) Parameterized constructor

3) Which of the following concepts is used to implement late binding ?

A) Virtual function B) Operator function

C) Const. function D) Static function

4) Which of the following problem causes an exception ?

A) Missing semicolon in statement in main ()

B) A problem in calling function

C) A syntax error

D) A run-time error

P.T.O.

5) Which of the following is correct about class and structure ?

A) Class can have member functions while structure cannot

B) Class data members are public by default while that of structure are private

C) Pointer to structure or classes cannot be declared

D) Class data members are private by default while that of structure are public by default

6) Which of the following functions are performed by a constructor ?

A) Construct a new class B) Initialize objects

C) Construct a new function D) Construct a new object

7) Which of the following operators cannot be overloaded ?

A) [] B) -> C) ?: D) *

8) Which of the following cannot be used with the keyword virtual ?

A) Class B) Member functions

C) Constructor D) Destructor

9) I) All operators can be overloaded in C++.

II) We can change the basic mening of an operator in C++.

A) Only I is true B) Both I and II are false

C) Only II is true D) Both I and II are true

10) Which of the following is not a type of inheritance ?

A) Multiple B) Multilevel

C) Distributive D) Hierarchical

B) State whether the following statements are **True** or **False** : **4**

1) A constructor has the different name as that of a class.

2) A class can inherit properties from more than one class which is known as multilevel inheritance.

3) A static function can have access to only other static member (functions or variables) declared in the same class.

4) An inline function is a function that is expanded in line when it is invoked.

2. A) Attempt the following questions : **8**

   i) What are the basic concepts of OOPs ? Explain in short.

   ii) What do you mean by function prototyping ? Explain with example.

   B) Write a short note on following : **6**

   i) Structure and class

   ii) Algorithm.

3. Attempt the following questions : **14**

   A) What is difference between call by reference and return by reference ? Explain with suitable example.

   B) What is constructor ? Explain copy constructor with example.

4. Attempt the following questions : **14**

   A) What is operator overloading ? Explain the syntax for overloading operator with example.

   B) Write a C++ program to implement multiple constructor.

5. Attempt the following questions : **14**

   A) What is inheritance ? Explain different types of inheritances with syntax and example.

   B) Write a program in C++ to demonstrate the use of class and object.

6. Attempt the following questions : **14**

   A) What is Template ? Explain class template with syntax and example.

   B) Write a C++ program to create the class shape, and overload the function to return the perimeters of the different shapes.

   (e.g. Triangle P = s1 + s2 + s3, Square P = 4*b, Rectangle P = 2*l + 2*W)

7. Attempt the following questions : **14**

   A) What is manipulator ? Explain width (), precision (), and fill () manipulators with example.

   B) Explain the use of dynamic memory allocation operators new and delete with example.

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (New CBCS)
Algebra – I (Paper No. – II)

Day and Date : Wednesday, 18-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Figures to the **right** indicates **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks (**one** mark **each**) : **7**

1) The Class equation of group of order 7 is ______________
2) In a commutative ring with unity the associate of 0 is ______________
3) Let f(x) be an irreducible polynomial in F[x] then < f(x)> is ______________
4) Let M_1 and M_2 be any two R-modules. A module homomorphism f from M_1 into M_2 is one iff ______________
5) If G contains only one sylow p-subgroup then G is ______________
6) If F is a field then every ideal in F[x] is a ______________
7) Every group of order 15 is ______________

B) State **true** or **false** (**one** mark **each**) : **7**

1) Union of sub-modules of an R-module M is always a sub-module of M.
2) If G is an abelian group and H is subgroup of G then N[H] = H.
3) Every group of prime order is solvable.
4) x-2 is irreducible over Q.
5) 14 is irreducible element in Z.
6) The field /p is prime field for each prime integer p.
7) Existence of gcd for any pair (a, b) in a commutative ring is compulsory.

P.T.O.

2. a) Define module and illustrate with one example. **3**

b) Prove that A group G is a G-set under conjugation. **4**

c) Prove that field is PID. **3**

d) Prove or disprove $x^2 - 2$ is irreducible over Q using Eisensteins criteria. **4**

3. a) State and prove Burnside theorem. **8**

b) Show that symmetric group P_4 of degree 4 is solvable. **6**

4. a) Let X be any G-set then prove that |xG| = (G : Gx) for any $x \in X$. **7**

b) For the polynomial $f(x) = x^4 + 3x^3 + 2x + 4$ in $Z_5[x]$. Show that $f(x) = (x - 1)^3 (x + 1)$. **7**

5. a) If D is unique factorization domain then prove that D[x] is unique factorization domain. **10**

b) Prove that Ring of integers is an Euclidean domain. **4**

6. a) Define sub-module. If A and B are two sub-modules of an R-module M then prove that $A \cap B$. Is also a sub-module of M. **7**

b) Prove or disprove $x^3 + 3x^2 - 8$ is irreducible over Q. **7**

7. a) If $N \trianglelefteq G$ then prove that derived subgroup of N is also a normal subgroup of G. **7**

b) Check whether Group of order 60 is simple or not. **7**

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M.Sc. – I (Semester – I) (New-CBCS) Examination, 2015
Paper – III : MATHEMATICS
Real Analysis – I

Day and Date : Friday, 20-11-2015 Total Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Figures to the **right** indicates **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. No. **3** to **7**.*

1. A) Fill in the blanks (**One** mark **each**) : **7**

   1) In first mean value theorem, the condition of ______________ is necessary for the function to assume its mean value in the given interval.
   2) The total derivative of a ______________ function is function itself.
   3) A function whose second derivative is positive will be ______________
   4) ______________ is sufficient condition of integrability.
   5) A partition P* is said to be a refinement of P if ______________
   6) A function can have a finite directional derivatives $f'(c, u)$ for every u but may fail to be ______________ at c.
   7) Second derivative of a function f measures a ______________ of a graph.

   B) State **true** or **false** (**one** mark **each**) : **7**

   1) Every monotonic function on closed interval is integrable.
   2) Functions possessing primitive are necessarily continuous.
   3) A function f have a local extremum at an interior point c of an interval if $f'(c) = 0$.
   4) The total derivative of a constant function is function itself.

P.T.O.

5) If f is a real valued function then Jacobian matrix contains n rows.

6) The bounded function f having only one point of discontinuity is integrable.

7) If directional derivatives exist in every direction then partial derivatives exist.

2. a) Show that $\int_0^2 x^2 \, dx^2 = 8$. **3**

b) Define Upper and Lower integral. **3**

c) Check whether the equation $\int_a^b f'(x)\, dx = f(b) - f(a)$ is valid for $f(x) = \sqrt{x}$ in [0, 1]. **4**

d) If f is bounded and integrable on [a, b] and k is a number such that $|f(x)| \le k$ for all $x \in [a, b]$ then prove that $\left|\int_a^b f dx\right| \le k\,|b - a|$. **4**

3. a) State and prove Darboux's theorem. **7**

b) If a function f is bounded and integrable on [a, b] then prove that the function F defined as $F(x) = \int_a^x f(t)\, dt;\ a \le x \le b$ is continuous on [a, b]. Furthermore if f is continuous at a point c of [a, b] then prove that F is derivable at c and $F'(c) = f(c)$. **7**

4. a) Let B = B (a; r) be an n-ball in R^n and $\partial B = \{x / \|x - a\| = r\}$ and $\overline{B} = B \cup \partial B$ denote its closure. Let $f = (f_1, f_2, f_3 \, ... \, f_n)$ be continuous on $\overline{B}$ and assume that all the partial derivatives $D_j f_i(x)$ exist if $x \in B$. Assume further that $f(x) \ne f(a)$ if $x \in \partial B$ and that the Jacobian $J_f(x) \ne 0$ for each x in B. Then prove that f (B) the image of B under f contains an n-ball with center at f (a). **7**

b) Show that 6x + 7 is integrable on [1, 2] and find $\int_1^2 (6x + 7)\, dx$. **7**

5. a) If a function f is monotonic on [a, b] then prove that it is integrable on [a, b]. **7**

   b) State and prove Bonnett's theorem. **7**

6. a) Prove that : A function f is integrable with respect to α on [a, b] iff for every $\in > 0$ there exists a partition P of [a, b] such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$. **7**

   b) State and prove Taylors formula for functions from R^n to R^1. **7**

7. a) For some integer $n \geq 1$, let f have a continuous nth derivative in the open interval (a, b). Suppose that for some interior point c in (a, b) we have $f'(c) = f''(c) = f'''(c) = ... = f^{(n-1)}(c) = 0$ but $f^n(c) \neq 0$ then prove that for n even f has local minimum at c if $f^n(c) > 0$ and f has local maximum at c if $f^n(c) < 0$. If n is odd, there is neither a local maximum nor local minimum. **7**

   b) Prove that the function f defined on [0, 1] as f (x) = 2n, if $x = \frac{1}{n}$ where n = 1, 2 , ... = 0, otherwise is not Riemann integrable. **7**

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Seat No.

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (Paper – IV) (New CBCS)
Differential Equation

Day and Date : Monday, 23-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : 1) *Question no.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) *Attempt* ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) *Figures to the* ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative (**one** mark **each**) : **5**

1) If ϕ_1, ϕ_2 are two solutions of $L(y) = y'' + a_1y' + a_2y = 0$ then $W(\phi_1, \phi_2)(x) =$

a) $\phi_1\phi_2' - \phi_1'\phi_2$ b) $e^{-a_1(x - x_0)}$

c) (a) or (b) d) (a) and (b)

2) $x^2y'' + xy' + (x^2 - n^2)y = 0$ is

a) Euler equation b) Bessel equation

c) Legendre equation d) Wave equation

3) If r_1 is a root of multiplicity m of characteristic polynomial p(r) of n^{th} order LDE with constant coefficient then $p(r_1) = p'(r_1) = \ldots p^{(m-1)}(r_1) =$

a) 0 b) 1 c) n d) nr

4) Two functions $\phi_1(x) = x$, $\phi_2(x) = |x|$ for $-\infty < x < \infty$ are

a) Linearly dependent b) Linearly independent

c) Both (a) and (b) d) None of these

5) $\sum_{m=0}^{\infty} \frac{(-1)^m}{(m!)^2}\left(\frac{x}{2}\right)^{2m}$ is

a) Bessel function of zero order of second kind

b) Bessel function of zero order of first kind

c) Bessel function of order α of the first kind

d) Bessel function of order 3 of the first kind

P.T.O.

B) Fill in the blanks (**one** mark **each**) : **7**

1) The singular point of $x^2y'' - 5y' + 3x^2y = 0$ is ________________. It is __________ singular point.

2) The two solutions of $y'' - 16y = 0$ are $\phi_1(x) =$ ________ and $\phi_2(x) =$ __________

3) On an interval I containing x_0 there exists ____________ solution ϕ of the initial value problem $L(y) = 0,\ y(x_0) = \alpha,\ y'(x_0) = \beta$.

4) The functions 1, x, x^3 are linearly ____________

5) $\|\phi(x)\|$ is just a magnitude or length of vector with components __________ and ____________

6) The Euler equation is ___________

7) Lipschitz condition for f(x, y) on set S is ___________

C) State **true** or **false** (**one** mark **each**) : **2**

1) Solution of $y'' + w^2y$ is $c_1 \cos wx + c_2 \sin wx$.

2) If ϕ_1, ϕ_2 are linearly dependent solutions of L(y) = 0 on an interval I they are linearly dependent on any interval I contained inside I.

2\. a) Let $\alpha \pm i\beta$ be the roots of characteristic polynomial of constant coefficient equation $y'' + a_1y' + a_2y = 0$. Show that $e^{\alpha x} \cos\beta x$, $e^{\alpha x} \sin\beta x$ are solution of $y'' + a_1y' + a_2y = 0$. **4**

b) Give the geometrical interpretation of **4**

$$\|\phi(x_0)\| e^{-k|x - x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{k|x - x_0|}$$

c) Prove that $c_1\phi_1 + c_2\phi_2$ is solution of $L(y) = y'' + a_1y' + a_2y = 0$ if ϕ_1, ϕ_2 are solution of L(y) = 0. **3**

d) Find the general solution of $y'' + 4ky' - 12k^2y = 0$. **3**

3\. a) Solve $y''' - y' = x$. **7**

b) Derive Bessel function of zero order of the first kind. **7**

4. a) State and prove uniqueness theorem for second order linear differential equation. **7**

b) Solve $y^{(4)} - 16y = 0$. **7**

5. a) Prove that particular solution of $L(y) = y'' + a_1 y' + a_2 y = b(x)$ is

$$\int_{x_0}^{x} \frac{[\phi_1(t)\,\phi_2(x) - \phi_1(x)\,\phi_2(t)] \cdot bt}{W(\phi_1, \phi_2)(t)}\, dt$$

where b be continuous on I and ϕ_1, ϕ_2 are two solutions of L(y) = 0. **8**

b) Show that the function $f(x, y) = 4x^2 + y^2$ satisfy Lipschitz condition on $S : |x| \le 1$, $|y| \le 1$. **6**

6. a) Let ϕ_1, ϕ_2 be two solutions of L(y) = 0 on an interval I. Prove that every solution ϕ of L(y) = 0 can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$. Where c_1, c_2 are constants. **7**

b) Find that solution ϕ satisfying $y''' - 4y' = 0$, $\phi(0) = 0$ $\phi'(0) = 1$, $\phi''(0) = 0$. **7**

7. a) Prove that ϕ is solution of $y' = f(x, y)$, $y(x_0) = y_0$ on an internal I if and only if it is a solution of $y = y_0 + \int_{x_0}^{x} f(t \cdot y)\, dt$. **7**

b) Find all solutions of $y'' + 4y = \cos x$. **7**

Seat No.	

M.Sc. I (Semester – I) Examination, 2015
MATHEMATICS (New – CBCS)
Classical Mechanics (Paper – V)

Day and Date : Thursday, 26-11-2015 Max.Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*

*2) Attempt **any three** questions from Q. **3** to Q. **7**.*

*3) Figures to the **right** indicates **full** marks.*

1. Fill in the blanks (**one** mark **each**) : **14**

1) Degrees of freedom of a simple pendulum with variable length is/are _______

2) Number of types of motions involving in rigid body motion is/are _______

3) Methods of calculus of variations are useful to find _______

4) Potential energy of any system is independent of generalized _______

5) An expression which represents constraints having equity is called _______

6) If the Lagrangian L is not depend on a particular generalized co-ordinate q then q is called _______

7) In a Lagrangian $L(q_1, q_2, q_3, q_4, \dot{q}_1, \dot{q}_2, \dot{q}_3, \dot{q}_4, t)$, q_1 and q_2 are ignorable then number of Routh's equations are _______

8) Order and degree of Lagrange's equations of motion are _______

9) Number of generalized co-ordinates to describe a rigid body motion is/are _______

10) If $\bar{r} = \bar{r}(a_j, t)$ then the expression $\sum_{j=1}^{k} \frac{\partial \bar{r}}{\partial q_j} \dot{q}_j + \frac{\partial \bar{r}}{\partial t}$ represents _______

11) $\frac{\partial L}{\partial \dot{q}_j}$ is called as _______

P.T.O.

12) Generalized force Q_j ________

13) If $H = \frac{1}{2} ml^2 (P_\theta)^2 + mgl\cos\theta$ then $\dot{P}_\theta$ ________

14) A curve with fixed perimeter which encloses maximum area is a ________

2. a) State D'Alembert's principle. **(3+4+3+4)**

   b) Find generalized potential of a dynamical system.

   c) Determine generalized co-ordinates for rigid body motion.

   d) Find Euler's equation for the functional $I = \int_a^b F(x, y')dx$.

3. a) State and prove principle of least action. **10**

   b) Find the extremal of function $I[y(x)] = \int_1^2 \frac{x^3}{y'^2} dx$. **4**

4. a) Find the kinetic energy of a single particle moving in a polar plane. **6**

   b) Find Lagrange's equations of motion of a spherical pendulum. **8**

5. a) Find a necessary condition for the functional $I = \int_{x_1}^{x_2} F(x, y_1, y_2, \ldots, y_n, y_1', y_2', \ldots, y_n')dx$ is to be extremum. **8**

   b) Find the extremal of the functional $I[y(x)] = \int_0^1 \left[(y')^2 + 12xy\right]dx$, y(0) = 0 and y(1) = 1. **6**

6. a) Find Hamiltonian of a particle moving on the surface of the sphere. **6**

   b) Find components of angular velocity along the space set of axes. **8**

7. a) Deduce energy equation from the Lagrange's equations of motion for conservative field. **7**

   b) Prove that successive transformation of an orthogonal matrix is also orthogonal. **7**

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Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (Old CGPA)
Object Oriented Programming using C++ (Paper – I)

Day and Date : Monday, 16-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Question No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternatives : **10**

1) An object is ____________
   A) A variable of class data type
   B) Same as a class
   C) Just like a global variable
   D) Collection of data-members and member functions

2) Wrapping up of data and functions together in a class is known as ________
   A) Overloading B) Data Abstraction
   C) Polymorphism D) Encapsulation

3) Which of the following is not a type of constructor ?
   A) Copy constructor
   B) Friend constructor
   C) Default constructor
   D) Parameterized constructor

4) The mechanism of deriving a new class from base class is known as ____________
   A) Polymorphism B) Encapsulation
   C) Overloading D) Inheritance

5) Which of the following can replace a simple if-else construct ?
   A) Ternary operator B) While loop
   C) Do-while loop D) For loop

6) Which of following concepts means waiting until runtime to determine which function to call ?

A) Dynamic casting B) Data hiding

C) Data binding D) Dynamic loading

7) Which of the following operator is overloaded for object count ?

A) >> B) << C) ?: D) +

8) Which of the following cannot be used with the keyword virtual ?

A) Constructor B) Member function

C) Class D) Destructor

9) Which of the following operators cannot be overloaded ?

A) [ ] B) – > C) ?: D) *

10) The ability to take more than one form is known as ___________

A) Polymorphism B) Encapsulation

C) Constructor D) Inheritance

B) State whether following statements are **true** or **false** : **4**

1) A static class function can be invoked by simply using the name of the function alone.

2) Members declared as private in a class are accessible to all member functions of that class.

3) Inheritance provides the idea of reusability.

4) The mechanism of deriving class from another derived class is known as multiple inheritance.

2. A) Write a short note on following : **8**

i) Flowchart

ii) Default arguments.

B) Answer the following : **6**

i) Explain the use of Scope Resolution Operator with example.

ii) What do you mean by user defined data type ? Explain in short.

3. Answer the following :

A) What is Friend Function ? Explain with example. **7**

B) What is constructor ? Explain multiple constructor with example. **7**

4. Answer the following :

   A) Write a program to implement Arrays of 5 Objects of class named 'STUDENT' which should include two member functions input ( ) and display ( ) to read the student details (Name, Roll_no, Marks) and display ( ) to display the details of these students. **7**

   B) What is Function Overloading ? Explain with suitable example. **7**

5. Answer the following :

   A) Write a C++ program to implement single inheritance. **7**

   B) Explain the importance of virtual function with its characteristics. **7**

6. Answer the following :

   A) What is Template ? Explain function template. **7**

   B) What is manipulator ? Explain the use of width ( ), precision ( ) and fill ( ) manipulators. **7**

7. Answer the following :

   A) What is File ? Explain the different methods for opening the file. **7**

   B) Write a program to swap two number (integer and float numbers) by using Function overloading concept. **7**

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
MATHEMATICS (Old) (CGPA)
Algebra – I (Paper – II)

Day and Date : Wednesday, 18-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct answer (**one** mark **each**) : **4**

1) Let $x^2 = x$ for all $x \in R$, where R is a ring then

I) Char. of R = 2

II) R is commutative

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

2) If G is a finite group of order n and $a \in G$ then

a) $a^n \neq e$ b) $a^n = e$ c) $a^2 = a$ d) $a = \frac{1}{a}$

3) Consider the two statements

I) Every finite group has composition series

II) The group of integers z has a composition series

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

4) For a ring R consider the two statements.

I) If R is a commutative, than R[x] is commutative

II) If R[x] is a commutative, then R is not commutative

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

B) State whether the following statements are **true** or **false** (**one** mark **each**) : **5**

1) Any finite group of prime order is a simple.
2) The group S_3 has no a normal series.
3) Product of two primitive polynomials in F[x] is always a constant polynomial.
4) The polynomial $f(x) = x^{P-1} + x^{P-2} + ... + x + 1$ is irreducible over Q for any prime number P.
5) $\frac{z_5(x)}{\langle x^2 - 2 \rangle}$ is a field.

C) Fill in the blanks (**one** mark **each**) : **5**

1) Two elements $a, b \in D$ are associates in D if __________
2) Field contains only two ideals ______________
3) Let <G, .> be a group, S be any non-empty subset of G then normalizer of S is N(S) = ___________
4) An ideal N of R is prime if and only if $\frac{R}{N}$ is ________________
5) A nonzero element p that is not a unit of an integral domain D is an irreducible of D if in any factorization P = a.b in D ______________

2. a) Let $f(x) = x^3 + 2x + 3 \in z_5[x]$. Find all the zeros of f(x) in z_5.
b) Prove that a subgroup of a nilpotent group is nilpotent.
c) Let X be any G-set. Then prove that $|xG| = (G : Gx)$, for any $x \in X$.
d) If G is a finite group of order P^n, where P is a prime number and n > 0. Then show that G has a non trivial center. **(3+3+4+4)**

3. a) If H is a subgroup of a group G and N is a normal subgroup of a group. Then prove that $\frac{HN}{N} \cong \frac{H}{H \cap N}$.
b) State and prove Schreier's theorem. **(7+7)**

4. a) State and prove Cauchy theorem.

   b) Let G be a finite group and $P/|G|$, (P is any prime number). Let r = number of sylow P – subgroups in G. Then prove that

      i) $r \equiv 1 \pmod{p}$

      ii) $r \mid IG \mid$. **(8+6)**

5. a) If $|G| = 255$ then prove that G is abelian and not simple.

   b) Let F be a field, then prove that F(x) is a VFD. **(6+8)**

6. a) Let F be a field, $f(x) \in F[x]$ and degree of f(x) = n. Show that

      i) $a \in F$ is a zero of f(x) if and only if x – a) is a factor of f (x) in F [x].

      ii) f(x) has atmost n-zeros.

   b) Show that a commutative ring with unity is a field if and only if it has no proper nontrivial ideals. **(8+6)**

7. a) Prove that an ideal <P> in a PID is marimal if and only if P is an irreducible.

   b) Prove that if D if UFD, then D[x] is a UFD. **(7+7)**

SLR-MM – 425

Seat No.	

M.Sc. – I (Semester – I) (Old) (CGPA) Examination, 2015
MATHEMATICS (Paper – III)
Real Analysis – I

Day and Date : Friday, 20-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :

   1) The ____________ sum becomes smaller and smaller as partition becomes finer and finer.

   2) The usual Riemann integral occurs as the special case of Riemann Stieltjes integral in which ____________.

   3) The partition P' of [a, b] is said to be ____________ than P if $P \subseteq P'$.

   4) If f is real valued function then total derivative of f is ____________.

   5) Existence of total derivative of a function at a point implies ____________ at that point.

   6) The statement $\int_a^b f\,dx$ exists implies that function f is ____________ and ____________ over [a, b].

   7) The quantity $f'(C).h$ in Taylor's formula is a ____________ of h.

   B) State **True** or **False** (**one** mark **each**) :

   1) Every open connected set is polygonally connected.

   2) Every bounded function possesses primitive.

   3) The total derivative of non-linear function is function itself.

P.T.O.

4) For any function f(x, y), $D_{12}f(x, y) = D_{21} f(x, y)$.

5) If P* is a refinement of P then, $L(P^*, f, f) \leq L(P, f, \alpha)$.

6) Every closed and bounded set in R^n is compact.

7) Every bounded function is Riemann integrable.

2. a) For any two partitions P_1 and P_2 prove that $L(P_1, f) \leq U(P_2, f)$. **14**

b) Show that : The total derivative of a linear function is a function itself.

c) Define :

i) Primitive of a function

ii) Mesh of partition

iii) Riemann sum of F.

d) If f is bounded and integrable on [a, b] and $f(x) \geq 0 \ \forall x \in [a, b]$ then prove that, $\int_a^b f\,dx \geq 0$ if $b \geq a$.

3. a) Prove that : A necessary and sufficient condition for the integrability of a bounded function f is that to every $\in > 0$ there corresponds $\delta > 0$ such that for every partition P of [a, b] with norm $\mu(P) < \delta$ U(P, f) – L(P, f) < $\in$. **8**

b) Compute $\int_{-1}^{1} f(x)\,dx$ where $f(x) = |x|$. **6**

4. a) Find the directional derivative of f(x, y) = e^x siny at point $P(0, \pi/4)$ in the direction V = (1, –1). **6**

b) Assume that g is differentiable at a with total derivative $g'(a)$. Let b = g(a) and assume that f is differentiable at b with total derivative $f'(b)$. Then prove that the composite function h = f$\circ$g is differentiable at a and total derivative $h'(a)$ is given by $h'(a) = f'(b) \circ g'(a)$ the composition of the linear function $f'(b)$ and $g'(a)$. **8**

5. a) If f is differentiable at c then prove that f is continuous at c. **6**

   b) Let f = (f_1, f_2,...f_n) be a vector valued function defined on an open set S in R^{n+k} with values in R^n. Suppose $f \in C'$ on S. Let (x_0, t_0) be a point in S for which f(x_0, t_0) = 0 and for which the $n \times n$ determinant [Djfi (x_0; t_0)]$\neq$ 0. Then prove that there exists a K dimensional open set to containing to and one and only one vector valued function g defined on T_0 and having values in R^n such that

   i) $g \in C'$ on T_0

   ii) $g(t_0) = x_0$

   iii) f(g(t), t) = 0 for every t in T_0. **8**

6. a) If f_1, f_2 are two bounded and integrable functions on [a, b] and there exists a number $\lambda > 0$ such that $|f_2(x)| \geq \lambda \; \forall x \in [a, b]$ then prove that f_1/f_2 is bounded and integrable on [a, b]. **7**

   b) Show that x^2 is integrable on any interval [0, k]. **7**

7. a) If P* is refinement of P then prove that for a bounded function f,

   i) L(P*, f) $\geq$ L (P, f)

   ii) U (P*, f) $\leq$ U(P, f). **8**

   b) Verify first mean value theorem in [–1, 1] for f(x) = x. **6**

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Seat No.	

M.Sc. – I (Sem. – I) (Old) (CGPA) Examination, 2015
MATHEMATICS
Differential Equations (Paper No. – IV)

Day and Date : Monday, 23-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

*N. B. : 1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) Fill in the blanks **(one** mark **each)**. **7**

1) Consider the inequality $\|\phi(x_0)\| e^{-K|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{K|x-x_0|}$ geometrically the inequality says that $||\phi(x)||$ always remains between two curves y = ____________ and y = ___________

2) The initial value problem for L(y) = 0 is a problem of finding a solution ϕ satisfying _________ and __________

3) If ϕ_1 is a solution of L(y) = 0 the n^{th} order linear differential equation with variable co-efficients then we can reduce the order of differential equation by __________

4) The function g is analytic at x_0 if g can be expressed in power series about x_0 which has ___________ radius of convergence.

5) An equation of the form $C_0(x)(x - x_0)^n y^{(n)} + C_1(x)(x - x_0)^{n-1} y^{(n-1)} + C_2(x)(x - x_0)^{n-2} y^{(n-2)} + ... + C_n(x) y = 0$ has a regular singular point at x_0 if $C_0, C_1, ...C_n$ are ___________ at $x = x_0$ and $C_0(x_0)$ is __________

6) The expression for Legendre's polynomial is $P_n(x)$ ____________

7) The indicial polynomial of Eulers equation of order two is given by q(r) = ____________

B) Choose correct alternative **(one** mark **each)**. **3**

1) Bessel's function of zero order of first kind is given by $J_0(x)$ = ___________

a) $\sum_{m=0}^{\infty} \frac{(-1)^m}{2^{2m}} \frac{x^{2m}}{(m!)^2}$

b) $\sum_{m=0}^{\infty} \frac{(-1)^m}{2^{m}} \frac{x^{m}}{(m!)^2}$

c) $\sum_{m=0}^{\infty} \frac{(-1)^m}{2^{2m}} \frac{x^{m}}{(m!)^2}$

d) $\sum_{m=0}^{\infty} \frac{(-1)^m}{2^{m}} \frac{x^{2m}}{(m!)^2}$

2) The solution of Legendre's equation have convergent power series expansions on ___________

a) $|x| > 1$ b) $|x| < 1$ c) $|x| > 0$ d) $|x| < 0$

3) A polynomial solution P_n of degree n of $(1 - x^2) y'' - 2x y' + n(n + 1)y = 0$ satisfying _________ is called Legendre's polynomial.

a) $P_n(1) = 0$ b) $P_n(0) = 0$ c) $P_n(0) = 1$ d) $P_n(1) = 1$

C) State **true** or **false**. **4**

1) $x = 0$ is regular singular point of Bessel's equation.

2) If ϕ_1, ϕ_2 are linearly independent functions on an interval I, they are linearly independent on any interval J contained inside I.

3) There exists n linearly independent solutions of n^{th} order linear differential equation.

4) If ϕ_1, ϕ_2 are two solutions of 2^{nd} order LDE with constant co-efficients then $\phi_1 - \phi_2$ is also a solution.

2. a) Find the solution of, $y''' - i \cdot y'' + 4y' - 4iy = 0$. **3**

b) Find the Wronskian of, $\phi_1(x) = e^{ix}$, $\phi_2(x) = \sin x$, $\phi_3(x) = 2\cos x$. **3**

c) Prove with usual notations that $J_0'(x) = -J_1(x)$. **4**

d) Find regular singular points in a finite plane of,

$x(x - 1)^2 (x + 2) y'' + x^2 y' - (x^3 + 2x + 1)y = 0$. **4**

3. a) Show that $\phi_n(x) = \sin nx$ satisfies a boundary value problem $y'' + n^2y = 0$; $y(0) = 0$, $y(\pi) = 0$ when n = 1, 2, ... and show that $\int_0^{\pi} \sin nx \ \sin mx \, dx = 0$; $n \neq m$. **7**

   b) Define Legendre's polynomial and find its expression. **7**

4. a) If $\phi_1, \phi_2 \ldots\ldots \phi_n$ are n solutions of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y = 0$ on an interval I then prove that they are linearly independent iff $W(\phi_1, \phi_2 \ldots \phi_n)(x) \neq 0$ for all x in I. **7**

   b) If $\phi_1(x)$ is a solution of $L(y) = y'' + a_1(x)y' + a_2(x)y = 0$ on an interval I and $\phi_1(x) \neq 0$ on I then prove that the second solution $\phi_2(x)$ is given by,

$$\phi_2(x) = \phi_1(x) \int_{x_0}^{x} \frac{1}{[\phi_1(s)]^2} \exp\left[-\int_{x_0}^{s} a_1(t)dt\right] ds.$$ **7**

5. a) Show that there is a basis ϕ_1, ϕ_2 for the solutions of $x^2y'' + 4xy' + (2+x^2)y = 0$ $(x > 0)$ of the form. $\phi_1(x) = \frac{\psi_1(x)}{x^2}$, $\phi_2(x) = \frac{\psi_2(x)}{x^2}$. **7**

   b) If a function ϕ is the solution of initial value problem $y' = f(x, y)$; $y(x_0) = y_0$ on an interval I iff it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)dt$ on I. **7**

6. a) If ϕ_1, ϕ_2 are two solutions of $y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0 then prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x - x_0)} \cdot W(\phi_1, \phi_2)(x_0)$. **7**

   b) Find the power series solution of $y'' - xy = 0$. **7**

7. a) Find all solutions of $x^2y'' + xy' + 4y = 1$ for $x > 0$. **7**

   b) Let ϕ be any solution of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing a point x_0. Then prove that for all x in I.

$$\|\phi(x_0)\| e^{-K|x-x_0|} \leq \|\phi(x)\| \leq \|\phi(x_0)\| \cdot e^{K|x-x_0|}$$

where $\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{1/2}$ and $K = 1 + |a_1| + |a_2|$. **7**

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SLR-MM – 427

Seat No.	

M.Sc. – I (Semester – I) (Old) (CGPA) Examination, 2015
MATHEMATICS
Classical Mechanics (Paper – V)

Day and Date : Thursday, 26-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*
3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) If the constraints are rheonomic Hamiltonian H may be a constant of motion but does not represent ________

2) If the forces acting on the particle are conservative then total energy of the particle is _________

3) For cyclic co-ordinates the Routhian function R acts as ________

4) Modified Hamilton's principle states that, "The motion of dynamical system between two points is such that δ-variation in the line integral of Lagrangian is ___________ for path chosen by the system".

5) The line integral of twice the kinetic energy is called _________

6) In case of orthogonal transformation the inverse matrix is identified by its ________

7) Geodesic on the surface of sphere is the arc of __________

B) Choose correct alternative (**one** mark **each**) :

1) The determinant of orthogonal matrix is equal to

a) +1 b) –1 c) 0 d) ± 1

2) If the torque $\overline{N}$ is zero then _________ of the system is conserved.

a) Linear momentum b) Angular momentum

c) Kinetic energy d) Potential energy

3) To define transformation matrix in terms of Eulerian angles, how many rotations are required ?

a) 3 rotations

b) 2 rotations

c) 4 rotations

d) 3 successive rotations in specific sequence

C) State **true** or **false** (**one** mark **each**) :

1) Constraints are nothing but limitations

2) In Δ-variation Hamiltonian is conserved.

3) Finite rotations of rigid body are commutative.

4) When a particle moves along a closed path in a conservative field of force then work done is infinite.

2. a) Prove that A coordinate which is cyclic in Lagrangian L is also cyclic in Hamiltonian H. **3**

b) Find the plane curve of fixed perimeter and maximum area. **4**

c) Prove that two infinitesimal rotations are commutative. **4**

d) Find kinetic energy of a single particle. **3**

3. a) Prove that the Lagranges equation $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Qj$ can also be written as, $\frac{\partial \dot{T}}{\partial \dot{q}_j} - 2\frac{\partial T}{\partial q_j} = Qj$. **7**

b) Show that the curve is a catenary for which the area of the surface of revolution is minimum when revolved about y-axis. **7**

4. a) Derive Hamilton's Canonical equation of motion. **7**

   b) Obtain Euler's equation of motion of rigid body when one point of rigid body remains fixed. **7**

5. a) Show that addition of total time derivative of any function of the form $f(q_j, t)$ to the Lagrangian of a Holonomic system then the generalized momentum is given by $p_j + \frac{\partial f}{\partial q_j}$. **6**

   b) Find the Euler-Lagranges differential equation satisfied by y(x) for which the integral $I = \int_{x_1}^{x_2} f(y, y', x)dx$ has extremum value where y(x) is twice differentiable function satisfying $y(x_1) = y_1$, $y(x_2) = y_2$. **8**

6. a) State and prove basic lemma is calculous of variation. **7**

   b) Find extremal of the functional $I(y(x)) = \int_{x_0}^{x_1} \frac{1+y^2}{y'^2} dx$. **7**

7. a) If the internal and external forces acting on the system of particle are conservative then prove that the total energy of the system is conserved. **7**

   b) Prove that the product of two linear orthogonal transformations is again a linear orthogonal transformation. **7**

Seat No.	

M.Sc. – I (Semester – II) (New) (CGPA) Examination, 2015
MATHEMATICS (Paper – VI)
Algebra – II

Day and Date : Tuesday, 17-11-2015 Max. Marks :70
Time : 10.30 a.m. to 1.00 p.m.

*N.B. : 1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) Fill in the blanks **(one mark each)** :

1) A field is ______________ if every non-constant polynomial in F(x) has a zero in F.

2) A complex zeros of a polynomials with real co-efficients occur in ______________

3) If K is an extension of F and every element in K which is outside F is moved by some element in G (K, F) then K is ______________ extension.

4) The extension K of a field F is said to be simple extension if ______________

5) $\left[Q\left(\sqrt{2}+\sqrt{3}\right) : Q \right]$ = ______________

6) If [L : F] is a prime number then there can be no field K properly contained betn. ______________ and ______________

7) The fixed field of G (K, F) contains ______________

8) The non-zero elements of a field form an abelian group with respect to ______________

B) State **True** or **False (one mark each)** :

1) If [K : F] = m then each element a in K is algebraic over F and degree of a over F will be m.

2) By straight edge and compass it is possible to construct the regular Septagon.

P.T.O.

3) Ring of integers is not a field.
4) The set of algebraic number forms a field.
5) If a complex number z is a root of some polynomial p(x) then $\bar{z}$ (complex conjugate) is also a root of p(x).
6) There is a field with 10 elements.

2. a) Find the degree and basis for an extension $Q\left(2^{1/3}, w\right)$ over Q. **4**

   b) Prove with usual notations that, F (a, b) = F (b, a). **3**

   c) Construct a field with 8 elements. **3**

   d) Find the splitting field of $x^4 - x^2 - 2$ over Q. **4**

3. a) Prove that : The set of all constructible real numbers forms a subfield of field of real numbers. **7**

   b) If K is field and $\sigma_1, \sigma_2 ... \sigma_n$ are distinct automorphisms of K then show that they are linearly independent. **7**

4. a) Show that $x^5 - 9x + 3$ is not soluble by radicals. **6**

   b) If K is a finite extension of a field F then prove that G (K, F) is a finite group and its order satisfies the relation O (G (K, F)) $\leq$ [K : F]. **8**

5. a) If F is a field of characteristic O and a, b are algebraic over F then prove that F (a, b) is a simple extension of F. **7**

   b) Show that a finite field of p^n elements has exactly one subfield of p^m elements for each division m of n. **7**

6. a) Let F be a field and let f (x) $\in$ F [x] be such that $f'(x) = 0$ then prove that :

   i) If the characteristic of F = 0 then f (x) is a constant polynomial.

   ii) If the characteristic of F = P $\neq$ 0 then f (x) = g (x^p) for some polynomial g (x) $\in$ F [x]. **7**

   b) Let K be an extension of a field F then prove that the element a $\in$ K is algebraic over F iff F(a) is finite extension of F. **7**

7. a) P.T. A polynomial of degree n over a field can have at most n roots in any extension field. **7**

   b) Show that the polynomials $x^2 + 3$ and $x^2 - x + 1$ have the same splitting field over F the field of rational numbers. **7**

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Seat No.

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (New) (CGPA)
Real Analysis – II (Paper – VII)

Day and Date : Thursday, 19-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

***Instructions** : i) Q.No. **1** and **2** are **compulsory**.*
*ii) Attempt **any three** questions from Q.No. **3** to Q.No. **7**.*
*iii) Figures to the **right** indicate **full** marks.*

1. a) Fill in the blanks : **7**

i) If ϕ is empty set then $m^*(\phi) =$ ______________

ii) If f is a measurable function and f = g a.e. then g is ______________

iii) If A and B are any two sets then $\chi A \cup B =$ ______________

iv) If $\langle u_n \rangle$ is a sequence of non-negative measurable functions and if $f = \sum_{n=1}^{\infty} u_n$ then $\int f =$ ______________

v) If $T_a^b(f)$ is total variation of f over [a, b] and if $T_a^b(f) < \infty$ then the function f is of ______________

vi) The positive part f^+ of a function f is given by $f^+(x) =$ ______________

vii) The difference of two absolutely continuous functions is ______________

b) State wheather following is **True** or **False** : **7**

i) If $E \subseteq [0, 1)$ is measurable set, then for each $y \in [0, 1)$ then the set $E \dot{+} y$ is measurable.

ii) Union of countable collection of measurable sets is measurable.

iii) If f and g are two measurable real-valued function defined on same domain then f + g is not measurable.

iv) If $D^+ f(x) = D_+ f(x) = D^- f(x) = D_- f(x) \neq \infty$ then f is differentiable at x.

v) Monotone convergence theorem remains invalid if "Convergence a.e." is replaced by "convergence in measure".

vi) A function φ defined on interval (a, b) is convex if for each x, y $\in$ (a, b) and for each $0 \leq \lambda \leq 1$ such that $\varphi(\lambda x + (1-\lambda)y) \geq \lambda\varphi(x) + (1-\lambda)\varphi(y)$.

vii) If f is integrable over E and if A and B are disjoint measurable sets contained in E then $\int_{A \cup B} = \int_A f - \int_B f$.

2. a) Show that $D^+(-f(x)) = -D_+f(x)$. **3**

b) If f is absolutely continuous on [a, b] then prove that it is of bounded variation on [a, b]. **3**

c) If φ is a convex function on $(-\infty, \infty)$ and f is an integrable function on [0, 1] then prove that $\int \varphi(f(t))dt \geq \varphi\left(\int f(t)dt\right)$. **4**

d) If $\varphi = \sum_{i=1}^{n} a_i \chi E_i$, with $E_i \cap E_j = \phi$ for $i \neq j$. Suppose each set E_i is measurable set of finite measure then prove that $\int \varphi = \sum_{i=1}^{n} a_i m(E_i)$. **4**

3. a) Prove that the interval $(0, \infty)$ is measurable. **7**

b) If f is an integrable function on [a, b] and suppose that $F(x) = F(a) + \int_a^x f(t)\,dt$, then prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$. **7**

4. a) If E is a given set, then prove the following statements are equivalent : **7**

i) E is measurable

ii) There is a set G in G_δ with $E \subset G$, $m^*(G \sim E) = 0$.

iii) There is a set F in F_σ with $F \subset E$, $m^*(E \sim F) = 0$.

b) If f is a non-negative function which is integrable over a set E, then prove that for given $\epsilon > 0$ there is a $\delta > 0$ such that for every $A \subset E$ with $m(A) < \delta$ we have $\int_A f < \epsilon$. **7**

5. a) If $\langle f_n \rangle$ is a sequence of measurable functions that convergence in measure to f, then prove that there is a subsequence $\langle f_{n_k} \rangle$ that converges to f almost everywhere. **7**

b) Show that if E is measurable set, then each translate E + y of E is also measurable. **7**

6. a) If $\langle f_n \rangle$ is an increasing sequence of nonnegative measurable functions and if $f = \lim\limits_{n \to \infty} f_n$ a.e. then prove that $\int f = \lim\limits_{n \to \infty} \int f_n$. **7**

b) Prove that the outer measure of an interval is its length. **7**

7. a) If f is bounded and measurable function on [a, b] and $F(x) = \int_a^x f(t)dt + F(a)$, then prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$. **7**

b) If f is absolutely continuous function on [a, b] and if $f'(x) = 0$ a.e., then prove that f is constant. **7**

SLR-MM – 430

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS
General Topology (Paper – VIII) (New) (CGPA)

Day and Date : Saturday, 21-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions : 1) *Q.* ***1*** *and Q.* ***2*** *are* ***compulsory****.*
2) *Attempt* ***any three*** *questions from Q.* ***3*** *to Q.* ***7****.*
3) *Figures to the* ***right*** *indicate* ***full*** *marks.*

1. A) State whether followings are either **true** or **false** : **10**

1) Two topologies on X are identical if and only if they admit the same neighbourhoods.
2) Intersection of two topologies for X is not a topology for X.
3) Continuous image of first countable space is also first countable.
4) Every discrete topological space is a first countable.
5) Union of finite collection of compact subsets of a space is also compact.
6) Empty set ϕ is not connected.
7) If a function f is closed and continuous then f is a homeomorphism.
8) Any mapping $f : X \to Y$, where X is a discrete topological space is continuous.
9) A continuous mapping of a compact space into a Hausdorff space is open map.
10) Metric space is not a completely regular space.

B) Fill in the blanks : **4**

11) A topological space (X, τ) is said to be T_2 if __________
12) A mapping $f : X \to Y$ is said to be bicontinuous if and only if __________
13) Closure of $(A \cup B)$ = ____________
14) Co-finite topology on a finite set is a ____________

P.T.O.

2. a) If d(A) is a derived set of A then show that $d(A \cup B) = d(A) \cup d(B)$. **3**

   b) If A and B are separated subsets of a space X and $C \subset A$ and $D \subset B$ then show that C and D are separated. **4**

   c) Prove that every second countable space is first countable. **4**

   d) Prove that continuous image of separable space is separable. **3**

3. a) If f is a continuous mapping of a connected space onto an arbitrary topological space Y then prove that Y is connected. **6**

   b) Prove that IR is connected. **8**

4. a) If X is a compact space and Y is a Hausdorff space then prove that every bijective continuous mapping of X onto Y is a homeomorphism. **7**

   b) If τ is a co-countable topology on a uncountable set X then show that (X, τ) is not first countable space. **7**

5. a) Prove that every open continuous image of second countable space is second countable. **7**

   b) Prove that every closed subspace of a Lindelof space is Lindelof. **7**

6. a) Prove that a topological space (X, τ) is a T_1-space if and only if every singleton subset {x} of X is τ-closed. **7**

   b) If each point of a topological space (X, τ) possesses a closed neighbourhoods which is a hausdorff subspace of X then prove that (X, τ) is a Hausdorff space. **7**

7. a) State and prove Urysohn's lemma. **5**

   b) Prove that continuous image of compact species compact. **9**

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Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (New) CGPA
Complex Analysis (Paper – IX)

Day and Date : Tuesday, 24-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No .****2*** *are* ***compulsory****.*

2) Solve ***any three*** *questions from Q. No.* ***3*** *Q. No.* ***7****.*

3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct answer (**one** mark each). **5**

1) If f is analytic in 0< |z–a|< R, and $\lim_{z\to a}(z-a)^m .\ f(2) = l(\neq 0)$ then

a) f has a pole at z = a of order m

b) f has a removable singularity at z = a

c) f has essential singularity at z = a

d) none of these

2) Every Mobius transformation can have at most ______________ fixed points.

a) 0 b) 1 c) 2 d) 3

3) Which one of the following is false if f is an entire function ?

a) f is continuous b) f is differentiable

c) f is analytic d) none of these

4) Let G be connected open set and let f : G $\to$ $\mathbb{C}$ be an analytic function then

I) f = 0

II) { z$\in$G : f (z) = 0} has a limit point in G

a) (I) $\Rightarrow$ (II) b) (I) $\Leftrightarrow$ (II)

c) (II) $\Rightarrow$(I) d) None of these

5) The radius of converges of the series $\sum_{n=0}^{\infty} \frac{1}{n^n} z^n$ is =

a) 0 b) 1 c) ∞ d) $\frac{1}{2}$

B) State whether **true** or **false** (**one** mark **each**). **5**

1) If f and g are analytic function on a region G, then $f \equiv g$ on G if and only if $\{z \in G \mid f(z) \neq g(z)\}$ has a limit point in G.
2) If G be an open set and H(G) be the space of all analytic functions in G then H(G) is a complete metric space.
3) Non constant polynomial is bounded.
4) Let z = a is called as a pole if $\lim_{z \to a} f(z)$ is not exists.
5) Every entire function is differentiable.

C) Fill in the blanks (**one** mark **each**). **4**

1) Let z = a be an isolated singularity of f and let $f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n$ be its Laurentz series expansion in a_{nn} (a; 0, R) . Then $a_{-m} \neq 0$ and $a_n = 0$ for $n \leq -(m+1)$ iff z = a is a pole of order ____________
2) If $f(z) = \frac{e^z}{z^2+1}$ then Res (f ; – i) = ____________
3) If an isolated singularity is neither a pole or a removable singularity then it is called ___________
4) If p(z) is a non-constant polynomial there there is a complex number z_1 with $p(z_1)$ = ___________

2. a) Define the Mobius transformation. Also show that the mobius transformation is the composition of translation, dilation and inversion. **4**

b) State and prove Morera's theorem. **4**

c) Prove that if f is a bounded entire function than f is a constant. **3**

d) State and prove Hurwitz's theorem. **3**

3. a) If f be an analytic in B (a; R) then prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$, for $|z-a|<R$, where $a_n \frac{f^{(n)}(a)}{n!}$ and this series has radius of convergence $\geq$ R. **7**

   b) Let G be a connected open set and let $f : G \to \mathbb{C}$ be an analytic function. Then prove that the following statements are equivalent

   i) $f \equiv 0$

   ii) there is a point 'a' in G such that $f^{(n)}$ (a) = 0 for each $n \geq 0$.

   iii) $\{z \in G : f(z) = 0\}$ has a limit point in G. **7**

4. a) State and prove Cauchy's integral formula first version. **7**

   b) State and prove Goursat's theorem. **7**

5. a) Prove that $\int_r \left(\frac{z}{z-1}\right)^n dz = 2n\pi i$, where r (t) = $1+e^{it}$, $0 \leq t \leq 2\pi$. **4**

   b) State and prove Cauchy's Residue theorem. **10**

6. a) State and prove Casorati Weierstrass therorem. **7**

   b) Find the value of the integral $\int_0^{\pi} e^{\cos\theta}, \cos(\sin\theta + n\theta) d\theta$. **7**

7. a) If f is an analytic in a region G and a is any point in G with $|f(a)| \geq |f(z)|$, for all z in G then prove that f must be a constant function. **7**

   b) State and prove Schwartz lemma. **7**

Seat No.	

M.Sc. I (Semester – II) (CGPA) Examination, 2015
MATHEMATICS
Relativistic Mechanics (New) (Paper No. – X)

Day and Date : Friday, 27-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

N.B. : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) The comparison between the direction of propagation of wave motion as observed by the observers in s and s′ frames is called __________

2) The increase in kinetic energy of particle is direct consequence of __________ varies with velocity.

3) The space time is flat means __________

4) The velocity of light in vacuum is independent of the __________ and __________

5) The collision is elastic if e = __________

6) The velocity of a fluid is a __________ but not the acceleration.

7) The total charge in an isolated system is unchanged by the motion of its __________

B) Choose the correct alternative (**one** mark **each**) :

1) The relativistic expression for Hamiltonian is, H =

a) T – V b) T + V c) $T^* + V$ d) $T^* – V$

2) If A_{lm} is symmetric and B^{lm} is skew symmetric tensor then $A_{lm} \cdot B^{lm}$ =

a) 1 b) – 1 c) 0 d) finite

3) The effects of length contraction, time dilation, mass variation cannot be observed at

a) ordinary speed
b) velocity of light
c) approximately equal to c
d) none of these

4) Classical transverse Doppler effect is given by

a) $\gamma = \gamma' (1+\beta)$
b) $\gamma = \gamma' (1-\beta)$
c) No effect
d) None of these

C) State **true** or **false** :

1) The scalar product of two four vectors is invariant quantity.

2) Conservation of classical momentum is invariant under lorentz transformation.

3) The totality of all possible events is called space time continuum.

2. a) What is the importance of constancy of the speed of light ? **3**

b) A ball has velocity 4i – 5j + 10k m/s relative to a train moving with velocity 3i + 4j m/s relative to an observer on the ground. Calculate the velocity of the ball relative to the ground. **4**

c) Prove that : For finite velocities the Lorentz transformation reduces to Gallilean transformation. **3**

d) Prove that : The Kronecker delta symbol is a mixed tensor of rank 2. **4**

3. a) State postulates of special theory of relativity and hence deduce the Lorentz transformation equations. **8**

b) A particle moves with velocity represented by a vector u' = 3i + 4j + 12k m/s in frame S'. Find the velocity of the particle in frame S if S' moves with velocity 0.8c relative to S along x-axis. **6**

4. a) Derive the transformation rules for momentum and energy of a particle. **7**

b) Derive the expression for relativistic Hamiltonian. **7**

5. a) Show that the quantity $c^2\rho^2 - j^2$ is an invariant quantity and is equal to $c^2\rho_0^2$. **6**

b) P.T. the following Maxwell's equations are invariant under Lorentz transformations. **8**

6. a) Derive the expression $K = m_0c^2 \left\{ \frac{1}{\sqrt{1-u^2/c^2}} - 1 \right\}$ for the relativistic kinetic energy. **7**

b) Prove that the result of two successive Lorentz transformations is itself a Lorentz transformation. **7**

7. a) A particle of mass m_1 moving with velocity u makes a head on collision with a particle of mass m_2 initially at rest. If v_1 and v_2 are their velocities after collision along the same line then if the collision is elastic show that,

$$v_2 = \frac{2u}{1+\frac{m_2}{m_1}}.$$ **7**

b) Obtain the metric in spherical polar co-ordinates and hence find co-variant and contravariant component of the metric tensor. **7**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
MATHEMATICS (Paper – VI) (Old) CGPA
Algebra – II

Day and Date : Tuesday, 17-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *i) Q.**1** and Q.**2** are **compulsory**.*
*ii) Attempt **any 3** from **Q. 3** to **Q. 7.***
*iii) Figures to the **right** indicate **full** marks.*

1. A) Define the following. **5**
   1) Separable extension.
   2) Algebraic element.
   3) Fixed field.
   4) Constructible number.
   5) Solvable group.

   B) State **true** or **false**. **5**
   1) R is separable extension of Q.
   2) Every finite field is perfect.
   3) There exists a field with 10 elements.
   4) $x^2 - 3x + 3$ is irreducible over Q.
   5) If a > 0 is constructible then $\sqrt{a}$ is constructible.

   C) Fill in the blanks : **4**
   1) $Q\left(\sqrt{5}, \sqrt{7}\right) =$ ____________
   2) Any field of characteristic zero is ____________
   3) A subgroup of a solvable group is ____________
   4) Any two finite fields having same number of elements are ________

2. a) Show that the general polynomial of degree $n \geq 5$ is not solvable by radicals. **4**
   b) Construct a finite field with 9 elements. **4**
   c) Find the splitting field of the polynomial $x^2 - 2$ over Q. **3**
   d) Show that the polynomial $8x^3 - 6x - 1$ is irreducible over Q. **3**

3. a) Show that any finite extension of a field of characteristic zero is a simple extension. **7**
   b) Show that any field of characteristic zero is perfect. **7**

4. a) If f(x)t F [x] is solvable by radicals over F then prove that the Galois group of f(x) over F is solvable group. **8**
   b) If the finite field F has $q = p^m$ elements, then prove that every $a \in F$ satisfies the relation $a^q = a$. **6**

5. a) If L is a finite extension of K and K is a finite extension of F, then show that L is a finite extension of F. More over prove that, $[L : F] = [L : K][K : F]$. **8**
   b) If L is a finite extension of F and K is a subfield of L which contains F, then prove that [K : F] is a divisor of [L : F]. **6**

6. a) Let K be an extension of a field F. Then show that the element $a \in K$ is algebraic over F if and only if F(a) is a finite extension of F. **8**
   b) If $a, b \in k$ are algebraci over F of degrees m and n respectively, and if m and n are relatively prime, prove that F (a,b) is of degree mn over F. **6**

7. a) Let G be a subgroup of the group of all automorphisms of a field K. Then show that the fixed field of G is a subfield of K. **7**
   b) Show that $Q\left(\sqrt{2}, \sqrt{3}\right) = Q\left(\sqrt{2} + \sqrt{3}\right)$. **7**

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Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS
Real Analysis – II (Old) CGPA (Paper No. – VII)

Day and Date : Thursday, 19-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *i) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
ii) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
iii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks : **5**
   1) The Cantor set C is of measure __________
   2) Let A and B be any two sets, then $\chi_{A^c} = 1 -$ ___________
   3) Let $\{f_n\}$ be an increasing sequence of non-negative measurable functions on E. If $\{f_n\} \to f$ pointwise a.e. on E, then $\lim\limits_{n\to\infty} \int_E f_n =$ __________
   4) If f is measurable on (a, b), then it is _____________ a.e. on (a, b).
   5) For any set A and a real number y $m^*(A + y) =$ _________

   B) State whether **true** or **false** : **7**
   1) If A is measurable, then so is χ_A.
   2) The Cantor set C is countable set.
   3) There exists a strictly increasing function on [0, 1] which is continuous only at irrational numbers in [0, 1].
   4) The outer measure of an interval is its length.
   5) If f is of bounded variation on [a, b] then f' need not be integrable over [a,b].
   6) Linear combination of two functions of bounded variation is also of bounded variation.
   7) If ϕ is differentiable on (a, b) and ϕ' is increasing, then ϕ is convex.

   C) Define : **2**
   1) Lebesgue measure
   2) Simple function.

2. a) Let f be a continuous function on [0, 1] that is absolutely continuous on [ε, 1], for each $0 < \varepsilon < 1$. Show that f may not be absolutely continuous on [0, 1]. **4**

b) Let $\{u_n\}$ be a sequence of non-negative measurable function and let f $\sum_{n=1}^{\infty} u_n$,

then prove that $\int f = \sum_{n=1}^{\infty} \int u_n$. **3**

c) By giving suitable example, show that inequality in Fatou's lemma may be strict. **3**

d) Show that a continuous function on (a, b) is convex if and only if

$\phi\left(\frac{x_1 + x_2}{2}\right) \leq \frac{\phi(x_1) + \phi(x_2)}{2}, \forall x_1, x_2 \in (a, b)$. **4**

3. a) Show that a set E is measurable if and only if for each $\varepsilon > 0$, there is a closed set F and open set O for which $F \subseteq E \subseteq O$ and $m^* (O \sim F) < \varepsilon$. **7**

b) Let the function f have a measurable domain E. Then show that the following statements are equivalent.

i) For each real number c, the set $\{x \in E / f(x) > c\}$ is measurable.

ii) For each real number c, the set $\{x \in E / f(x) \geq c\}$ is measurable. **7**

4. a) Let f be a non-negative measurable function on E. Then prove that $\int_E f = 0$ if and only if f = 0 a.e. on E. **7**

b) State and prove Jensen's inequality. **7**

5. a) Show that every interval is measurable. **6**

b) State and prove bounded convergence theorem. **8**

6. a) Let f be a measurable function on E. Then prove that f^+ and f^- are integrable over E if and only if |f| is integrable over E. **7**

b) If ϕ is differentiable on (a, b) and ϕ' is increasing, then show that ϕ is convex. Further also show that ϕ is convex if it has a non-negative second derivative ϕ'' on (a, b). **7**

7. a) State and prove Lebesgue's dominated convergence theorem. **7**

b) Show that if $\{A_k\}_{k=1}^{\infty}$ is an ascending collection of measurable sets, then

$m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k \to \infty} m(A_k)$. **7**

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Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
(Old) (CGPA)
MATHEMATICS (Paper – VIII)
General Topology

Day and Date : Saturday, 21-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
*2) Q. No.**(1)** and Q.No.**(2)** are* ***compulsory****.*
3) Attempt ***any three*** *from Q.No.**(3)** to Q.No.**(7)**.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose correct alternative. (**1** mark for **each**) :

1) If τ_1 and τ_2 are two topologies for a set X and consider following statements :

P : $\tau_1 \cap \tau_2$ is also topology for X.

Q : $\tau_1 \cup \tau_2$ is also topology for X.
then

a) Only P is true b) Only Q is true
c) Both P,Q are true d) Both P,Q are false

2) The empty set and the set X itself in a topological space (X, τ) are always

a) open b) closed
c) open and closed d) open but not closed

3) Every finite topological space is

a) Lindelof b) Compact
c) Neither (a) nor (b) d) Both (a) and (b)

4) If τ is the usual topology on a set of real numbers R then relative topology for the set of integers is always

a) usual topology b) discrete topology
c) Indiscrete topology d) Never exist such a topology.

B) Fill in the blanks (**1** mark for **each**) :

5) Continuous image of compact topological space is _________ .

6) If f(G) is open set whenever G is a open set then f is __________ .

7) If $f^{-1}(G)$ is closed set whenever G is a closed set then f is __________ .

8) A topological space (X, τ) is said to be T_2 – space if __________.

9) A topological space (X, τ) is said to be compact if _________ .

C) State whether the following statements are **true** or **false** :

10) Every empty set is connected.

11) An open interval (0,1) is compact.

12) Indiscrete topological space is not T_0 – space.

13) Every Lindelof space is compact space.

14) $\overline{A \cap B} = \overline{A} \cap \overline{B}$

2. a) Prove that any set containing a dense set D is also dense set. **4**

b) Prove that closed subspace of a normal space is a normal space. **4**

c) Prove that subspace of T_0 – space is again T_0 – space. **3**

d) If $N_1(x)$ and $N_2(x)$ are neighbourhoods of x then prove that $N_1(x) \cap N_2(x)$ is also neighbourhood of x. **3**

3. a) If X is a compact space and $A \subset X$ is closed in X then A, in its relative topology is also compact. **6**

b) Prove that a topological space (X, τ) is compact if and only if any family of closed sets having finite intersection property has a non-empty intersection. **8**

4. a) Prove that the real line R is connected. **7**

b) Prove that a topological space (X, τ) is connected if and only if no non-empty proper subset of X is both open and closed. **7**

5. a) Prove that a topological space (X, τ) is a T_0 – space if and only if for every distinct arbitrary points $x, y \in X$, the closures of {x} and {y} are distinct. **8**

   b) Prove that a convergent sequence in a Housdorff space has the unique limit. **6**

6. a) Prove that every Tychonov space is a T_3 – space. **7**

   b) Prove that a normal space X is completely regular if and only if is a regular. **7**

7. a) If (X, τ) and (Y, τ^*) are two topological spaces then prove that a one to one mapping f on X onto Y is a homeomorphism if and only if $f(\overline{E}) = \overline{f(E)}$ for every $E \leq X$. **8**

   b) If A is a subset of a topological space (X, τ) and $\mathscr{B}$ is a basis for τ then prove that $\{B \cap A : B \in \mathscr{B}\}$ is a basis for the subspace topology on A. **6**

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Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (Old) (CGPA)
Complex Analysis (Paper No. – IX)

Day and Date : Tuesday, 24-11-2015 Max. Marks : 70

Time : 10.30 a.m. to 1.00 p.m.

Instructions : *1) Q. No. **1** and Q. No. **2** are **compulsory**.*
*2) Solve **any three** questions from Q. No. **3** to Q.No. **7***
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks : **6**

1) A mobius transformation has '0' and '∞' as it's fixed points iff ________

2) If $\gamma : [a, b] \to \mathbb{C}$ is rectifiable path and f is a function and continuous on the trace γ then the line integral of f along γ is ______________

3) If $f : G \to \mathbb{C}$ is analytic then f is ____________

4) If f is entire function then f has ____________

5) If f has an isolated singularity at a then the point z = a is a removable singularity iff ___________

6) Suppose f has a pole of order m at z = a and put $g(z) = (z - a)^m f(z)$ then Res (f ; a) = _____________

B) Choose correct alternative : **6**

1) If $f : G \to \mathbb{C}$ is analytic function in $\overline{B}(a ; r) \subset G$ then $f^{(n)}(a) =$ _________

a) $\frac{n!}{2\pi}\int_\gamma \frac{f(w)}{(w-a)^{n+1}}dw$

b) $\frac{n}{2\pi i}\int_\gamma \frac{f(w)}{(w-a)^{n}}dw$

c) $\frac{n!}{2\pi i}\int_\gamma \frac{f(w)}{(w-a)^{n}}dw$

d) $\frac{n!}{2\pi i}\int_\gamma \frac{f(w)}{(w-a)^{n+1}}dw$

P.T.O.

2) Let f be analytic in B(a; R) then $f(z) = \sum_{n=0}^{\infty} an(z-a)^n$ for |z – a| < R where a_n =

a) $\frac{1}{n!} f^{(n)}(a)$ b) $\frac{1}{(n-1)!} f^{(n)}(a)$

c) $\frac{1}{(n+1)!} f^{(n)}(a)$ d) $\frac{1}{n!} f^{(n-1)}(a)$

3) If f : G $\rightarrow \mathbb{C}$ is analytic and not constant a $\in$ G and f(a) = 0 then there is R > 0 such that B(a; R) $\subset$ G and ____________ for 0 < |z – a| < R.

a) f(z) = 0 b) |f(z)| $\leq$ M

c) f(z) # 0 d) |f(z)| $\geq$ 0

4) An entire function has removable singularity at infinity iff it is

a) Bounded b) Constant

c) Both a) and b) d) None of these

5) Let D = {z/|z| < 1} and suppose f is analytic on D with |f(z)| $\leq$ 1 for z $\in$ D and f(0) = 0 then ______________ $\forall$ z $\in$ D.

a) |f(z)| $\leq$ 1 and |f(z)| $\leq$ |z| b) |f(z)| $\leq$ 1 and |f(z)| $\geq$ |z|

c) |f(z)| $\geq$ 1 and |f(z)| $\geq$ |z| d) None of the above

6) Annulus involves ________curves.

a) One curve b) Two curves

c) Atleast one curve d) Atleast two curves

C) Define the term : **2**

i) Entire function

ii) Smooth curve

2. a) Evaluate the following cross ratios **4**

i) (7 + i, 1, 0, ∞)

ii) (2, 1, – i, 1, 1 + i)

b) T be a mobius transformation with fixed points z_1 and z_2. If S is a mobius transformation show that S^{-1} T s has fixed points $S^{-1}z_1$ and $S^{-1}z_2$. **4**

c) Evaluate the integral $\int_\gamma \frac{1}{z-a} dz$ where $\gamma(t) = a + re^{it}$ $0 \leq t \leq 2\pi$. **3**

d) State and prove Liouville's theorem. **3**

3. a) If $\gamma : [0,1] \to \mathbb{C}$ is closed rectifiable curve and $a \notin \{\gamma\}$ then prove that

$$\frac{1}{2\pi i}\int_{\gamma} \frac{1}{z-a}\,dz$$ is an integer. **7**

b) State and prove fundamental theorem of algebra. **7**

4. a) Let f be analytic in disk B(a ; R) and suppose γ is a closed rectifiable curve in B(a; R) then prove that $\int_{\gamma} f = 0$. **7**

b) Let G be an open subset of the plane and $f : G \to \mathbb{C}$ be an analytic function. If $\gamma_1, \gamma_2, \ldots \gamma_m$ are closed rectifiable curves in G such that $n(\gamma_1; w) + n(\gamma_2; w) + \ldots + n(\gamma_m; w) = 0 \;\forall\; w \in \mathbb{C} - G$ then show that for 'a' in $G = -\{\gamma\}$

$$f(a)\sum_{k=1}^{m} n_{(\gamma k;a)} = \sum_{k=1}^{m} \frac{1}{2\pi i}\int_{\gamma k} \frac{f(z)}{(z-a)}\,dz.$$ **7**

5. a) If f has simple pole at z = a and $f(z) = \frac{h(z)}{k(z)}$ then prove that Res (f ; a) = $\frac{h(a)}{k'(a)}$. **6**

b) State and prove Goursat's theorem. **8**

6. a) Evaluate $\int_{0}^{\infty} \frac{1}{1+x^2}\,dx$. **6**

b) State and prove Casorati-Weier strass theorem. **8**

7. a) Let z_1, z_2, z_3, z_4 be four distinct points in $(\infty$ then show that (z_1, z_2, z_3, z_4) is real number iff all four points lie on the circle. **7**

b) Find Laurent series expansion of $\frac{1}{z(z-1)(z-2)}$ in ann $(0; 2, \infty)$ **7**

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Seat No.	

M.Sc. – I (Semester – II) (CGPA) Examination, 2015
MATHEMATICS
Relativistic Mechanics (Old) (Paper – X)

Day and Date : Friday, 27-11-2015 Max. Marks : 70
Time : 10.30 a.m. to 1.00 p.m.

N. B. : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) : **10**

1) If v<<<c then $u' \oplus v$ = ______________

2) The transformation of Lorentz contractive factor is given by, $\frac{1}{\sqrt{1-u'^2/c^2}}$ = _______

3) The collision is inelastic if e = ______________

4) The space-time metric of Minkowski space is ds^2 = ______________

5) Velocity of fluid is a ______________ but not acceleration.

6) An index which is ______________ in single term is called real index.

7) Relativity of ______________ is the Einsteins special theory of relativity.

8) Transverse Doppler effect is purely ______________

9) A region where in a small magnet or a loop of wire carrying current will experience a force is called ______________

10) An unaccelerated frame is ______________ frame.

B) State **true** or **false** (**one** mark **each**) : **4**

1) Gravity is ignored in special theory of relativity.

2) The fundamental concepts mass and energy are identical and inter-convertible.

3) The value of co-efficient of restitution 'ρ' depends on the velocity of s'-frame.

4) If the charges are at rest then charge density is zero.

P.T.O.

2. a) Prove that : The Kronecker delta symbol is a mixed tensor of rank 2. **3**

b) What is twin paradox in special relativity ? **3**

c) Prove that : For small velocities Lorentz transformations reduces to Galilean transformations. **4**

d) Derive the relativistic longitudinal Doppler effect. **4**

3. a) Derive the expression for Longitudinal and Transverse mass. **7**

b) Prove that if the rod moves with velocity V relative to the observer then its measured length is contracted in the direction of motion by the factor. **7**

4. a) A particle moves with velocity represented by a vector $u' = 3i + 4j + 12k$ m/s in frame s'. Find the velocity of the particle in frames if s' moves with velocity 0.8c relative to s along +ve X-axis. **7**

b) Find the relativistic expression for Lagrangian. **7**

5. a) Derive the transformation rules for momentum and energy of a particle. **7**

b) Show that the electromagnetic wave equation is invariant under Lorentz's transformation. **7**

6. a) Explain geometrical interpretation of Lorentz's transformations. **7**

b) Show that the quantity $x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformations. **7**

7. a) A contravariant tensor A^i has components (3, 5, 6) in a rectangular Cartesian co-ordinates. Find its components in spherical polar co-ordinates. **6**

b) Derive transformation equations for electric field. **8**

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Seat No.	

M.Sc. – II (Semester – III) (New – CGPA) Examination, 2015
MATHEMATICS
Functional Analysis (Paper – XI)

Day and Date : Monday, 16-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *i) Q. no.* ***1*** *and* ***2*** *are* ***compulsory****.*

ii) Attempt ***any three*** *questions from Q. no.* ***3*** *to Q. no.* ***7****.*

iii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Fill in the blanks : **7**

i) The elements of N*, the conjugate space of N are known as _________

ii) The conjugate space of l_p^n is _________

iii) A closed linear subspace M of a Hilbert space H is said to be invariant under T if _________

iv) The set of all continuous linear transformations of a normed linear space N into a normed linear space is denoted by _________

v) If we define $\|x\|_p = \left\{\sum_{i=1}^{n} |x_i|^p\right\}^{\frac{1}{p}}$ then the Minkowski's inequality can be written as _________

vi) If S is non-empty subset of a Hilbert space then the orthogonal compliment of S is defined by $S^{\perp}$ = _________

vii) If E is a projection on a linear space L then the set $\{E(x) : x \in L\}$ is known as _________

b) State whether following is **True** or **False** : **7**

i) An idempotent continuous linear transformation E on a Banach space is known as projection.

ii) If x and y are any two vectors in a Hilbert space then $|(x, y)| \geq \|x\|\|y\|$.

iii) A normed linear space N is separable if its conjugate space N* is separable.

P.T.O.

iv) The parallelogram law is not true in l_1^n.

v) The conjugate space N* of a normed linear space N cannot be a Banach space under the norm $\| f \| = \sup \{ |f(x)| : \| x \| \leq 1 \}$.

vi) For any arbitrary operator T on Hilbert space H both TT* and T*T are positive.

vii) Two vectors x and y in a Hilbert space H are orthogonal if (x, y) $\neq 0$.

2. a) State uniform boundedness theorem. **3**

b) If N is a normal operator on Hilbert space H, then prove that $\| N^2 \| = \| N \|^2$. **3**

c) If T, $T' \in \mathscr{B}$(N) then prove that $\| T.T' \| \leq \| T \| \| T' \|$. **4**

d) If H is complex inner product space then prove that $(x, \alpha y + \beta z) = \overline{\alpha}(x, y) + \overline{\beta}(x, y)$ for all x, y, z $\in$ H. **4**

3. a) If N and N' are normed linear spaces and let $\mathscr{B}(N, N')$ denote the set of all continuous transformations of N into N'. Then prove that $\mathscr{B}(N, N')$ is itself a normed linear space with respect to point-wise linear operations

(T + U) (x) = T(x) + U(x),

$(\alpha T)(x) = \alpha T(x)$

and the norm defined by $\| T \| = \sup \{ \| T(x) \| : \| x \| \leq 1 \}$. **7**

b) If M is a linear subspace of a normed linear space N, and let f be linear functional defined on M, then prove that f can be extended to a functional f_0 defined on the whole space N such that $\| f_0 \| = \| f \|$. **7**

4. a) If T is continuous linear transformation of a normed linear space N into a normed linear space N', and if M is null space (kernel of T), then show that T induces a natural linear transformation T' of N/M into N' and that $\| T' \| = \| T \|$. **7**

b) If B and B' are Banach spaces, and T is a linear transformation of B into B' then prove that T is continuous if and only if its graph is closed. **7**

5. a) If M is a closed linear subspace of a Hilbert space H, if x is a vector not in M, and if d is the distance from x to M then prove that there exists a unique vector y_0 in M such that $\|x - y_0\| = d$. **7**

b) If H is a Hilbert space and if f is an arbitrary functional in H* then prove that there exists a unique vector y in H such that f(x) = (x, y) for every x in H. **7**

6. a) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, then prove that linear subspace M + N is also closed. **7**

b) The space L_2 associated with a measure space X with measure m with inner product of two functions f and g defined by $(f, g) = \int f(x)\overline{g(x)}\, dm(x)$
Show that L_2 is inner product space. **7**

7. a) State and prove Banach fixed point theorem. **7**

b) If x and y are any to vectors in a Hilbert space then prove that $|(x, y)| \le \|x\|\|y\|$. **7**

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Seat No.	

M.Sc. – II (Semester – III) (New-CGPA) Examination, 2015
MATHEMATICS (Paper – XII)
Advanced Discrete Mathematics

Day and Date : Wednesday, 18-11-2015 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Figures to the **right** indicates **full** marks.*
*2) Q.No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q.No. **3** to **7**.*

1. A) State **true** or **false** (**One** mark **each**).
   i) A partially ordered set with greatest element I is complete if every non empty set has Least upper bound.
   ii) A vertex of degree one is called pedant vertex.
   iii) Every Boolean ring is a commutative ring.
   iv) In a complete graph with n vertices then degree of each vertex is n (n-1)
   v) In any graph the number of vertices of odd degree is always even.
   vi) A connected graph is called as tree.
   vii) A expression for geometric series is $\frac{1}{(1+x)^n} = \sum_{r=0}^{\infty}(n-1+r)\, C_r X^r$
   viii) A pedant vertex is called leaf of a tree.
   ix) A walk is always a path.
   x) In any poset, maximal and minimal elements are always unique.

   B) Fill in the blanks (**One** mark **each**)
   i) Every finite lattice is _________
   ii) Let A and B be finite sets then ________
   iii) The total degree of graph G is ______ the number of edges in a graph G.
   iv) If (n+1) objects are put into n boxes then at least one box contains _______

2. a) Draw all the spanning trees of K_4 graph. **(4+3+4+3)**

   b) Give the short note on isomorphism of graph.

   c) If five points are chosen randomly in the interior of equilateral triangle with each side of two units. Then show that atleast one pair of point has separation less than one unit.

   d) Give the short note on complete graph with examples.

3. a) Show that a graph G is connected if and only if given any pair u and v of vertices there is a path From u to v. **(7+7)**

   b) Let $(L, \wedge, \vee)$ be a triplet with a non empty set L and meet and join are binary operations on L which Satisfy associative, commutative, idempotent and absorption law then show that L is lattice.

4. a) Show that the lattice of normal subgroups of a group is a modular lattice. **(7+7)**

   b) If G is tree with vertices then show that it has precisely (n–1) edges.

5. a) Let a graph G be a non empty graph with at least two vertices then G is bipartite graph if and only If it has no odd cycle. **(7+7)**

   b) Find the coefficient of x^{27} in $(x^4 + x^5 + x^6 + \cdots)^5$

6. a) Find the general solution of $a_r = -4\, a_{r-1} - 3\, a_{r-2}$; $r \geq 2$ with the conditions $a_0 = 2$ and $a_1 = 8$. **(7+7)**

   b) Let G be a connected graph. Then show that G is a tree if and only if for every edge e of G the sub graph G – e has two components.

7. a) Find the general solution of $a_r - 3\, a_{r-1} - 4\, a_{r-2} = 4^r$ **(7+7)**

   b) Use the method of iteration to find an explicit formula for the sequence $\{b_n\}$ defined by the Recurrence relation $b_n = 5\, b_{n-1} + 3$ for $n \geq 2$ and initial condition $b_1 = 2$.

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Seat No.	

M.Sc. (Part – II) (Semester – III) (New – CGPA) Examination, 2015
MATHEMATICS (Paper – XIII) (Elective – I)
Linear Algebra

Day and Date : Friday, 20-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative (**one** mark **each**) :

i) The characteristic polynomial of $A = \begin{pmatrix} 2 & -3 \\ 5 & 1 \end{pmatrix}$ is equal to

a) $t^2 - 3t + 17$ b) $t^2 + 3t - 17$

c) $t^2 + 17$ d) $t^2 + t + 17$

ii) Let V and W be finite dimensional inner product spaces over the same field, having the same dimensions. If T is an vector space isomorphism from V onto W, then

I) T preserves inner products

II) T does not carries every orthonormal basis for V onto an orthonormal basis for W

a) Only I is false b) Only II is false

c) Both I and II are false d) Both I and II are true

iii) A complex $n \times n$ matrix D is diagonal then

a) D is normal b) D is adjoint

c) D is unitary d) None of these

iv) If T is a linear operator on an inner product space V. If T is Skew-Hermitian then

a) T^2 is Skew-Hermitian
b) T^2 is self adjoint
c) T^2 is not normal
d) $(T^2)^* \neq T^2$

v) Let V be a vector space over F and let $T : V \to V$ be a linear operator. If zero is an eigen value of T then

a) T is one-one
b) T is onto
c) T is singular
d) T is no-singular

B) Fill in the blanks (**one** mark **each**) :

i) If V is a vector space of dimension n over the field F then dim V^* = __________, where V^* is the dual space of V.

ii) A real $n \times n$ matrix A is said to be orthogonal, if ____________

iii) Let dim V (F) is finite and $T : V \to V$ be a linear operator. If characteristic polynomial of T is $(x-3)(x-5)(x-7)$ then the minimal polynomial of T is ____________

iv) A complex $n \times n$ matrix A is called unitary if ____________

v) Let W be a subspace of a vector space V (F). Then A (W) = ____________, where A (W) is an anni-hilator of W.

C) Define the following terms (**one** mark **each**) :

i) Linear functionals.

ii) Invariant subspace.

iii) Elementary Jordan matrix with characteristic value C.

iv) Normal matrix.

2. a) If V is a finite dimensional vector space and $V \neq \overline{o} \in V$, then show that there exists a functional $f \in V^*$ such that $f(V) \neq 0$. **4**

b) Show that minimal polynomial divides the characteristic polynomial for T. **4**

c) If A is a complex 5×5 matrix with characteristic polynomial $f(x) = (x-2)^3 \cdot (x+7)^2$. Find all possible Jordan canonical forms. **3**

d) For any linear operator T on a finite dimensional inner product space V, then prove that there exists a unique linear operator T* on V such that $(T(\alpha), \beta) = (\alpha, T^*(\beta))$ for all α, β in V. **3**

3. a) Let dim V (F) is a finite and $1B = \{\alpha_1, \alpha_2, ..., \alpha_n\}$ be a basis for V. Then show that there is a unique dual basis $1B^* = \{f_1, f_2, ..., f_n\}$ for V* such that $f_i(\alpha_j) = \delta_{ij}$ and for each linear functional f on V, $f = \sum_{i=1}^{n} f(\alpha_i) f_i$. Also show that for each α in V, $\alpha = \sum_{i=1}^{n} f_i(\alpha_i)\alpha_i$. **7**

b) Let T be a linear operator on an n-dimensional vector space V over F. Then show that the characteristic and minimal polynomials for T have the same roots, except for multiplication. **7**

4. a) Find the dual basis of the basis set, $1B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for $V_3(R)$. **7**

b) If $V = W_1 \oplus W_2 \oplus ... \oplus W_k$, then show shot there exists k linear operators $E_1, E_2, ..., E_k$ on V such that

i) each E_i is a projection ;

ii) $E_i : E_j = 0$ if $i \neq j$; i, j = 1, 2, ..., k ;

iii) $I = E_1 + E_2 + ... + E_k$;

iv) the range of E_i is W_i. **7**

5\. a) If W_1 and W_2 are subspaces of a vector space V (F) and if $V = W_1 \oplus W_2$ then prove that $V^* = A(W_1) \oplus A(W_2)$. **7**

b) Let a, b and c be elements of a field F, and let A be the following 3×3 matrix over F ; $A = \begin{bmatrix} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{bmatrix}$.

Prove that the characteristic and minimal polynomial for A is $x^3 - ax^2 - bx - c$. **7**

6\. a) Find all possible rational canonical forms for 6×6 matrices with minimal polynomial $m(t) = (t^2 + 3)(t + 1)^2$. **7**

b) Let M be an $m \times n$ matrix with entries in the polynomial algebra F [x]. Then prove that M is equivalent to a matrix N which is in normal form. **7**

7\. a) Let V be a finite dimensional complex inner-product space, and let T be a linear operator on V. Prove that T is self adjoint if and only if $(T(\alpha), \alpha)$ is real for every α in V. **7**

b) Let V be a complex vector space and f a form on V such that $f(\alpha, \alpha)$ is real for every α. Then prove that f is Hermitian. **7**

Seat No.	

M.Sc. – II (Semester – III) (New – CGPA) Examination, 2015
MATHEMATICS (Paper – XIV) (Elective – II)
Differential Geometry

Day and Date : Monday, 23-11-2015 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Figures to the right indicates **full** marks.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** questions from Q. **3** to Q. **7**.*

1. A) Fill in the blanks (**one** mark **each**) :

1) For a plane surface principal curvatures $k_1 = k_2 =$ ________

2) If a vector field Y is parallel then its Euclidean co-ordinate functions are ______

3) A reparametrization α(h) of a curve α is said to be orientation preserving if ______

4) Different positions of the profile curve as it is rotated are called as the ______ of the surface M.

5) A curve α in $M \subset E^3$ is a geodesic of M if its acceleration is always ______

6) For a surface M : z = xy, the normal vector field on M is given by ______

7) The quadratic approximation of the curve β near $\beta(0)$ is ______

B) State **true** or **false** (**one** mark **each**) :

1) A regular curve can have cusps.

2) Unit speed reparametrization is always orientation preserving.

3) If a unit speed curve has non zero curvature then it is a straight line.

4) Frenet apparatus of a given curve are independent of choice of parameter.

5) Orthogonal transformation preserves length of vectors and angle between them.

6) To find covariant derivative of a vector field we find the directional derivative of its coordinate functions in the direction of tangent vector at point.

7) Geodesics on sphere are circles.

2. a) Evaluate one form $\phi = yzdx - x^2 dz$ on the vector field $V = yzU_1 + xzU_2 + xyU_3$. **4**

   b) Find the arc of a circle $\alpha(t) = (acost, asint, O), 0 \leq t \leq 2\pi$. **4**

   c) Write note on Torsion of a curve. **3**

   d) Show that M : $(x^2 + y^2)^2 + 3z^2 = 1$ is a surface. **3**

3. a) Define directional derivative of a function f with respect to tangent vector and derive its properties. **7**

   b) Let β be a unit speed curve in E^3 with k > 0. Then prove that β is a plane curve iff its torsion is zero. **7**

4. a) Describe geographical patch $X : D \to E^3$ in the sphere Σ. Show that X is a parametrization of X(D) in Σ. **7**

   b) Let α be a curve cut from a surface $M \subset E^3$ by a plane P. If the angle between M and P is constant along α then prove that α is a principal curve of M. **7**

5. a) Compute Frenet apparatus of the curve $\alpha(t) = (3t - t^3, 3t^2, 3t + t^3)$. **7**

   b) If F is an isometry of E^3 such that F(0) = 0 then show that F is an orthogonal transformation. **7**

6. a) Show that a surface obtained by rotating a curve is a surface. **6**

   b) Let $V = -yU_1 + xU_3$ and $W = \cos x\, U_1 + \sin x\, U_2$. Express the following covariant derivatives in terms of U_1, U_2, U_3.

   i) $\nabla_v W$ ii) $\nabla_w V$ iii) $\nabla_v z^2 W$ iv) $\nabla_v(\nabla_v W)$ **8**

7. a) Prove that $u.v \times w \neq 0$ iff u, v, w are linearly independent. **7**

   b) Show that : A mapping $X : D \to E^3$ is regular iff $X_u \times X_v \neq 0$ for all $u, v \in D$. **7**

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Seat No.	

M.Sc. – II (Semester – III) Examination, 2015
MATHEMATICS (Paper – XV) (Elective – III)
Numerical Analysis (New – CGPA)

Day and Date : Thursday, 26-11-2015 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*
4) ***Use*** *of calculator is* ***allowed****.*

1. A) Choose the correct alternative (**one** mark **each**) : **6**

1) In the Gauss elimination method for solving a system of linear algebraic equations, triangularization leads to

a) Diagonal matrix b) Lower triangular matrix
c) Upper triangular matrix d) Singular matrix

2) First approximation to the root of the equation $x^3 - 2x - 5 = 0$ using method of false position is

a) 2.05882 b) 2.5882
c) 2.15882 d) 2.882

3) The backward difference operator is

a) $\nabla f(x) = f(x + h) - f(x)$ b) $\nabla f(x) = f(x) - f(x - h)$
c) $\nabla f(x) = f(x - h) - f(x)$ d) $\nabla f(x) = f(x) + f(x - h)$

4) If $f(x) = \frac{1}{x^2}$ then the value of first divided difference of the argument 2 and 3 is equal to

a) $-3/4$ b) $5/36$
c) $-5/36$ d) $2/3$

P.T.O.

5) The relation between ∇ and E is given by

a) $E = (1 - \nabla)^{-1}$ b) $E = (1 + \nabla)^{-1}$

c) $\nabla = 1 + E^{-1}$ d) $\nabla = E - 1$

6) An approximate value of π is 3.1428571 and its true value is 3.1415926 then the absolute error is

a) – 0.0012645 b) 0.012645

c) – 0.000402 d) – 0.00012645

B) Fill in the blanks (**one** mark **each**) : **6**

1) The error in Simpson's $1/3$ rule over $[x_0\ x_2]$ is ______________

2) The Newton Raphson method fails when $f'(x)$ is ______________

3) If A is upper triangular then A^{-1} is ______________

4) Householders method is used to obtain eigen values of __________ matrices.

5) The third forward difference $\Delta^3 y_0$ is ______________

6) Newtons divided interpolating formula a is ___________

C) State **True** or **False** (**one** mark **each**) : **2**

1) The method of false position is also known as method of chords.

2) Error in trapezoidal rule is $-\frac{1}{12}h^3\left(y_0'' + y_1'' + ... + y_{n-1}''\right)$.

2. a) Prove that $\left[x_0, x_1 ... x_n\right] = \frac{1}{h^n \cdot n!}\Delta^n y_0$. **3**

b) Show that :

$$e^x\left(u_o + x\Delta u_o + \frac{x^2}{2!}\Delta^2 u_o + ...\right) = u_0 + u_1 x + u_2\frac{x^2}{2!} + ...$$ **4**

c) Explain Absolute, Relative and percentage errors. **3**

d) Find the cubic polynomial which takes the values y(0) = 1, y(1) = 0, y(2) = 1 and y(3) = 10. **4**

3. a) Derive Newton's forward difference interpolation formula. **7**

   b) Solve $y' = -y$, $y(0) = 1$ using Euler's method. **7**

4. a) Prove that Newton Raphson method converges quadraticaly. **7**

   b) Determine the largest eigen value and the corresponding eigen vector of **7**

$$\begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

5. a) Reduce the matrix $\begin{pmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{pmatrix}$ to tridiagonal form. **7**

   b) Show that, using method of separation of symbols. **7**

$$\Delta^n u_{x-n} = u_x - nu_{x-1} + \frac{n(n-1)}{2} u_{x-2} + ... + (-1)^n u_{x-n}.$$

6. a) Deduce Newton's general interpolation formula with divided differences. **7**

   b) Evaluate $\int_0^1 \frac{1}{1+x} dx$ by trapezoidal rule with h = 0.5, correct to three decimal places. **7**

7. a) Show that lag range interpolating polynomial is unique. **7**

   b) A real root of $x^3 - 5x + 1 = 0$ lies in the interval (0, 1). Perform four iterations of secant method to obtain this root. **7**

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Seat No.	

M.Sc. – II (Semester – III) (Old) (CGPA) Examination, 2015
MATHEMATICS
Functional Analysis (Paper – XI)

Day and Date : Monday, 16-11-2015 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

*N.B. : 1) Q. 1 and Q. 2 are **compulsory**.*
*2) Attempt **any three** questions from Q. 3 to Q. 7.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) : **5**

1) Every complete subspace of a normed linear space is ____________

2) A non empty subset $\{e_i\}$ of a Hilbert space H is said to be____________ if $||e_i|| = 1 \forall i$ and $i \neq j \Rightarrow (e_i, e_j) = 0$.

3) Let T be an operator on a Hilbert space H then there exist a unique operator T* on H such that (Tx, y) = _________

4) An operator T on a Hilbert space H is said to be self adjoint if ___________

5) An operator U on a Hilbert space H is said to be Unitary if __________

B) State whether following statements are **true** or **false** (**one** mark **each**) : **4**

1) Every normal operator is unitary operator.

2) P is projection on a closed linear subspace M of H then I – P is projection on M.

3) A complete normed linear space is called Banach space.

4) In a normed linear space every Cauchy sequence is convergent.

P.T.O.

C) Choose the correct alternative (**one** mark **each**) : **5**

1) Which of the following statement is false ?

a) Every normed linear space is metric space

b) Every normed linear space is Hilbert space

c) Every Hilbert space is Banach space

d) Every Banach space is a normed linear space

2) If P is projection on a Hilbert space H with range M and Null space N. Then $M \perp N \Leftrightarrow$

a) $N = M^{\perp}$ b) $N = M$

c) $N = M^{\perp\perp}$ d) None of these

3) If H is a finite dimensional Hilbert space then every isometric isomorphism of H into itself is

a) Self adjoint b) Normal

c) Unitary d) Both a) and b)

4) If A is positive operator on H then which of the following is true ?

a) I + A is singular b) I + A is non singular

c) $I + AA^*$ is non singular d) Both a) and b)

5) If Y is complete then B(X, Y) is

a) Complete b) Not complete

c) Compact d) Bounded

2. a) If N is normed linear space then show that the operations of addition and scalar multiplication in N are jointly continuous. **4**

b) If $T_1, T_2 \in B(N)$ then prove that **3**

$\| T_1 . T_2 \| \leq \| T_1 \| . \| T_2 \|$

c) Show that the space $l_2^{(n)}$ is an inner product space. **4**

d) If x and y are two vectors in a Hilbert space then show that **3**

$\| x + y \|^2 + \| x - y \|^2 = 2 (\| x \|^2 + \| y \|^2)$

3. a) Let B and B' are Banach spaces and let T be a linear transformation of B into B' then show that T is continuous if and only if its graph is closed. **8**

   b) If M is closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$. **6**

4. a) Prove that the set of all normal operators on a Hilbert space H is a closed subspace of B(H) which contains the set of all self adjoint operators and it is closed under scalar multiplication. **8**

   b) If T is an arbitrary operator on a Hilbert space H and if α and β are scalars such that $|\alpha| = |\beta|$ then show that $\alpha T + \beta T^*$ is normal. **6**

5. a) Let N and N' are normed linear spaces and let T be linear transformation of N into N' show that the following statements are equivalent. **7**

      i) T is continuous

      ii) T is uniformly continuous on N.

      iii) T is continuous at some point of N.

   b) Let N be a normed linear space and x_0 a non zero vector in N then show that there exist a functional F in N* such that **7**

      $F(x_0) = \|x_0\|$ and $\|F\| = 1$

6. a) Let N be an arbitrary normed linear space then show that each vector x in N induces a functional F_x on N* defined by $F_x(f) = f(x)\ \forall f \in N^*$ s.t. $\|F_x\| = \|x\|$. **7**

   b) Let B and B' are Banach spaces. If T is a continuous linear transformation of B onto B' then prove that T is an open mapping. **7**

7. a) Let $\langle x, d \rangle$ be a complete metric space, and let f be a contraction mapping on X. Then prove that there exist one and only one point x in X such that $f(x) = x$. **8**

   b) If A_1 and A_2 are self adjoint operators on H then prove that $A_1 A_2$ is self adjoint iff $A_1A_2 = A_2A_1$. **6**

Seat No.	

M.Sc. (Part – II) (Semester – III) (CGPA) Examination, 2015
MATHEMATICS (Paper – XII)
Advanced Discrete Mathematics (Old)

Day and Date : Wednesday, 18-11-2015 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Choose correct answer : **7**

i) If graph G is simple then all diagonal entries are __________

a) 0 b) 1 c) 2 d) None of these

ii) A pedant vertex of tree is called __________

a) Seed b) Leaf c) Branch d) Chord

iii) Every semi modular __________

a) Is modular b) Need not be modular
c) Not modular d) None of these

iv) A posset with lowest element __________ is a complete lattice, its every non empty subset has l.u.b.

a) 1 b) 0 c) –1 d) 2

v) In a graph G(V, E) the number of vertices of ________________ degree is always ______________

a) odd, even b) even, odd
c) odd, odd d) even, even

vi) Among 13 people there are at least __________ people having there birthday in the same month.

a) 1 b) 2 c) 3 d) 4

vii) The number of distinct simple graphs with upto three nodes is __________

a) 15 b) 10 c) 7 d) 9

b) Fill in the blanks : **7**

i) The number of edges in a graph with six vertex two of degree 5 and 4 of degree 4 are ________

ii) The number of objects of a set S which have the name of the property $P_1, P_2, ..., P_m$ is given by, $\left|\overline{A}_1 \cap \overline{A}_2 \cap \overline{A}_3 \cap ..... \cap \overline{A}_m\right|$ = ________

iii) A lattice is distributive if $a \wedge (b \vee c)$ = ________

iv) The formal power series expansion of the $1/(1+5x)$ is ________

v) Lattice is defined as ________

vi) A weight of graph is ________

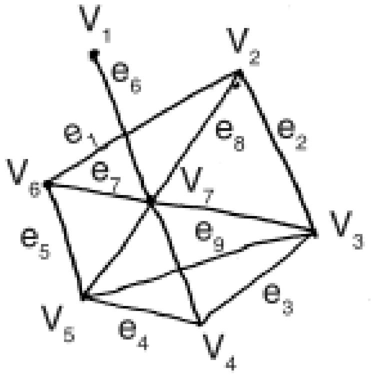
vii) Let X = {2, 3, 6, 12, 24}, Let $\leq$ be partial order defined by X $\leq$ y if x dividesY number of edges in the Hasse diagram of (X, $\leq$) is ________

2. a) If graph contain exactly two odd degree vertices then show that there is path between these two vertices. **4**

b) Show that any totally ordered set is distributive lattice. **3**

c) Find solution of $a_r = 3a_{r-1} + 3a_{r-2} - a_{r-3} = 0$. **3**

d) Calculate the both matrices of given graph : **4**



3. a) i) If u and v are distinct vertices of tree then show that there is precisely one path from u to v.

ii) Let G be a graph without any loop if for every pair of distinct vertices u and v of graph G there is precisely one path from u to v show that G is tree. **7**

b) Prove that an edge ‘e’ of graph G is bridge if and only if ‘e’ is not part of any cycle in G. **7**

4. a) Let G be a graph with 'n' vertices and q edges and W(G) denotes number of connected component of graph G then show that G has at least n–w(G) edges. **7**

   b) Solve the Recurrence relation $a_r - 4a_{r-1} + 4a_{r-2} = (r + 1)2^r$ with a = 0, $a_1 = 1$. **7**

5. a) A chess master who has 11 weeks to prepare for the tournament decides to play at least one game per day but not to play more than 12 games during any week show that there exist a succession of days during which chess master will play exactly 21 games. **7**

   b) State and prove Bridge theorem. **7**

6. a) G be a non-empty graph with atleast two vertices then prove that G is bipartite graph if and only if it has no odd cycle. **7**

   b) If B is Boolean algebra then show that :

      i) complement of every element is unique

      ii) $(a')' = a$

      iii) $(0') = 1, (1)' = 0$

      iv) $(a \wedge b)' = a' \vee b'$ and $(a \vee b)' = a' \wedge b'$. **7**

7. a) Let $(B, +, \cdot, 0, 1)$ be Boolean ring then $(B, \wedge, \vee, 0, 1)$ is Boolean algebra where
   $x \vee y = x + y - x \cdot y$
   $x \wedge y = x \cdot y$. **7**

   b) Show that the lattice of normal subgroup of a group is modular. **7**

SLR-MM – 445

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
MATHEMATICS (Paper – XIII) (Old) CGPA
Linear Algebra (Elective – I)

Day and Date : Friday, 20-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. No. **1** and Q. No. **2** are **compulsory**.*
*2) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) A linear operator E on a vector space V is called projection if ________

2) What is the minimal polynomial of $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ ________

3) A linear operator T on V is called normal if ____________

4) If W is a subspace of V and W^0 is the annihilator of W then $\dim W^0$ = ________

B) State whether following are **true** or **false** (**one** mark **each**) :

5) Similar matrices have same characteristic polynomial.

6) If T is diagonalizable then minimal polynomial of T has distinct roots.

7) Row rank of A need not be equal to column rank of A.

8) For any vector space V, V is isomorphic to V**, where V is finite dimensional.

C) Define the following (**two** marks **each**) :

9) Annihilator of a set.

10) Self adjoint operator.

11) Minimal polynomial.

2. a) Show that an orthogonal set of non zero vector is linearly independent.

   b) Let V be a finite dimensional vector space over the field F. Show that each basis for V* is the dual of some basis for V.

   c) If V is an inner product space, then for any $\alpha, \beta \in V$, show that $\|\alpha + \beta\| \leq \|\alpha\| + \|\beta\|$.

   d) Find a 3 × 3 matrix for which the minimal polynomial is x^2. **(4+4+3+3)**

3. a) Let W be the subspace of R^5 which is spanned by the vectors $\alpha_1 = (2, -2, 3, 4, -1)$, $\alpha_3 = (0, 0, -1, -2, 3)$, $\alpha_2 = (-1, 1, 2, 5, 2)$, $\alpha_4 = (1, -1, 2, 3, 0)$. Describe the annihilator of W i.e. W^0.

   b) Let V be a finite dimensional vector space over the field F. For each vector 'α' in V define $L_\alpha(f) = f(\alpha)$, f in V*. Show that the mapping $\alpha \to L_\alpha$ is an isomorphism of V onto V**. **(7+7)**

4. a) State and prove Cayley-Hamilton theorem.

   b) If V is a finite dimensional vector space over the field F and T be a linear operator on V. Show that T is triangulable iff the minimal polynomial for T is a product of linear polynomials over F. **(7+7)**

5. a) Find the possible Jordan forms of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ a & 2 & 0 \\ b & c & -1 \end{bmatrix}$. Show that A is similar to a diagonal matrix iff a = 0.

   b) Let T and U be linear operators on a finite dimensional inner product space V. Let 'C' be any scalar then show that

      i) $(T + U)^* = T^* + U^*$

      ii) $(CT)^* = \overline{C}\ T^*$

      iii) $(TU)^* = U^*T^*$

      iv) $(T^*)^* = T$. **(7+7)**

6. a) Let T be a linear operator on an n-dimensional vector space V. Show that the characteristic and minimal polynomials for T have the same roots, except for multiplicities.

   b) Show that the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is diagonalizable. **(7+7)**

7. a) State and prove spectral theorem.

   b) Apply the Gram-Schmidt process to the vectors $\beta_1 = (1, 0, 1), \beta_2 = (1, 0, -1), \beta_3 = (0, 3, 4)$ to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product. **(7+7)**

SLR-MM – 446

Seat No.	

M.Sc. – II (Semester – III) Examination, 2015
MATHEMATICS (Old CGPA) (Elective – II)
Paper – XIV : Modeling and Simulation

Day and Date : Monday, 23-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *i) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*

ii) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*

iii) Figures to the ***right*** *indicate* ***full*** *marks.*

iv) Use of simple or scientific calculator is ***allowed****.*

1. A) Select most correct alternative : **10**

i) If in a Markov Chain of two states j and k with one step transition probabilities $p_{jj} = 0$, $p_{kj} = 1$ then value of $p_{kk}^{(3)}$ is

a) 0.5 b) 2 c) 0 d) 1

ii) If a customer, on arriving at the service system stays in the system until served, no matter how he has to wait for service is called ____________ customer.

a) a regular b) an irregular c) a patient d) an impatient

iii) In M/M/1:∞/FCFS Queue model if λ is mean customer arrival rate an μ is mean service rate then the probability of server being stand idle is equal to __________.

a) $\frac{\lambda}{\mu}$ b) $1 - \frac{\lambda}{\mu}$ c) $1 - \frac{\mu}{\lambda}$ d) $\frac{\mu}{\lambda}$

iv) A manufacturer has to supply his customers 600 units of his product per year. Shortages are not allowed and the storage cost amounts to Rs. 0.60 per unit per year. The set up cost per order is Rs. 80. The optimal number of orders n is______ per year.

a) 3 b) 2 c) 1 d) 1.5

P.T.O.

v) What will be the corresponding random observation generated on continuous uniform distribution over (0,15) when a random number generated between 0 and 1 is 0.1 ?

a) 1.5 b) 0.15 c) 22.5 d) 15.15

vi) X is a binomial variate with parameters (n, p). If n = 1, the distribution of X reduces to

a) Bernoulli distribution b) Geometric distribution

c) Poisson distribution d) Discrete uniform distribution

vii) As a simulation is not an analytical mode, therefore, result of simulation must be viewed as

a) simplified b) exact c) unrealistic d) approximation

viii) In Monte-Carlo simulation

a) Randomness is the key requirement

b) The model is of deterministic nature

c) Random numbers can be used to generate the value of input variables only, if the sampled distribution is uniform

d) None of the above

ix) When there are more than one servers, customer behaviour in which he moves from one queue to another is

a) balking b) reneging c) jockeying d) alternating

x) The objective of network analysis is to

a) Minimize total project cost

b) Minimize total project duration

c) Minimize production delays, interruption and conflicts

d) All of the above

B) Fill in the blanks : **4**

i) The long form of CPM is ______________.

ii) In queue model completely specified in the symbolic form (a/b/c):(d/e), the third symbol c specifies ______________.

iii) The time gap between placing of an order and its actual arrival in the inventory is known as ______________.

iv) Simulation of systems in which the state changes abruptly at discrete points in time are called ______________.

2. A) i) Define continuous uniform distribution. State its cumulative distribution function. **4**

ii) Let $\{X_n, n \geq 0\}$ be a Markov Chain with three states 0, 1, 2 and with one step transition matrix

$$P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$$ and the initial distribution $P(X_0 = i) = \frac{1}{3}$ for i = 0, 1, 2

Find i) $P(X_2 = 0, X_1 = 1, X_0 = 2)$ ii) $P(X_3 = 2, X_2 = 1 / X_1 = 1)$ **4**

B) i) Define Geometric distribution and find the P (X = 3) if X follows Geometric distribution with parameter P = 0.45 **3**

ii) What do you mean by movement inventories ? **3**

3. A) Explain objectives of Scientific Inventory Control. **7**

B) A project schedule has the following activities and the time (in weeks) of completion of each activity is as follows :

Activity	1-2	2-3	2-5	3-4	3-5	4-5
Time	5	15	8	15	6	10

Draw the network diagram and find the minimum time of completion of the project, slack times for each activity and critical path. **7**

4. A) What are the advantages and limitations of using simulation ? **7**

B) Solapur Bakery keeps stock of a popular brand of cake. Previous experience indicates the daily demand as given here :

Daily Demand	0	10	20	30	40	50
Probability	0.02	0.25	0.10	0.40	0.22	0.01

Consider the following sequence of random numbers :
0.18, 0.68, 0.29, 0.11, 0.56, 0.79, 0.05, 0.34, 0.58, 0.09.

Using this sequence, simulate the demand for the next 10 days. Find out the stock situation if the owner of the bakery decides to make 25 cakes every day. Also estimate the daily average demand for the cakes on the basis of simulated data. **7**

5. A) Give the rules for constructing the network diagram in network analysis. **7**

B) Generate a random sample of size 5 from binomial distribution with parameters n = 1 and p = 0.51 using the sequence of random numbers 0.2261, 0.9907, 0.5053, 0.7470, 0.3864. **7**

6. A) Explain briefly the important characteristics of queueing system. **7**

B) Generate a random sample of size 5 from exponential distribution with mean 2 using the sequence of random numbers 0.472, 0.85, 0.294, 0.999, 0.423. **7**

7. A) Give the steps of Monte-Carlo simulation technique. **7**

B) Explain generation of a random sample from Normal distribution. **7**

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Seat No.	

M.Sc. II (Semester – III) Examination, 2015
MATHEMATICS
Elective – III – Numerical Analysis
(Paper No. – XV) (CGPA) (Old)

Day and Date : Thursday, 26-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q.No. **1** and Q.No. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q.No. **3** to Q.No. **7**.*
*3) Figures to the **right** indicate **full** marks.*
*4) Use of calculator is **allowed**.*

1. A) Fill in the blanks (**one** mark **each**) :

   i) Newton Raphson method has _____ order of convergence.

   ii) The second forward difference $\Delta^2 y_0$ is given by _____.

   iii) _______ rule is obtained by setting n = 1 in Newton-Cotes formula.

   iv) ________ method is accurate than Euler's method.

   B) Choose correct alternative (**one** mark **each**) :

   i) Power method is used to obtain ___________

      a) real root of the equation f (x) = 0

      b) solution of system of equations

      c) largest eigen value and corresponding eigen vector

      d) none

   ii) The Lagranges interpolating formula is given by ________

      a) $p(x) = \sum_{i=0}^{n} l_i(x)f(x_i)$ b) $p(x) = \sum_{i=0}^{\infty} l_i(x)f(x_i)$

      c) $p(x) = \sum_{i=0}^{n} l_i(x)f(x)$ d) none

iii) The iteration method converges iff _____

a) $|\varphi'(x_k)| \le c < 1$ b) $|\varphi'(x_k)| \le 1$

c) $|\varphi'(x_k)| \le 2$ d) none

iv) Which of the following is useful when arguments unequally spaced ?

a) Newton's forward interpolation formula
b) Newton's backward interpolation formula
c) Newton's divided difference interpolation formula
d) None

v) Simpson's $3/8^{th}$ rule is obtained from Newton-Cotes general integration formula by putting n = ________

a) 0 b) 1 c) 3 d) 4

C) **True** or **false** (**one** mark **each**) :

i) Every matrix can be reduced to symmetric tridiagonal form using Householder's method.

ii) Bisection method converges faster than Newton Raphson method.

iii) A set of tabular values having n + 1 points of x and f (x) is interpolated by a polynomial of degree n.

iv) Newton's forward difference interpolation formula is derived form Newton Cotes formula.

v) If f(a) f(b) > 0 then the root of the equation f(x) = 0 does not lies in the interval (a b).

2. i) Obtain the relation between shift operator E and Derivative operator D. **3**

ii) Explain iteration method. **4**

iii) Construct the divided difference table for the following tabular values. **4**

x	2	4	6	8	10
y = f(x)	10	20	30	40	50

iv) Prove that $\mu^2 = 1 + \frac{\delta^2}{4}$. **3**

3. i) Perform five iterations of Newton Raphson method to find a real root of the equation $x^3 - 5x - 4 = 0$. **8**

ii) Explain Secant method. **6**

4. i) Reduce the following matrix in to tridiagonal form using Householder's method. **7**

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

ii) Solve using Gauss elimination method. **7**

$2x + 3y - z + 2u = 7$

$x + y + z + u = 2$

$x + y + 3z - 2u = -6$

$x + 2y + z - u = -2.$

5. i) Value of x (in degrees) and tanx are given in the following table **8**

x	15	20	25	30	35	40
y = tan x	0.2679	0.3640	0.4663	0.5774	0.7002	0.8391

Find the value of tan 18.

ii) Using Lagrange's interpolation formula find y(4) from following data. **6**

x	0	5	10	15
y = f(x)	1	3	31	73

6. i) Find the largest eigen value and corresponding eigen vector of the following matrix. **7**

$$A = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

ii) Find the cubic polynomial which takes the following values. **7**

y (1) = 24, y (3) = 120, y (5) = 336, y (7) = 720.

7. i) Solve by Euler's modified method, the problem $\frac{dy}{dx} = x + y, \ y(0) = 0$ choose h = 0.2 and compute y (0.2), y (0.4). **7**

ii) Evaluate $\int_0^1 \frac{1}{1+x^3} dx$ using Trapezoidal rule with $h = 0.1$. **7**

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Seat No.	

M.Sc. (Part – II) (Sem. – IV) (CGPA) Examination, 2015
MATHEMATICS (Paper – XVI)
Measure and Integration

Day and Date : Tuesday, 17-11-2015 Max. Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct answer (**one** mark **each**) : **5**

1) Consider the statements

I) Every σ-finite measure is finite.

II) Every σ-finite measure is saturated.

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

2) Consider the statements

I) If γ is a measure $\Rightarrow \gamma$ is a signed measure.

II) If γ is a signed measure $\Rightarrow \gamma$ is a measure.

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

3) Lebesgue measure on [0, 1] is

a) Complete b) Finite

c) Both complete and finite d) None of these

4) If {A, B} is a Hahn decomposition for γ and $\gamma^+(E) = \gamma\,(E \cap A)$, $\gamma^-(E) = -\gamma\,(E \cap B)$ then

a) γ^+ is positive and γ^- is negative

b) γ^+ is negative and γ^- is positive

c) γ^+ and γ^- both are positive

d) γ^+ and γ^- both are negative

5) Consider the statements

I) Union of countable collection of a positive set is positive.

II) Every subset of a positive set is positive.

a) Only I is false
b) Only II is false
c) Both I and II are false
d) Both I and II are true

B) State **true** or **false** (**one** mark **each**) : **5**

1) The union of countable collection of sets of σ – finite measure is again of σ – finite measure.

2) Condition of σ – finiteness is not an essential in Radon Mikodym theorem.

3) Hahn decomposition is unique except for null sets.

4) The class $\mathbb{B}$ of μ^*– measurable sets is an algebra.

5) If $x \in X$ and $E \in \mathbb{R}_{\sigma\delta}$ then E_x is a measurable subset of X.

C) Fill in the blanks (**one** mark **each**) : **4**

1) A measure μ is called σ – finite if __________

2) Two measures μ and γ are mutually singular if __________

3) If $\langle X, \mathbb{B}, \mu \rangle$ is a measure space, a subset E of X is locally measurable if __________

4) Let μ and γ are signed measures then μ is absolutely continuous w. r. t. γ if ____________

2. a) State and prove monotone convergence theorem.

b) State Fubini's theorem.

c) Define a signed measure, show that the countable union of positive sets w. r. t. the signed measure γ is a positive set.

d) Let γ_1 and γ_2 be signed measure and let μ be a measure defined on a measurable space $\langle X, \mathbb{B} \rangle$. If $\gamma_1 \perp \mu$ and $\gamma_2 \perp \mu$ then show that $c_1\gamma_1 + c_2\gamma_2 \perp \mu$ for any two constants C_1 and C_2. **(4+3+4+3)**

3. a) Let $\langle X, \mathbb{B}, \mu \rangle$ be a measure space. If $E_i \in \mathbb{B}, \forall i$, then show that $\mu\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} \mu(E_i)$.

   b) State and prove Lebesgue convergence theorem. **(6+8)**

4. a) State and prove Jordan decomposition theorem.

   b) Show that if E is any measurable set, then $-\gamma^{-}(E) \leq \gamma(E) \leq \gamma^{+}(E)$ and $|\gamma(E)| \leq |\gamma|(E)$. **(8+6)**

5. a) State and prove Radon Mikodym theorem for a finite measure space $\langle X, \mathbb{B}, \mu \rangle$.

   b) State and prove Tonelli's theorem. **(6+8)**

6. a) Give an example that condition of integrability of f is must in Fubini's theorem.

   b) Let {Ai} be a disjoint sequence of measurable sets. Prove that

$$\mu_*\left(E \cap \bigcup_{i=1}^{\infty} Ai\right) = \sum_{i=1}^{\infty} \mu_*(E \cap Ai).$$ **(7+7)**

7. a) Prove that if $\mu^*(E) < \infty$ then E is measurable iff $\mu_*(E) = \mu^*(E)$.

   b) Define the outer measure μ^* on a power set of X. Show that the set $\mathbb{B}$ of all μ^* – measurable sets forms a σ – algebra and the restriction $\bar{\mu}$ of μ^* to $\mathbb{B}$ is a complete measure. **(6+8)**

____

Seat No.	

M.Sc. II (Semester – IV) Examination, 2015
MATHEMATICS (Paper – XVII) (CGPA)
Partial Differential Equations

Day and Date : Thursday, 19-11-2015 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Question No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q.No. **3** to Q.No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative (**1** mark **each**).

i) Equation ptany + qtanx = $\sec^2 z$ is of order

a) 1 b) 2 c) 0 d) None of these

ii) The equation (2x + 3y) p + 4xq – 8pq = x + y is

a) linear b) non-linear c) quasi-linear d) semi-linear

iii) A function f(x, y) is said to be homogeneous of degree n if it satisfies.

a) $f(\lambda x, \lambda y) = \lambda^n f(x, y)$ b) xfx + yfy = nf

c) Both (a) & (b) d) None of these

iv) Eliminating a, b from z = (x + a) (y + b) gives

a) pq = z b) p/q = z

c) p + q = z d) None of these

v) A necessary condition for the existence of solution of Neumann problem

$\nabla^2 u = 0$ in D, $\frac{\partial u}{\partial n} = f(s)$ on B is

a) $\int_B f(s)\,ds = 0$ b) $\int_B f(s)\,ds \neq 0$

c) $\int_B |f(s)|^2\,ds \neq 0$ d) $\int_B |f(s)|^2\,ds = 0$

vi) The one dimensional heat equation is given by

a) $\frac{\partial u}{\partial t} = k\nabla^2 u$ b) $\frac{\partial^2 u}{\partial t^2} = k\nabla^2 u$

c) $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$ d) $\frac{\partial^2 u}{\partial t^2} = k\frac{\partial^2 u}{\partial x^2}$

B) Fill in the blanks (**one** mark **each**).

i) For any constant a parametric equations x = a sinu cosv, y = a sinu sinv z = a cos u represents __________

ii) The corresponding partial differential equation for $z = x + ax^2y^2 + b$ is ______

iii) The complete integral of the partial differential equation z = px + qy + log pq ________

iv) Wave equation is an example of __________

v) The Pfaffian differential equation P(x, y, z) dx + Q(x, y, z) dy + R(x, y, z) dz = 0 is integrable if and only if __________

vi) The condition that the surfaces f(x, y, z) = c forms a family of equipotential surfaces is that the quantity ___________ is of function f only.

C) State **true** or **false** (**1** mark **each**).

i) Eliminating arbitrary function from z = F(xy/z) gives a Partial differential equation.

ii) There always exists integrating factor for a Pfaffian differential equation in two variables.

2. a) Eliminating parameters a and b find the corresponding partial differential equation from $2z = (ax + y)^2 + b$. **3**

b) Find the general integral of $z_t + zz_x = 0$. **3**

c) If $\overline{X} \cdot \text{curl}\overline{X} = 0$ where $X = P_i + Q_j + R_k$ and μ is an arbitrary differential function of x, y and z then prove that $\mu\overline{X} \cdot \text{curl}(\mu\overline{X}) = 0$. **4**

d) Find the partial differential equation satisfied by all surfaces of the form F(u, v) = 0 where u = u(x, y, z) v = v(x, y, z) and F is an arbitrary function. **4**

3. a) Explain Charpit's method of solving first order partial differential equation of the form f(x, y, z, p, q) = 0. **7**

   b) Find the general integral of $z(xp - ya) = y^2 - x^2$. **7**

4. a) Reduce the equation $u_{xx} + 2u_{xy} + 17u_{yy} = 0$ into canonical form. **7**

   b) Solve $z^2 + zu_z - u_x^2 - u_y^2 = 0$ by using Jacobi's method. **7**

5. a) Find the integral surface of first order partial differential equation $(x - y)\,p + (y - x - z)\,q = z$ passing through the circle $z = 1,\ x^2 + y^2 = 1$. **7**

   b) Show that necessary and sufficient condition that there exist between two functions u(x, y) and v(x, y) a relation F(u, v) = 0 not involving x or y explicitely is $\frac{\partial(u, v)}{\partial(x, y)} = 0$. **7**

6. a) Describe the analytic expression for the Monge cone at (x_0, y_0, z_0). **7**

   b) Find particular solution of $f(x, y, z, p, q) = z - px - qy - p^2 - q^2$. **7**

7. a) Show that the solution of the following problem, if it exists, is unique.

   $u_{tt} - c^2 u_{xx} = F(x, t),\ 0 < x < l,\ t > 0$

   $u(x, 0) = f(x),\ 0 \le x \le l$

   $u_t(x, 0) = g(x),\ 0 \le x \le l$

   $u(0, t) = u(l, t) = 0,\ t \ge 0.$ **7**

   b) Find complete integral of $f = z^2 - pqxy$ by Charpit's method. **7**

Seat No.	

M.Sc. – II (Semester – IV) (CGPA) Examination, 2015
MATHEMATICS
Elective – I : Integral Equations (Paper – XVIII)

Day and Date : Saturday, 21-11-2015 Max. Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

N. B. : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select correct alternative : **5**

i) An integral equation is called linear if, only linear operations are performed in it upon the ________

a) Known function b) Unknown function
c) Both a) and b) d) None of these

ii) A given function is said to be square function if ________

a) $\int_a^b |y(x)|^2 dx < \infty$ b) $\int_a^b |y(x)| dx < \infty$

c) $\int_a^b |y(x)|^2 dx > \infty$ d) $\int_a^b |y(x)| dx > \infty$

iii) If Kernel of integral equation is symmetric then it has __________ eigen values.

a) Zero b) One c) At least one d) At most one

iv) Solution of $y(x) = 1 + \int_0^x y(t)dt$ is ________

a) e^{x^2} b) e^x c) x d) x^2

v) The sequence of eigen functions of a symmetric Kernel can be made ____

a) Orthogonal b) Convergent c) Divergent d) Orthonormal

B) Fill in the blanks : **5**

i) A necessary and sufficient condition for a symmetric L_2 Kernel has only finite number of eigen-values is __________

ii) The multiplicity of any non-zero eigen value is finite for every symmetric Kernel for which $\int_a^b \int_a^b |k(x, t)|^2 dxdt$ is __________

iii) If $\int_0^\infty F(x)\cos px dx = e^x$ then F(x) = ____________

iv) Iterated Kernel is defined as __________

v) General integral equation is given by _____________

C) State **true** or **false** : **4**

i) The homogeneous volterra integral equation of second kind is $y(x) = f(x) + \lambda \int_a^x k(x, t)\, y(t)dt$.

ii) The integral equation $y(x) = F(x) + \lambda \int_a^b k(x, t)\, y(t)dt$ where k(x, t) is continuous and symmetric and λ is not eigen value has unique solution if λ does not take on an eigen value.

iii) By solving initial value problem we obtain Fredholm integral equation.

iv) If k(x, t) = 3(x – t) then it is symmetric Kernel.

2. a) Show that the function $y(x) = (1 + x^2)^{(-3/2)}$ is solution of the V.I.E.

$$y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2}\, y(t)dt .$$ **4**

b) Convert the following I.V.P. into integral equation.
$y'' + y = 0$ when $y(0) = y'(0) = 0$. **4**

c) Define Green's function. **3**

d) Show that the integral equation $y(x) = \lambda \int_0^1 (3x - 2)t\, y(t)dt$ has no characteristic numbers and eigen functions. **3**

3\. a) Prove that if Kernel is symmetric then all its iterated Kernels are also symmetric. **7**

b) Solve $y(x) = f(x) + \lambda\int_0^1 (x + t)\, y(t)dt$. **7**

4\. a) Convert the given B.V.P. to integral equation $y''(X) + \lambda y(X) = 0$ with end conditions $y(0) = 0 = y(l)$. **7**

b) Let $R(x, t\,;\lambda)$ be the resolvent Kernel of a V.I.E. $y(x) = f(x) + \lambda\int_a^x k(x, t)\, y(t)dt$ then show that the resolvent Kernel satisfies the integral equation

$R(x, t\,;\lambda) = K(x, t) + \lambda\int_a^x k(x, z)\cdot R(z, t\,;\lambda)dz$. **7**

5\. a) Find the Neumann series for solution of integral equation

$y(x) = 1 + x + \lambda\int_0^x (x - t)\, y(t)dt$ in particular case if $\lambda = 1$ what we get solution. **7**

b) Find the resolvent Kernel when $k(x, t) = e^{x^2} - t^2$. **7**

6\. a) Solve $y(x) = (x + 1)^2 + \int_{-1}^1 (xt + x^2t^2)\, y(t)dt$. **7**

b) Solve $y(t) = t^2 + \int_0^t y(u)\sin(t - u)du$. **7**

7\. a) Find the Green's function of $\frac{d^2y}{dx^2} + \mu^2 y = 0$ with y(0) = y(1) = 0. **7**

b) By using Green's function Reduce the B.V.P.

$y'' + y = x$ with $y(0) = 0$

$y'(1) = 0$. **7**

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Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
MATHEMATICS (Paper – XIX) (CGPA)
Elective – II : Operations Research

Day and Date : Tuesday, 24-11-2015 Total Marks : 70
Time : 2.30 p.m. to 5.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No.**1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q.No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative. **5**

1) A necessary and sufficient condition for a b.f.s. to a minimization LPP to be an optimum is that (for all j)

a) $(z_j - c_j) \geq 0$ b) $(z_j - c_j) \leq 0$
c) $(z_j - c_j) = 0$ d) $(z_j - c_j) > 0$ or $(z_j - c_j) < 0$

2) Which of the following is not true ? Dual simplex method is applicable to those LPPs that start with

a) an infeasible solution

b) a feasible solution

c) an infeasible but optimum solution

d) a feasible and optimum solution

3) Consider the LPP

Maximize $Z = 3x_1 + 5x_2$

subject to the constraints,

$x_1 + 2x_2 \leq 4,\ 2x_1 + x_2 \geq 6$

and $x_1, x_2 \geq 0$

This problem represents :

a) zero-one IPP b) pure IPP c) mixed IPP d) non-IPP

4) For a two person game with A and B, the minimizing and maximizing players, the optimum strategies are

   a) minimax for A and maxmin for B

   b) maximax for A and minimax for B

   c) minimin for A and maxmin for B

   d) maximin for A and minimax for B

5) The quadratic form X^TQX is said to be negative semi-definite if

   a) $X^TQX > 0$ b) $X^TQX < 0$

   c) $X^TQX \geq 0$ d) $X^TQX \leq 0$

B) Fill in the blanks : **5**

1) A set of vectors $X_1, X_2, \ldots X_n$ which satisfies the constraints of LPP is called _______

2) An optimum solution is considered the __________ among feasible solutions.

3) In dual simplex method the starting basic solution is always_________

4) A Quadratic programming problem is based on __________ simplex method.

5) A pair of strategies (p, q) for which $\underline{V} = \overline{V} = V$ is called _________ of E (p,q).

C) State whether the following statements are **True** or **False.** **4**

1) Linear programming problem is probabilistic in nature.

2) The solution to maximization LPP is not unique if $(z_j - c_j) > 0$ for each of the non- basic variables.

3) Dual simplex method is an alternative method to Big M method.

4) In a two person zero sum game, a game is said to be fair if both the players have equal number of strategies.

2. a) i) Show that dual of the dual of an LPP is primal.

   ii) Explain the graphical method of solving $m \times 2$ game. **6**

b) Write short notes on the following :

   i) Artificial variables ii) Unrestricted variables. **8**

3. a) Use two phase method to solve the LPP : **6**

Minimize $Z = x_1 + x_2 + x_3$

$x_1 + x_2 + x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 = 4$

and $x_1, x_2, x_3, x_4 \geq 0$

b) Use simplex method to solve the problem : **8**

Maximize $Z = 3x_1 + 2x_2 + x_3$

subject to the constraints,

$2x_1 + 5x_2 + x_3 = 12$

$3x_1 + 4x_2 = 11$

$x_1, x_3 \geq 0$, x_2 is unrestricted.

4. a) State and prove basic duality theorem. **8**

b) Obtain an optimum solution, if any, to the following LPP **6**

Maximize $Z = 5x_1 + 8x_2 + 10x_3$

subject to the constraints,

$x_1 + x_2 + 2x_3 \leq 20$

$3x_1 - 2x_2 - x_3 \geq 90$

$2x_1 + 4x_2 + 2x_3 = 100$

and x_1, x_2, x_3 is ≥ 0

5. a) Describe branch and bound method of solving Integer Programming Problem. **7**

b) Solving the following IPP using Gomorey's cutting plane algorithm. **7**

Maximize $Z = 110x_1 + 100x_2$

subject to the constraints,

$6x_1 + 5x_2 \leq 29$

$4x_1 + 14x_2 \leq 48$

and $x_1, x_2 \geq 0$ and integers

6. a) Use Beale's method to solve the following problem. **6**

Minimize $Z = -4x_1 + x_1^2 - 2x_1 x_2 + 2x_2^2$

subject to the constraints,

$2x_1 + x_2 \geq 6$

$x_1 - 4x_2 \geq 0$

and $x_1, x_2 \geq 0$.

b) Describe the Wolfe's method for solving Quadratic Programming Problem. **8**

7. a) Obtain optimal strategies for both players and value of game from the following payoff matrix. **6**

Player B

		B_1	B_2	B_3	B_4	B_5	B_6
Player A	A_1	1	3	−1	4	2	−5
	A_2	−3	5	6	1	2	0

b) Explain the following terms : **8**

i) Two person zero sum game.

ii) Pure and mixed strategy.

iii) Principle of dominance

iv) Supporting and separating hyper planes.

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Seat No.	

M.Sc. (Part – II) (Semester – IV) (CGPA) Examination, 2015
MATHEMATICS (Paper – XX)
Elective III : Probability Theory

Day and Date : Friday, 27-11-2015 Total.Marks : 70

Time : 2.30 p.m. to 5.00 p.m.

***Instructions** : 1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) Let $F_1 = \{A, A^C, \phi, \Omega\}$ and $F_2 = \{B, B^C, \phi, \Omega\}$ then

a) $F_1 \cup F_2$ is field

b) $F_1 \cap F_2$ is field

c) Only F_1 is field

d) Only F_2 is field

2) σ - field is closed under

a) Complementation and finite intersection

b) Complementation and finite union

c) Complementation and countable union

d) Countable union and finite intersection

3) The C.L.T. states that for large n, the sample mean has approximately _________ distribution.

a) Bernaulli

b) Poisson

c) Cauchy

d) Normal

P.T.O.

4) Convergence is probability implies
   a) Almost sure convergence
   b) Convergence in distribution
   c) Convergence in r^{th} mean
   d) All the above

5) If $\{X_n, n \geq 1\}$ are non-negative random variables then
   a) $E(\underline{\lim} X_n) = \underline{\lim} E(X_n)$
   b) $E(\underline{\lim} X_n) \leq \underline{\lim} E(X_n)$
   c) $E(\underline{\lim} X_n) \geq \underline{\lim} E(X_n)$
   d) None of these

B) Fill in the blanks : **5**

1) Characteristic function of degenerated random variable at X = 0 is __________
2) A __________ linear combination of indicator sets is called elementary function.
3) Minimal field containing Ω is __________
4) A sequence of sets $\{A_n\}$ is said to be monotonic decreasing if __________
5) A sample space Ω contains n points then number of subsets of Ω are __________

C) State whether the following statements are **True** or **False** : **4**

1) Arbitrary union of σ-fields is always a σ-field.
2) A monotone field is a σ-field.
3) A probability measure is a non-decreasing function.
4) If $|X|$ is integrable does not implies X is integrable.

2. a) i) Give two definitions of field and establish their equivalence. **6**

ii) Define limit of sequence of sets. Show that if lim A_n exists then $\lim A_n^c$ also exists.

b) Write short notes on the following : **8**

1) Conditional probability measure.
2) Bernoulli's WLLN.

3. a) State and prove necessary and sufficient condition for convergence in probability.

   b) Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$.

   Show that :

   i) $X_n + Y_n \xrightarrow{P} X + Y$

   ii) $X_n Y_n \xrightarrow{P} XY$ **(6+8)**

4. a) State and prove Fatou's lemma.

   b) i) If X $\geq$ Y a.s. then show that $E(X) \geq E(Y)$.

   ii) Show that X is integrable iff $|X|$ is integrable. **(6+8)**

5. a) Define monotone field. Prove that every monotone field is a σ-field.

   b) Let $A_n = \left\{\omega : 0 < \omega < 1 + \frac{1}{n}\right\}$. Find lim A_n. **(7+7)**

6. a) State and prove continuity property of probability measure.

   b) State and prove Borel-Cantelli lemma. **(7+7)**

7. a) Define characteristic function of a random variable X. Examine the effect of change of origin and scale on characteristic function.

   b) Find the distribution of random variable X when characteristic function is :

   i) $\phi_x(t) = \frac{1}{1 + t^2}$

   ii) $\phi_x(t) = e^{-|t|}$ **(6+8)**
