SLR-BP – 468

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – II)
Real Analysis (New)

Day and Date : Friday, 17-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

***Instructions** : 1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative. **5**

1) A set may have
   a) No limit point
   b) A unique limit point
   c) Finite or infinite number of limit points
   d) All the above

2) The limit points of $S_n = 1 + (-1)^n$ are
   a) 1, 0 b) 0, 2
   c) 1, 1 d) 2, 1

3) The function $f(x) = x^2$ is
   a) Continuous b) Discontinuous
   c) Uniformly continuous d) None of these

4) The improper integral $\int_{-\infty}^{\infty} e^x dx =$
   a) 0 b) 1
   c) π d) ∞

5) The function f is bounded and integrable on [a, b] then f is
   a) Continuous on [a, b] b) Differentiable on [a, b]
   c) Both a) and b) d) Neither a) nor b)

B) Fill in the blanks : **5**

1) A set of all limit points of a set is called ________ set.
2) A set is closed if and only if its complement is _________
3) Every convergent bounded sequence has _______ limit.
4) If a power series converges for all values of x, then it is called _________ convergent.
5) The radius of convergence of series $1 + 2x + 3x^2 + 4x^3 + ...$is _________

C) State whether the following statements are **true** or **false** : **4**

1) The limit point of a set is always a member of that set.
2) A sequence cannot converge to more than one limit points.
3) Every power series is convergent for x = 0.
4) The function $f(x) = \frac{1}{2}$ is uniformly convergent on (0, 1].

2. a) State the following : **6**
   i) Taylor's theorem
   ii) Heine-Borel theorem
   iii) Bolzano-Weierstrass theorem.

   b) Write short notes on the following : **8**
   i) Countable and uncountable sets.
   ii) Radius of convergence.

3. a) Define open set. Give an example of an open set and other one which is not open set with justifications.
   b) Prove that finite intersection of open sets is an open set.
   c) Show that the set of real numbers in [0, 1] is uncountable. **(5+5+4)**

4. a) Define Cauchy sequence. Prove that every Cauchy sequence is convergent.
   b) Examine the convergence of following sequence.

   i) $S_n = \frac{1}{1!} + \frac{1}{2!} + ... + \frac{1}{n!}, \forall n \in N$

   ii) $S_n = \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{n+n}, \forall n \in N$ **(6+8)**

5. a) Describe any four tests for convergence of series.

b) Show that the series $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ converges absolutely for all values of x.

c) Show that for any fixed value of x, $\sum_{n=1}^{\infty} \frac{\sin nx}{n^2}$ is convergent. **(8+3+3)**

6. a) Define Riemann integral. Prove that every continuous function is integrable.

b) Find the radius of convergence of the following series.

i) $1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \ldots$

ii) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ **(8+6)**

7. a) Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = 3a$.

b) Show that the function $f(x) = x^2$ is uniformly continuous on $[-1, 1]$.

c) Test the convergence of $\int_0^1 \frac{dx}{\sqrt{1 - x^3}}$. **(6+4+4)**

____

SLR-BP – 469

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – III)
Linear Algebra (New)

Day and Date : Monday, 20-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : 1) *Attempt **five** questions.*

2) *Q.No. **1** and Q. No. **2** are **compulsory**.*

3) *Attempt **any three** from Q. No. **3** to Q. No. **7**.*

4) *Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative :

i) The rank of $A = \begin{bmatrix} 2 & 0 & 0 \\ 3 & 3 & 6 \\ 2 & 2 & 4 \end{bmatrix}$ is

A) 1 B) 2 C) 3 D) None of these

ii) The characteristic of a real symmetric orthogonal matrix are

A) 0 and 1 B) – 1 and 1

C) – 1 and 0 D) None of these

iii) The quadratic form $X_1^2 - X_2^2$ is

A) Positive definite B) Negative definite

C) Indefinite D) None of these

iv) A square matrix A is called skew-symmetric matrix if

A) $A = A^T$ B) $A = A^{-1}$ C) $A = A^T A$ D) $A = -A^T$

v) Let $V = \{X, X, X \mid X \in R\}$ be a vector space then dimension of V is

A) 1 B) 2 C) 3 D) None of the above

P.T.O.

B) Fill in the blanks :

I) If $A_{n \times n}$ is a non-singular matrix, then rank (A) = ____________

II) The system of equation : $2x + 2y = 6, 3x - y = 5, 2x + y = 5$ has ____________ solution.

III) If the trace and determinant of a 2× 2 matix are 10 and 16, then the largest characteristic root is ____________

IV) The matrix A of the quadratic form $X_1^2 + X_2X_3$ is ____________

V) The trace of a matrix is ____________ of diagonal elements of a matrix.

C) State whether the following statements are **True** or **False** :

I) If A is a positive semidefinite matrix then $|A|$ is zero.

II) Let A = $[1, 2, 3]^T$ then G = [1 0 0] is a g-inverse of A.

III) Moore-Penrose inverse is not unique.

IV) The symmetric matrix A of the quadratic form $(X_1 - X_2)^2$ is $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

2. a) Answer the following : **(5+5+4)**

I) Discuss classification of quadratic form.

II) Define Moore-Penrose inverse and state its properties.

b) Write short notes on the following :

I) Choleskey decomposition.

II) Vector space and subspace. **(6+8)**

3. a) Explain linearly independent set of vectors. Let X and Y be n-component linearly independent vectors. Show that $X + \alpha Y$ and $X + \beta Y$ are also linearly independent if $\alpha \neq \beta \neq 0$.

b) Describe Gram-Schmidt orthogonalization process using this construct an orthonormal basis for the vector space spanned $\underline{a}_1$ and $\underline{a}_2$ as given below

$a_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $a_2 = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$. **(7+7)**

4. a) Explain the following terms with one illustration :
   I) Trace of a matrix
   II) Inverse of a matrix
   III) Kronekar product.

   b) Define rank of a matrix. Show that rank (AB) $\leq$ min {rank (A), rank (B)}. **(6+8)**

5. a) Define homogenous system of equations. Explain trival and non-trivial solution for the same system. Also, prove that there are n – r linearly independent solutions to this system, where rank (A) = r.

   b) Define g-inverse of a matrix. Find the g-inverse of $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \end{bmatrix}$ and verify the same. **(7+7)**

6. a) Define characteristic roots and vectors of a matrix. If λ be a characteristic root of matrix A prove that

   I) $\frac{1}{\lambda}$ is the characteristic root of A^{-1}

   II) λ^K is the characteristic roots of A^K.

   b) Derive the spectral decomposition of a real symmetric matrix. Obtain for the same $\begin{bmatrix} 1 & 0.5 \\ 0.5 & 1 \end{bmatrix}$. **(7+7)**

7. a) Show that necessary and sufficient condition for a real quadratic form X^TAX to be positive definite is that gi $\geq$ 0 for i = 1,2---n, where

$$gi = \begin{vmatrix} a_{11} & a_{12} --- & a_{1i} \\ a_{21} & a_{22} --- & a_{2i} \\ a_{i1} & a_{i2} --- & a_{ii} \end{vmatrix}$$

   b) Reduce the quadratic form $2x_1^2 + x_2^2 - 3x_3^2 - 8x_2x_3 - 4x_1x_3 + 12x_1x_2$ to a canonical form. **(8+6)**

SLR-BP – 470

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – IV)
Distribution Theory (New)

Day and Date : Wednesday, 22-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*

*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*

*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*

*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Suppose $(X_1, X_2, \ldots X_k)$ is a multinomial random variate then $Cov(X_i, X_j)$, i = j = 1, 2, . . . , k, $i \neq j$ is

a) np_i b) np_ip_j c) $-np_ip_j$ d) $n^2 p_ip_j$

2) If X is standard normal variate the mean of X^2 is

a) 0 b) 0.5 c) 0.75 d) 1

3) Suppose X is Poisson (λ) random variate and we define Y = 2 X. Then distribution of Y is

a) Poisson (λ) b) Poisson (2λ)

c) Poisson $\left(\frac{\lambda}{2}\right)$ d) Not Poisson

4) Suppose X is U(0, 1) random variable then Y = – log X has _______ distribution.

a) uniform b) exponential c) normal d) none of these

5) Which of the following is not a scale family ?

a) $U(0, \theta)$ b) U(0, 1)

c) $N(0, \sigma^2)$ d) Exp (θ)

P.T.O.

B) Fill in the blanks : **5**

1) The moment generating function of $N(0, \sigma^2)$ random variable is ________

2) Probability generating function of B(n, p) random variable is ________

3) Let $X_1, X_2, \ldots X_n$ are i.i.d. N(0, 1) random variables. The distribution of $\sum_{i=1}^{n} X_i^2$ is ________

4) Let X is uniformly distributed over (0, 1) then distribution of 1-X is ________

5) Let $X_1, X_2, \ldots X_n$ are i.i.d. $N(0, \sigma^2)$ random variables and $\overline{X}$ is sample mean. Then distribution of $\overline{X}$ is ________

C) State whether the following statements are **True** or **False**. **4**

1) For r = 2, Markov inequality reduces to Liapounov inequality.

2) If X is symmetric about α then $(X - \alpha)$ is symmetric about zero.

3) If X > 0 then $E[\log X] \geq \log E[X]$.

4) If F(x) is distribution function then $[F(x)]^2$ is also a distribution function.

2. a) Answer the following : **6**

i) Define scale family. Illustrate it with one example.

ii) Let X has N(0, 1) distribution. Find the distribution of $F_x(x)$.

b) Write short notes on the following : **8**

i) Bivariate exponential distribution.

ii) Non-central chi-square distribution.

3. a) Using Jensen's inequality derive the relationship between A. M., G. M. and H. M.

b) Decompose the following distribution function into discrete and continuous components.

$$F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x+1}{4}, & 0 \leq x < 1 \\ \dfrac{2+x}{4}, & 1 \leq x \leq 2 \\ 1, & x > 2 \end{cases}$$

(7+7)

4. a) State and prove Holder's inequality.

   b) Define power series distribution. Show that Geometric distribution is power series distribution. Obtain m.g.f. of geometric distribution using m.g.f. of power series distribution. **(7+7)**

5. a) Define distribution function of bivariate random variate (X, Y). State and prove its important properties.

   b) Let X and Y be independent random variables each with densities

   $f_x(x) = \frac{1}{\pi}\frac{1}{\sqrt{1-x^2}}$, $|x| < 1$ and $f_y(y) = \frac{y}{\sigma^2} e^{-\frac{y^2}{2\sigma^2}}$. Show that Z = X Y is $N(0, \sigma^2)$.

   (7+7)

6. a) Define order statistics. Derive the joint distribution of r^{th} and s^{th} order statistics.

   b) Obtain the probability generating function of the negative binomial distribution and hence find its mean. **(7+7)**

7. a) Define truncated normal distribution truncated below a. Obtain its mean.

   b) If X and Y are jointly distributed with probability density function (p.d.f.)

   $f(x, y) = 24\,xy,\ x \geq 0,\ y \geq 0$ and $x + y \leq 1$.

   Find :

   i) Marginal distributions of X and Y.

   ii) Conditional distribution of Y given X = x.

   iii) E(Y/X = x). **(7+7)**

____

SLR-BP – 474

Seat No.	

M.Sc. (Part – I) (Sem. – I) Examination, 2015
STATISTICS (Paper – III) (Old)
Linear Algebra

Day and Date : Monday, 20-4-2015 Max. Marks : 70

Time :11.00 a.m. to 2.00 p.m.

Instructions : *i) Attempt **any five** questions.*
*ii) Q.No. **(1)** and Q.No. **(2)** **compulsory**.*
*iii) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*iv) Figures to **right** indicates **full** marks.*

1. A) Select the correct alternative :

i) If $\underline{X}$ and $\underline{Y}$ are linearly independent, then $\underline{X} + \alpha\underline{Y}$ and $\underline{X} + \beta\underline{Y}$ are linearly dependent if

A) $\alpha = \beta$ B) $\alpha < \beta$ C) $\alpha > \beta$ D) $\alpha \neq \beta$

ii) The characteristic roots of a real symmetric orthogonal matrix are

A) 0 or 1 B) –1 or 1 C) 0 or –1 D) None of these

iii) The rank of $A = \begin{bmatrix} 4 & 0 & 0 \\ 6 & 6 & 12 \\ 4 & 4 & 8 \end{bmatrix}$ is

A) 2 B) 1 C) 3 D) None of these

iv) Let A be an idempotent matrix. Then the value of $\max_{X} \dfrac{X'AX}{X'X}$ is

A) 0 B) 1

C) Cannot be determined D) None of these

v) The determinant and trace of 2×2 matrix A are 12 and 8 respectively, then characteristic roots are

A) 2 and 6 B) 3 and 4 C) 12 and 1 D) 8 and 1

P.T.O.

B) Fill in the blanks :

i) If λ is characteristic root of A, then the characteristic root of (A + I) is ____

ii) The dimension of the vector space $V = \{(x, y, x + 2y) : x, y \in R\}$ is ______

iii) The rank of a $K \times K$ orthogonal matrix is ______

iv) The quadratic form $x_1^2 + x_2^2$ is ________ definite.

v) The system of equations $2x + 2y = 6$, $x - y = 1$, $4x + 2y = 10$ has ______ solution.

C) State **true** or **false** :

i) Moore Penrose (M – P) inverse is not unique.

ii) A matrix $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ is positive semidefinite matrix.

iii) P is an idempotent matrix if $P = P^2$.

iv) The g-inverse of (1, 1, 1) is $(1, 1, 1)^T$. **(5+5+4)**

2. a) i) Define inverse of matrix. Find the inverse of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

ii) Obtain g-inverse of $\begin{bmatrix} 1 & 0 & 2 \\ 1 & 2 & 2 \\ 2 & 0 & 4 \end{bmatrix}$.

b) Write short notes on the following :

i) Row and column space of a matrix.

ii) Classification of a quadratic form. **(6+8)**

3. a) Define and illustrate giving one example each (i) Vector space (ii) Canonical form of a quadratic form.

b) Describe Gram-Schmidt orthogonalization process. Using this method obtain an orthogonal basis for R^2 starting with vector $a_1 = (2, 4)$ and $a_2 = (2, 8)$. **(7+7)**

4. a) Define rank of a matrix. Prove that rank (AB) $\leq$ min {rank (A), rank (B)}.

b) Let X and Y be n-component linearly independent vectors. Show that $X + \alpha Y$ and $X + \beta Y$ are also linearly independent if $\alpha \neq \beta$. **(7+7)**

5. a) Define (i) trace of a matrix (ii) symmetric matrix (iii) skew-symmetric matrix. Give an example each.

   b) Let A and B be two square matrices. Then prove or disprove AB and BA have the same characteristic roots. **(7+7)**

6. a) State and prove a necessary and sufficient condition for a system of linear equations AX = b to be consistent.

   b) Examine for the definiteness of the quadratic form (i) $4x_1^2 - 4x_1x_2 + x_2^2 + x_3^2$

   (ii) $\sum_{i=1}^{n} x_i^2$. **(7+7)**

7. a) Explain the spectral decomposition of a symmetric matrix. Give an illustration.

   b) Prove that a necessary and sufficient condition for a quadratic form $X'AX$ to

   be positive definite is that $\begin{vmatrix} a_{11} \cdots & a_{1i} \\ \vdots & \\ a_{i1} \cdots & a_{ii} \end{vmatrix} > 0$ for i = 1, 2, . . .n. **(7+7)**

____

SLR-BP– 475

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – IV)(Old)
Distribution Theory

Day and Date : Wednesday, 22-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **Five** questions.*

*2) Q. No. (**1**) and Q. No. (**2**) are **Compulsory**.*

*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**).*

*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Which of the following distribution is not a member of scale family ?

a) $U(0, \theta), \theta > 0$ b) U (0, 1)

c) $N(0, \sigma^2)$ d) Exponential with mean θ

2) The probability density function of a random variable X is symmetric about zero. The value of distribution function F(x) at x = – 1 is

a) 1 – F(1) b) $F(1) - \frac{1}{2}$

c) $F(1) + \frac{1}{2}$ d) F(–1)

3) If X > 0 then

a) E [log X] = log [E (X)] b) E [log X] $\geq$ log [E(X)]

c) E [log X] $\leq$ log [E(X)] d) None of these

4) Let X be a random variable with probability generating function P_x (S). The probability generating function of 2 X will be

a) P_x (2 S) b) P_x (S/2)

c) P_x (S + 2) d) P_x (S^2)

P.T.O.

5) Let X be a r.v. with p.d.f. $f(x) = \frac{1}{\sqrt{\pi}} \exp\left\{-\frac{1}{\sqrt{2}} x^2\right\}, x \in R$.

The mean and standard deviation of X are

a) 1 and 1 b) 0 and 1

c) 0 and $\frac{1}{\sqrt{2}}$ d) 1 and $\frac{1}{\sqrt{2}}$

B) Fill in the blanks : **5**

1) If x is continuous random variable with distribution function F (x) then F (x) has ___________ distribution.

2) Probability generating function of Poisson distribution with mean λ is __________.

3) Let X has standard exponential distribution then $P\left[F(x) \leq \frac{1}{4}\right] =$ _________.

4) Let X has B (1, p) distribution then (1 –X) has ________ distribution.

5) If Z is standard normal variate the variance of Z^2 is ________.

C) State whether the following statements are **True** or **False**. **4**

1) If X has Poisson distribution then 2 X has Poisson distribution.

2) The probability generating function is used for discrete variables only.

3) The moment generating if exists is unique.

4) $\{N(\theta,1), \theta \in R\}$ is a scale family.

2. a) i) Define location-scale family. Given an example of the same.

ii) Show that Binomial distribution is a particular case of power series distribution. **6**

b) Write short notes on the following : **8**

1) Mixture of distribution.

2) Moment generating function.

3. a) State and prove the relation between distribution function of a continuous random variable and uniform variable.

   b) State and prove Holder's inequality. **(6+8)**

4. a) State and Prove the result of generating geometric random variables from U (0,1) variates.

   b) Let X be $U(0, \theta)$, where θ is an integer greater than one. Find the distribution of Y = [X]. **(7+7)**

5. a) Define multinomial distribution. Obtain its moment generating function. Hence obtain the p.m.f. of trinomial distribution.

   b) If X and Y are jointly distributed with p.d.f. $f(x, y) = x + y; 0 \leq x \leq 1, 0 \leq y \leq 1$.

   Find $P[X > \sqrt{y}]$. **(8+6)**

6. a) Define bivariate Poisson distribution. Describe the method of generating random observations from it.

   b) If X and Y are independent N (0,1) variates. Show that

   $E[\text{Max}(X, Y)] = \frac{1}{\sqrt{\pi}}$. **(7+7)**

7. a) Define order statistics. Derive the joint distribution of r^{th} and s^{th} order statistics.

   b) If (X, Y) is a bivariate random vector having joint p.d.f.

   $f(x, y) = \theta^2 e^{-\theta y}, 0 < x < y < \infty$. Find E [X/y] and E [Y/x]. **(7+7)**

____

Seat No.	

M.Sc. (Part – I) (Semester – I) Examination, 2015
STATISTICS (Paper – V) (Old)
Theory of Estimation

Day and Date : Friday, 24-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select the correct alternative :

1) _______ is not a power series distribution.

a) B(n, p) b) C(1, 0)
c) N(0, 1) d) N(1, 1)

2) Suppose $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ are two independent random samples from two populations having equal variance σ^2.

Let $A = \sum_{i=1}^{m}(X_i - \overline{X})^2$ and $B = \sum_{i=1}^{n}(Y_i - \overline{Y})^2$. Then an unbiased estimator of σ^2 is

a) $\frac{A+B}{m+n-2}$ b) $\frac{A+B}{m+n}$
c) $\frac{A+B}{m+n-1}$ d) $\frac{A}{m-1}+\frac{B}{n-1}$

3) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from $N(\mu, \sigma^2)$ distribution, where both μ and σ^2 are unknown. Then, . . . is a complete sufficient statistics.

a) $\sum_{i=1}^{n} X_i$ b) $\frac{1}{n-1}\sum_{i=1}^{n} X_i$
c) $\sum_{i=1}^{n} X_i^2$ d) $\left(\sum_{i=1}^{n} X_i, \sum_{i=1}^{n} X_i^2\right)$

4) Let X ~ b(1, p), where p $\in$ (1/4, 3/4). The MLE of p on the basis of a single observation X is

a) X
b) $\frac{2X+1}{4}$
c) $1 - X$
d) $1 + X$

5) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with finite mean μ. The U-statistic for estimating μ is

a) $\sum_{i=1}^{n} X_i$
b) $\frac{1}{n}\sum_{i=1}^{n} X_i$
c) $\sum_{i=1}^{n} (X_i - \overline{X})$
d) None of above **(1×5)**

B) Fill in the blanks :

1) __________ is a function of every sufficient statistic.
2) Suppose T is an unbiased estimator of θ. Then g(T) is unbiased for g(θ) if g is a _____ function.
3) __________ statistic is independent of every complete sufficient statistic.
4) An unbiased estimator that attains C-R lower bound is called ____________
5) Suppose $X_1, X_2, \ldots, X_n$ is a random sample from a distribution with finite variance σ^2, then MME of σ is ___________ **(1×5)**

C) State **true** or **false** :

1) An unbiased estimator always exists.
2) MLE is not necessarily unique.
3) Bayes estimators are functions of sufficient statistics.
4) Symmetric kernel does not exist for every estimable parameter. **(1×4)**

2. a) i) Show that every one-to-one function of a sufficient statistic is also sufficient.
ii) Describe the method of minimum Chi-square. **(3+3)**

b) Write short notes on the followings :
i) Bootstrap method
ii) U statistic **(4+4)**

3. a) State and prove a necessary and sufficient condition for an estimator of a parametric function $\psi(\theta)$ to be UMVUE.

   b) Derive UMVUE of $1/\theta$ based on a random sample from $U(0, \theta)$ distribution. **(7+7)**

4. a) State Factorization theorem to determine a sufficient statistics and prove it for a discrete family of distributions.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(0, \sigma^2)$ distribution. Show that $\sum_{i=1}^{n} X_i^2$ is a minimal sufficient statistics and $\sum_{i=1}^{n} X_i$ is not a sufficient statistic for σ^2. **(7+7)**

5. a) Define completeness. Prove or disprove that $\{U(0, \theta), \theta \in (0, \infty)\}$ is a complete family.

   b) Define ancillary statistic. Suppose X_1 and X_2 are iid observations from pdf $f_\alpha(x) = \alpha x^{\alpha-1} e^{-x\alpha}$; $x > 0$, $\alpha > 0$. Show that $\log X_1 / \log X_2$ is an ancillary statistic. **(7+7)**

6. a) Define MLE. State and prove the invariance property of MLE.

   b) Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with location parameter μ and scale parameter σ. Obtain MLE of $\mu + \sigma$. **(7+7)**

7. a) State and prove Chapman-Robbins-Kiefer inequality.

   b) Define Fisher information contained in a single observation and in n (> 1) independent and identically distributed observations. Obtain Fisher information matrix in case of $N(\mu, \sigma^2)$ distribution. **(7+7)**

____

SLR-BP – 477

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI)
Probability Theory (New)

Day and Date : Thursday, 16-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) Let P(.) is a probability measure defined on (Ω, IF). Then P(Ω) = 1 is ______ property of the measure.

a) Normed b) Non-negativity
c) Finite additivity d) Sigma additivity

2) If A_n is equal to A or B according as n is odd or even, the $\overline{\lim} A_n =$

a) 0 b) 1
c) $A \cap B$ d) $A \cup B$

3) Let X be a random variable defined on (Ω, IF) then

a) X^2 is a random variable b) 1 – X is a random variable
c) |X| is a random variable d) All the above

4) If X_1 and X_2 are independent random variables then $\phi_{x_1+x_2}(t)$ =

a) $\phi x_1(t) + \phi x_2(t)$ b) $\phi x_1(t)\ \phi x_2(t)$
c) $\phi x_1(t)$ d) $\phi x_2(t)$

5) Which of the following are Borel sets of real line ?

a) Single point sets b) Open intervals of type (a, b)
c) Closed intervals of type [a, b] d) All the above

P.T.O.

b) Fill in the blanks : **5**

1) If E(X) is finite then X said to be
2) A finite linear combination of indicators of sets is called ________ function.
3) The minimal σ-field induced by indicator function I_A is
4) The number of points in a set is called
5) A set A is called co-finite set if

c) State whether the following statements are **true** or **false**. **4**

1) The counting measure is a finite measure.
2) Mutual independence implies pairwise independence.
3) If Ω is the set of convergence then $\{X_n\}$ is said to be converge nowhere.
4) Mapping preserves all the set relations.

2. a) Answer the following : **6**

i) For a non-negative random variable X, prove that $E(X) = \int_0^\infty [1 - F(x)]dx$.

ii) Define lim inf and lim sup of sequence of sets $\{A_n\}$.

b) Write short notes on the following : **8**

i) Lebesgue measure.

ii) Indicator function.

3. a) Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence.

b) Find lim inf and lim sup of following sequence of sets.

i) $A_n = \left(1 + \frac{1}{n},\ 2 + \frac{1}{n}\right)$

ii) $A_n = \left[a - \frac{1}{n},\ a\right]$ **(6+8)**

4. a) Define field. Examine for the class of finite or co-finite sets to be a field.

b) Define probability measure. State and prove monotone property of probability measure. **(7+7)**

5. a) If X and Y are simple random variables then prove that $E(X + Y) = E(X) + E(Y)$.

   b) Let E be an experiment having two outcomes 'success' S and 'failure' F respectively. Let $\Omega = \{S, F\}$ and $IF = \{\phi, S, F, \Omega\}$. Define $X(\omega) = \begin{cases} 1, & \text{if } \omega = S \\ 0, & \text{if } \omega = F \end{cases}$. Examine whether X is random variable with respect to IF. **(7+7)**

6. a) Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

   b) State Lindberg-Feller form of central limit theorem and deduce the Liapunov's theorem. **(7+7)**

7. a) Define characteristic function of random variable X. Suppose X is Poisson (λ) random variable. Obtain characteristic function of X.

   b) Find the distribution of random variable X when characteristic function is

      i) $\phi_X(t) = \dfrac{1}{1+t^2}$

      ii) $\phi_X(t) = e^{-|t|}$. **(6+8)**

SLR-BP – 478

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII)
Linear Models (New)

Day and Date : Saturday, 18-4-2015 Total Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*

*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*

*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*

*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) In general linear model, $y = X\beta + \varepsilon$, ______________

a) y is known and X is unknown b) y and X both are unknown

c) y is known and β is unknown d) X is unknown and β is known

2) In two-way ANOVA model $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$; i = 1, 2, ... p; j = 1, 2, ... q, ______________ is estimable.

a) $\mu + \alpha_1$ b) $\alpha_1 - \alpha_2$ c) $\alpha_1 + \alpha_2$ d) $\alpha_1 + \beta_2$

3) In a connected block design with v treatments and b blocks, rank of D matrix is ______________

a) v b) b c) b – 1 d) v – 1

4) For a BIBD with usual notation, bk = ______________

a) rv b) r(v – 1) c) r(k – 1) d) λ(v – 1)

5) A connected design is ______________ balanced.

a) sometimes b) always c) never d) generally **(1×5)**

B) Fill in the blanks :

1) In general linear model $y = X\beta + \in$, a particular solution of the normal equations is ______________

2) The rank of the estimation space in two-way ANOVA with interaction model with p rows and q columns and with one observation per cell is ___________

3) The BLUE of a treatment contrast $\sum_i c_i\alpha_i$ in a two-way ANOVA model is _________

4) The physical variables other than the response variable involved in ANOCOVA model are called ______________

5) A connected block design is orthogonal if and only if ______________ **(1×5)**

P.T.O.

C) State **true** or **false** :

1) In general linear model, not every solution of normal equations minimizes residual sum of squares.
2) In two-way ANOVA model with interaction and one observation per cell the degrees of freedom of SSE is 1.
3) In general linear model, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations.
4) In a connected block design, all elementary treatment contrasts are estimable. **(1×4)**

2. a) i) Show that in general linear model, the normal equations are consistent.
   ii) Define complete block design, connected block design and orthogonal block design. **(3+3)**

   b) Write short notes on the following :
   i) Estimation space.
   ii) C matrix in a general block design. **(4+4)**

3. a) State and prove Gauss-Morkoff theorem.

   b) Define error space for general linear model $y = X\beta + \in$. Prove that a linear function of observations $a'y$ belongs to the error space if and only if the coefficient vector a is orthogonal to the columns of X. **(7+7)**

4. a) Describe error rates in multiple comparisons.

   b) Derive a test for testing a general linear hypothesis in a general linear model. **(7+7)**

5. a) Obtain the rank of the estimation space and a complete set of linearly independent estimable linear parametric function in one-way ANOVA model and show that only contrasts of treatment effects are estimable.

   b) Derive the test for testing the hypothesis of the equality of column effects in two-way ANOVA without interaction model with one observation per cell. **(7+7)**

6. a) Describe ANOCOVA model in general and obtain an expression for error SS.

   b) Describe two-way ANOCOVA model and obtain the least square estimates of its parameters. **(7+7)**

7. a) State and prove a necessary and sufficient condition for orthogonality of a general block design.

   b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. **(7+7)**

———————

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
STATISTICS (Paper – VIII)
Stochastic Processes (New)

Day and Date : Tuesday, 21-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the most correct answer :

1) Number of accidents upto time t (>0) is an example of ___________ time, ___________ state space stochastic process.

a) Discrete, discrete

b) Discrete, continuous

c) Continuous, discrete

d) Continuous, continuous

2) Let $\{X_n, n \geq 0\}$ be a Markov chain with state space {0, 1, 2} and tpm

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & 0 \end{bmatrix} \end{array}$$

which of the following is correct ?

a) State 1 is transient b) State 2 is periodic

c) State 1 and 2 are not communicative d) All states are recurrent

3) If $\{X_n, n \geq 0\}$ be a M.C. with state space {0, 1} and tpm $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ then

$P(X_4 = 0 | X_2 = 0)$__________

a) 1

b) 0.5

c) 0

d) Can't be calculated with available information

4) If $\{X(t), t \geq 0\}$ is a Poisson process with rate $\lambda > 0$ then $P(X(s) = k | X(t) = n)$ for s < t and k < n is

a) $B\left(n, \frac{s}{t}\right), k = 0, 1, \ldots n$

b) $P(\lambda(t - s))$

c) $B\left(k, \frac{s}{t}\right)$, for any k

d) None of these

5) The probability generating function of the offspring distribution of a discrete time branching process is Q(s) = 0.4 + 0.6S. The probability of extinction is

a) 0 b) 0.3

c) 0.7 d) 1

B) Fill in the blanks :

1) In a MC both state space and index set are __________

2) If $\{X_n, n \geq 0\}$ is a MC then $P(X_{10} = i | X_0 = 0, X_1 = 1, X_2 = 2, \ldots, X_9 = 9) =$ __________

3) If a chain is irreducible, __________, and if there exist a unique stationary distribution for the chain, then the chain is ergodic.

4) For a poisson process $\{N(t), t \geq 0\}$ with intensity parameter λ, the variance of N(t) is __________

5) As $t \to \infty$, $\frac{M(t)}{t} \to$ __________, where M(t) is the renewal function.

C) State whether following statements are **true** or **false** :

1) A matrix whose row sum is one is called as Stochastic matrix.

2) $\lim_{n \to \infty} P_{ij}^{(n)}$ always exists.

3) Birth death process is continuous time, discrete state space stochastic process.

4) In a delayed renewal process, inter occurrence times are not i.i.d. **(5+5+4)**

2. A) Let $\{X_n, n \geq 0\}$ be a MC with state space {0, 1}, tpm $P = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}$ and initial distribution is (0.5, 0.5) find,

i) Marginal distribution of X_2

ii) $f_{11}^{(3)}$. **(3+3)**

B) Write short notes on the following :

i) Finite dimensional distributions of stochastic processes.

ii) M/M/1 queuing system. **(4+4)**

3. A) Define :

i) Stochastic processes

ii) Processes with independent increments

iii) Non-homogeneous Markov chain

iv) Period of the state.

B) State the postulates of poisson process. Show that the inter-occurrence times of events of a poisson process are i.i.d. exponential r.v.s. **(7+7)**

4. A) Explain Gambler's ruin problem in detail.

B) Prove that state j is persistent iff

$$\sum_{n=0}^{\infty} P_{jj}^{(n)} = \infty.$$ **(7+7)**

5. A) Define stationary distribution. Obtain the stationary distribution of MC $\{X_n, n \geq 0\}$ with state space {0, 1, 2} and t.p.m.

$$\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

B) State and prove Chapman-Kolmogorov equations. **(7+7)**

6. A) Define Birth and Death Process. Obtain differential equations of the same.

B) Define BGW branching process and in usual notations establish

$\phi_{n+1}(s) = \phi_n(\phi(s))$ **(7+7)**

7. A) Define a renewal process. Show that the renewal function M(t) satisfies

the equation $M(t) = F(t) + \int_0^t M(t-X)dF(X)$.

B) Write down the algorithm for

i) Simulation of MC

ii) Simulation of Poisson process. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – X)
Sampling Theory (New)

Day and Date : Saturday, 25-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative. **5**

1) If n units are selected in a sample from N population units, then sampling fraction is ____________

a) $\frac{1}{N}$ b) $\frac{1}{n}$ c) $\frac{n}{N}$ d) $1 - \frac{n}{N}$

2) In a stratified sampling with strata sizes N_1 and N_2, stratum variances S_1^2, S_2^2 under Neyman allocation the ratio of sample size $\frac{n_1}{n_2}$ is ________

a) $\frac{N_1}{N_2}$ b) $\frac{N_1 S_1}{N_2 S_2}$ c) $\frac{S_1}{S_2}$ d) $\frac{N_1 S_1^2}{N_2 S_2^2}$

3) In simple random sampling the ratio estimator is ____________

a) always biased
b) always unbiased
c) minimum variance unbiased
d) none of these

4) If 100 students are selected out of 500, and 15 students are then selected from the 100 selected students. The procedure adopted is __________

a) cluster sampling b) systematic sampling

c) two-stage sampling d) stratified sampling

5) Hurwitz-Hansen technique is used to deal with __________

a) non response errors b) non sampling errors

c) sampling errors d) none of these

b) Fill in the blanks : **5**

1) Cluster sampling helps to __________ cost of survey.

2) A basic principle of stratifying a population is that the strata should be internally ____________

3) Under SRSWR, the sample unit can occur __________ times in the sample.

4) In Midzuno sampling scheme, the unit at first draw is selected with _____________ probabilities.

5) Failure to measure some of the units in the selected sample is _______ error.

c) State whether the following statements are **true** or **false** : **4**

1) Regression estimators are generally biased.

2) Deep stratification is a technique used to deal with non sampling errors.

3) Systematic sampling is equal probability sampling.

4) In PPS sampling some units may be selected with probability one.

2. a) Answer the following : **6**

i) What are basic principles of sample survey ? Write in brief advantages of sampling over census method.

ii) Define circular systematic sampling. Give an example.

b) Write short notes on the following : **8**

i) Cumulative total method

ii) Midzuno system of sampling.

3. a) Explain and illustrate the benefits of stratifying a population before sampling.

   b) Describe any two methods for allocating a sample of size n to different strata of population. **(6+8)**

4. a) Explain the concept of systematic sampling. Derive the sampling variance of unbiased estimator of population mean under the linear systematic sampling.

   b) Explain cluster sampling and clearly specify the advantages of the scheme. **(7+7)**

5. a) Explain the ratio and regression methods of estimation. When are these methods considered to be efficient ?

   b) Define unbiased and almost unbiased ratio-type estimators. **(8+6)**

6. a) Define PPSWR sampling design. Obtain an unbiased estimator of the population mean and its variance when a PPSWR sample of size n is drawn from a population of size n.

   b) Define Horvitz-Thompson estimator of population mean and establish its unbaisedness under an arbitrary sampling design. Also derive its sampling variance. **(7+7)**

7. a) Explain the problem of non response and any one technique to deal with the non response.

   b) What is double sampling ? Explain any one practical situation where double sampling is appropriate. **(8+6)**

____

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VI) (Old)
Probability Theory

Day and Date : Thursday, 16-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) Expectation of random variable $X = X^+ - X^-$ is said to exist if ___________
a) at least one of $E(X^+)$ or $E(X^-)$ is finite
b) both $E(X^+)$ and $E(X^-)$ are finite
c) both $E(X^+)$ and $E(X^-)$ are infinite
d) none of these

2) Which one of the following statement is correct ?
a) every field is a σ-field
b) union of fields is a field
c) intersection of fields is a field
d) $\{A, A^C\}$ is a field, where A is proper non-empty subset of Ω .

3) Which one of the following statement is correct ?
a) $X_n \xrightarrow{a.s.} X \Rightarrow X_n \xrightarrow{P} X$
b) $X_n \xrightarrow{L} X \Rightarrow X_n \xrightarrow{P} X$
c) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{a.s.} X$
d) $X_n \xrightarrow{P} X \Rightarrow X_n \xrightarrow{r} X$

P.T.O.

4) The trivial field is ______________

a) $\{\phi\}$ b) $\{\Omega\}$

c) $\{\phi, \Omega\}$ d) none of these

5) Let X Poisson (λ) variate. Then characteristic function of X is ____________

a) $e^{-\lambda(1-e^{it})}$ b) $e^{\lambda(1-e^{-it})}$

c) $e^{-\lambda(e^{it}-1)}$ d) $e^{\lambda(e^{-it}-1)}$

B) Fill in the blanks : **5**

1) If $\phi_x(t)$ is real then X is ____________

2) A sequence of sets $\{A_n\}$ is said to be monotonic increasing if ____________

3) If $X_n \xrightarrow{r} X$ then $E|X_n|^r \to$ __________

4) If A is finite set with n elements then power set of A contains __________ elements.

5) If P $(\cdot)$ is a probability measure then $P(A^C)$ = ____________

C) State whether the following statements are **TRUE** or **FALSE**. **4**

1) Almost sure convergence always implies convergence in probability.

2) Every field is a σ-field.

3) Mutual independence implies Pairwise independence.

4) Every field contains empty set ϕ.

2. a) Answer the following : **6**

i) Define field and σ-field.

ii) Prove that every σ-field is a field.

b) Write short notes on the following : **8**

i) Mutual and Pairwise independence.

ii) Strong law of large numbers.

3. a) If F_1 and F_2 are fields. Show that

   i) $F_1 \cap F_2$ is a field.

   ii) $F_1 \cup F_2$ is not a field.

   b) Find lim A_n if exist. $A_n = \left(1+\frac{1}{n}, 2+\frac{1}{n}\right)$ **(8+6)**

4. a) State and prove continuity property of probability measure.

   b) If $X_n \leq Y$ and Y is integrable then show that $E(\overline{\lim} X_n) \geq \overline{\lim} E(X_n)$. **(7+7)**

5. a) Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$ as $n \to \infty$.

   b) Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and c be a constant. Show that

   i) $X_n + c \xrightarrow{L} X + c$

   ii) $c X_n \xrightarrow{L} cX, c \neq 0$. **(6+8)**

6. a) State Kolmogrov's three series criterion for almost sure convergence.

   b) Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Show that

   $P(\overline{\lim} A_n) = 0$. **(6+8)**

7. a) Define characteristic function. Suppose X is B(n, p) random variable. Obtain characteristic function of X.

   b) State inversion formula and obtain the probability distribution of random variable corresponding to characteristic function $\phi_X(t) = \frac{1}{1+t^2}$. **(6+8)**

____

SLR-BP – 483

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – VII) (Old)
Linear Models and Design of Experiments

Day and Date : Saturday, 18-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) In general linear model, any linear function belonging to the error space and any BLUE are

a) Positively correlated b) Uncorrelated
c) Negatively correlated d) Correlated

2) The rank of the estimation space in one-way ANOVA model with *N* observations and *k* levels of treatment is

a) N – k b) k – 1 c) k d) N – k – 1

3) For a BIBD with usual notation, $\lambda(\nu - l)$ =

a) k (r – 1) b) k (r + 1) c) r (k + 1) d) r (k – 1)

4) In a connected block design with ν treatments and b blocks, rank of C matrix is

a) $\nu - 1$ b) $\nu + 1$ c) ν d) $\nu b - 1$

5) In a general linear model, the normal equation are

a) always consistent b) not always consistent
c) always inconsistent d) not always in consistent

P.T.O.

B) Fill in the blanks :

1) In general linear model $y = X\beta + \in$, a particular solution of the normal equations is __________

2) The rank of the estimation space in two-way ANOVA without interaction model with p rows and q columns and with one observation per cell is __________

3) A block design is __________ if and only if $CR^{-\delta} N = 0$.

4) The degrees of freedom of error SS in two-way ANOVA with interaction model with p rows and q columns and with r > 1 observation per cell is ________

5) The degrees of freedom of error SS in two-way without interaction ANOCOVA model with p rows, q columns, m observation per cell and m covariate is ____________ **(1×5=5)**

C) State **True** or **False** :

1) In a general linear model, if S^- is g-inverse of $S = X'X$, its transpose is not in general g-inverse of S.

2) $\mu + \alpha_i$, i = 1, 2, ..., k, are estimable in one-way ANOVA model with k levels of treatment.

3) In a general linear model $y = X\beta + \in$, the quantity XS^-X' is invariant under the choice of g-inverse of $S = X'X$.

4) A balanced design is always connected. **(1×4=4)**

2. a) i) Show that any solution of normal equations minimizes residual sum of squares.

ii) Prove or disprove that a connected design is always balanced. **(3+3)**

b) Write short notes on the following :

i) Estimation space.

ii) Tuckey's procedure of multiple comparisons. **(4+4)**

3. a) Show that in general linear model $y = X\beta + \epsilon$

   i) $H = H^2$

   ii) $SH = H$

   iii) rank (H) = trace (H) = rank (S) = rank (X), where $S = X'X$, $H = S^- S$, S^- being g-inverse of S. **(2+2+3)**

   b) Prove that in a general linear model $y = X\beta + \epsilon$, the BLUE of every estimable linear parametric function is a linear function of the LHS of normal equations, and conversely, any linear function of the LHS of normal equations is the BLUE of its expected value. **7**

4. a) Describe one-way ANOVA model and obtain the least square estimates of its parameters.

   b) Derive the test for testing the hypothesis of the equality of row effects in two-way ANOVA without interaction model with one observation per cell. **(7+7)**

5. a) Show that in general block design, adjusted treatment totals and block totals are uncorrelated.

   b) State and prove a necessary and sufficient condition for orthogonality of a connected block design. **(7+7)**

6. a) Describe ANOCOVA model in general and obtain the least square estimates of its parameters.

   b) Describe two-way with interaction ANOVA model with $r > 1$ observations per cell and obtain the least square estimates of its parameters. **(7+7)**

7. a) State and prove Gauss-Morkoff theorem.

   b) Prove that in a BIBD, the number of blocks is greater than or equal to the number of treatments. **(7+7)**

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
STATISTICS (Paper – VIII) (Old)
Stochastic Processes

Day and Date : Tuesday, 21-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the most correct answer :

1) Poisson process has
   a) Discrete state space
   b) Discrete index set
   c) Continuous state space
   d) None of the above

2) Suppose $\{X_n, n \geq 0\}$ be a Markov chain, then state j is persistent iff
   a) $\sum P_{jj}^{(n)} = 1$
   b) $\sum P_{jj}^{(n)} = \infty$
   c) $\sum P_{jj}^{(n)} > 1$
   d) $\sum P_{jj}^{(n)} < \infty$

3) Let X(t) be a Poisson process with rate λ. Let P be the probability that an event is marked. Let Y(t) be a marked process then
   a) Y(t) is branching process
   b) Y(t) is birth and death process
   c) Y(t) is Poisson process
   d) None of the above

4) In branching process, if $E(X_1) = m$, then $E(X_n) =$
   a) m^n
   b) n^m
   c) $n * m$
   d) none of the above

5) In a renewal process M(t) = E{N(t)} is called
   a) Renewal equation
   b) Renewals number
   c) Renewal times
   d) Renewal function **5**

P.T.O.

B) Fill in the blanks :

1) In an finite irreducible Markov chain all states are ____________

2) If state K is aperiodic persistent non-null then $\lim_{n \to \infty} P_{kk}^{(n)} \to$ ____________

3) A state j is called ergodic if it is ______________

4) In M/M/S queuing system, 'S' represents ______________

5) If {N (t), t $\geq$ 0} is a Poisson process then interarrival time has ___________ distribution. **5**

C) State whether following statements are **true** or **false** :

1) Period of any state must be greater than one.

2) If initial distribution is known, distribution of Markov chain can be determined completely.

3) State space of birth process is discrete.

4) In usual notations, the elementary renewal theorem asserts, $\frac{M(t)}{t} \to \frac{1}{\mu}$ as $n \to \infty$. **4**

2. A) Define stochastic process. Give two examples. What do you mean by finite dimensional distribution of stochastic processes.

B) Define Markov chain. Show that Markov chain is completely specified by initial distribution and one step transition probability matrix. **(7+7)**

3. A) State and prove first entrance theorem.

B) Define stationary distribution of Markov chain and mean recurrence time of its state. State the relation between two. **(7+7)**

4. A) Let {X_n, n $\geq$ 0) be a Markov chain with state space S = {0, 1, 2}, t.p.m.

$$P = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{4}{5} & \frac{1}{5} & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

and initial distribution (1, 0, 0).

Obtain :

1) $P[X_2 = 2 | X_0 = 1]$

2) $P(X_2 = 1)$

3) $E(X_1)$.

B) Define first passage time. Let $\{X_n, n \geq 0\}$ be a Markov chain with state space $S = \{0, 1, 2\}$ and t.p.m.

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1 & 0 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

Find first passage time distribution for state 1 given that system starts with 2. **(8+6)**

5. A) Define Poisson process. Show that the inter-arrival times of a Poisson process are exponential random variables.

B) Derive the steady state probability distribution of population size in a Birth Death process. **(7+7)**

6. A) Define a renewal process. Show that the renewal function M(t) satisfies the equation $M(t) = F(t) + \int_0^t M(t-x)\, d_F(x)$.

B) Define branching process. In a branching process with the off spring distribution having the p.g.f. P, show that the probability of eventual extinction is the smallest positive root of P(S) = S. **(7+7)**

7. A) Write an algorithm for the simulation of branching process.

B) Explain in detail M/M/S queuing model. **(7+7)**

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – IX) (Old)
Theory of Testing of Hypothesis

Day and Date : Thursday, 23-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q.No. (**1**) Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative

i) Type I error is defined as
A) reject H_0 when H_0 is false
B) reject H_0 when H_0 is true
C) both (A) and (B)
D) none of the above

ii) If the Test function $\phi(x) = \begin{cases} 1 & \text{if } x > c \\ 0 & \text{otherwise} \end{cases}$ then the test is
A) randomised B) non-randomised
C) both (A) and (B) D) neither (A) nor (B)

iii) Testing a simple hypothesis H_0 against a simple alternative H_1, let the power of the MP tests at level α and α' be β and β'. Then always
A) $\beta \geq \beta'$ B) $\beta \leq \beta'$ C) $\beta = \beta'$ D) $\beta \neq \beta'$

iv) For testing $H_0 : \theta \geq \theta_0$ vs $H_1 : \theta < \theta_0$ or $H_0 : \theta \leq \theta_0$ vs $H_1 : \theta > \theta_0$, the UMP test exists for the family of distribution
A) belongs one parameter exponential family
B) has an MLR property
C) either (A) or (B)
D) none of the above

v) Which statement is true ?

A) Every similar test has a Neyman-structure

B) Tests with Neyman-structure is a similar test

C) Both (A) and (B)

D) Neither (A) nor (B)

B) Fill in the blank :

i) The family of U(0, θ) distribution has MLR in _______________, when sample of size 'n' is available from U(0, θ).

ii) Likelihood ratio test for testing $H_0 : \theta \in \Theta_0$ vs $H_1 : \theta \in \Theta_1$ is defined as ___________

iii) MLR property of the distribution is used to obtain _____________ tests.

iv) If $\lambda(x)$ denotes the likelihood ratio statistic, then the asymptotic distribution of $-2 \log \lambda(x)$, under certain regularity conditions is ______________

v) UMP test leads to _____________ confidence intervals.

C) State whether the following statements are **true** or **false**.

i) UMP test always exist

ii) Test with Neyman-structure is a subset of similar test.

iii) There is no difference between level and size of a test.

iv) If ϕ is a test function then $(1- \phi)$ is also a test function. **(5+5+4)**

2. a) Answer the following.

i) Define simple and composite hypothesis. Give one example each.

ii) Explain shortest length confidence interval.

b) Write short notes on the following.

i) Chi-square test for contingency table

ii) Likelihood ratio test and MP test. **(6+8)**

3. a) State and prove the sufficiency part of Neyman-Pearson lemma.

b) Let x be a random sample with p.d.f.'s

$f_0(x) = 1 \quad 0 \le x \le 1$

$\quad = 0 \quad$ otherwise

and

$f_1(x) = 4x \quad 0 \le x \le 1/2$

$\quad = 4 - 4x \quad \frac{1}{2} \le x \le 1$

$\quad = 0 \quad$ otherwise

on the basis of one observation, obtain the MP test of $H_0 : f = f_0$ against $H_1 : f = f_1$ at level $\alpha = 0.05$. What is the power of M.P. test ? **(7+7)**

4. a) Show that for p.d.f.'s $f_\theta(x)$ which have MLR property in $T(x)$, there exist an UMP test of size α for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

b) Let $X_1, X_2 ..... X_n$ be iid $N(\theta, \sigma^2)$ where σ^2 is known consider the testing problem $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$. Show that a UMP size-$\alpha$ test for testing this problem does not exists. **(7+7)**

5. a) Define the terms.

I) Confidence coefficient of a confidence set

II) UMA confidence set

III) Unbiased confidence set

b) Obtain a UMA confidence interval for θ based on a random sample of size n from $U(0, \theta)$. **(6+8)**

6. a) Describe the Wilcoxon Signed -Rank test for single sample of size n.

   b) Define

      i) Similar test

      ii) Test with Neyman-structure

      iii) UMP α -similar test. Describe the method of obtaining similar test. **(6+8)**

7. a) Let X_1, X_2.... X_n be a random sample of size n from a normal population with mean μ and unknown variance σ^2. Derive LRT test to test $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$.

   b) Write short notes on the following.

      i) Generalised N-P lemma

      ii) MLR property. **(8+6)**

____

SLR-BP – 486

Seat No.	

M.Sc. (Part – I) (Semester – II) Examination, 2015
STATISTICS (Paper – X)
Sampling Theory (Old)

Day and Date : Saturday, 25-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. a) Choose the correct alternative : **5**

1) If an investigator selects districts from a state and farmers from selected districts then such a sampling procedure is ______________

a) systematic sampling b) double sampling
c) two-stage sampling d) cluster sampling

2) Systematic sampling means _____________

a) selecting n continuous units
b) selecting n units situated at equal intervals
c) selection of n largest units
d) selection of any n units

3) Under proportional allocation, the sample size for i^{th} stratum is proportional to ________________

a) N_i b) N_iS_i c) $N_iS_i^2$ d) $\frac{N_i}{S_i}$

4) Which of the following estimators is generally biased ?

a) Horvitz-Thompson Estimator
b) Des Raj estimator
c) Heartly-Ross estimator
d) Ratio estimator

5) Non sampling errors occurs in ________________

   a) only sample surveys

   b) only complete enumeration

   c) sample surveys as well as complete enumeration

   d) none of these

b) Fill in the blanks : **5**

1) Horvitz-Thompson estimator is used when sample selection is done with ___________ probabilities.

2) Errors introduced in editing, coding and tabulating the results are _________ errors.

3) Strata in stratified sampling should be internally ____________

4) Under SRSWOR, the sample unit can occur ____________ in the sample.

5) If 30 units are drawn in a population of 300 units then sampling fraction is ________________

c) State whether the following statements are **True** or **False** : **4**

1) Des Raj estimators are unbiased.

2) In PPSWR sampling design, an unbiased estimator of the population means does not exist.

3) In two-stage sampling, second stage units should be always equal sized.

4) Lahiri's method is convenient for PPSWR sampling.

2. a) Answer the following : **6**

i) Explain sampling method and census method.

ii) In SRSWOR, show that the probability of drawing a specified unit at every draw is same.

b) Write short notes on the following : **8**

i) Circular systematic sampling.

ii) Cumulative total method.

3. a) In SRSWOR examine whether sample mean is an unbiased estimator of population mean. Derive its variance.

   b) Define linear systematic sampling. Derive the sampling variance of the traditional unbiased estimator of a population mean under this scheme. **(6+8)**

4. a) Explain two stage sampling. Give a practical situation where such a design can be used.

   b) Explain a cluster sampling. In SRSWOR of n clusters each containing M elements from a population of N clusters. Show that sample mean is unbiased estimator of population mean. **(7+7)**

5. a) Describe stratified random sampling. Explain various sample allocation criteria in stratified sampling.

   b) Explain the concept of formation of strata. Derive the proportional allocation for the best value of the boundary point Y_h of h^{th} stratum. **(7+7)**

6. a) Explain the ratio and regression methods of estimation.

   b) Make a comparison between the ratio and regression estimators in terms of MSE and state when the ratio estimator can be more efficient than regression estimator. Justify your answer. **(7+7)**

7. a) What is the problem of non response ? Discuss Hansen-Hurwitz technique for dealing this problem.

   b) Define ordered and unordered estimators. Develop Murthy's unordered estimator for n = 2. **(7+7)**

____

SLR-BP – 487

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XI)
Asymptotic Inference

Day and Date : Wednesday, 15-4-2015 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

Instructions : 1) *Attempt **five** questions.*
2) *Q.No. (**1**) and Q.No. (**2**) are **compulsory**.*
3) *Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
4) *Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) An estimator $\hat{\mu}_Y$ of population value μ_Y is more efficient when compared with another estimator $\tilde{\mu}_Y$ if

a) $E(\hat{\mu}_Y) > E(\tilde{\mu}_Y)$

b) It has smaller variance

c) Its cdf is flatter than that of the other

d) Both estimators are unbiased and $Var(\hat{\mu}_Y) < Var(\tilde{\mu}_Y)$

2) The variance stabilizing transformation for binomial population is

a) square root b) logarithmic c) $\sin^{-1}$ d) $\tanh^{-1}$

3) In case of $N(\mu, \sigma^2), \mu \in R, \sigma^2 > 0$, the MLE of σ^2 is

a) unbiased and consistent b) biased and consistent

c) unbiased and not consistent d) biased and not consistent

4) IF T_n is consistent estimator of θ then $\varphi(T_n)$ is consistent estimator of $\varphi(\theta)$ if

a) φ is linear function b) φ is continuous function

c) φ is differentiable function d) none of these

5) The asymptotic distribution of Rao's statistic is

a) normal b) t c) chi-square d) F

P.T.O.

B) Fill in the blanks. **5**

1) Let $X_1, X_2, ..., X_n$ be iid with $E\left(X_i^2\right) = Var\ (X_i) = \sigma^2$. The asymptotic distribution of $\overline{X}_n$ is _________ .

2) Let $X_1, X_2, ..., X_n$ be iid $B(1, \theta)$. CAN estimator of $P_\theta\ (X = 1)$ is _________.

3) Cramer class is __________ than exponential class of distributions.

4) For Laplace (θ, 1) distribution, asymptotic variance of $\overline{X}_n$ is __________.

5) Let $X_1, X_2, ..., X_n$ be iid $N(\theta, 1)$. Then CAN estimator of θ^2 is _________.

C) State whether the following statements are **true** or **false**. **4**

1) Cauchy distribution is a member of Cramer family.

2) Every CAN estimator is consistent.

3) Consistency of estimator is always unique.

4) Consistent estimator based on MLE need not be CAN.

2. a) Answer the following. **6**

i) State Cramer-Huzurbazar results.

ii) Let $X_1, X_2, ..., X_n$ be iid $U(0, \theta)$. By computing the actual probability, show that $X_{(n)}$ is consistent estimator for parameter θ.

b) Write short notes on the following. **8**

i) Super efficient estimator.

ii) Strong consistency.

3. a) Define consistent estimator. State and prove invariance property of consistent estimator.

b) Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with location parameter θ. Find two consistent estimators of θ. Examine the CAN property of the suggested estimators. **(6+8)**

4. a) State Cramer regularity conditions in one parameter set up. Give an example of a distribution which satisfies Cramer regularity conditions. Justify your answer.

   b) Let $X_1, X_2, \ldots, X_n$ be a sample from $N(\theta, \sigma^2)$ distribution. Obtain CAN estimator of σ^2. **(7+7)**

5. a) Define CAN estimator. Show that sample distribution function at a given point is CAN for the population distribution function at the same point.

   b) Let $X_1, X_2, \ldots, X_n$ be iid $N(\theta, \sigma^2)$ random variables. Find the variance stabilizing transformation for S^2 and obtain $100(1-\alpha)\%$ confidence interval for σ^2 based on the transformation. **(7+7)**

6. a) Describe Bartlett test for homogeneity of variances.

   b) Let $X_1, X_2, \ldots X_n$ be a random sample of size n form the distribution with pdf,

   $f(x;\mu,\lambda) = \frac{1}{\lambda}\exp\left[-\left(\frac{x-\mu}{\lambda}\right)\right], x \geq \mu, \lambda > 0$. Obtain moment estimator of

   (μ, λ) and its variance-covariance matrix. **(6+8)**

7. a) Define m-parameter exponential family of distributions. Show that $\left\{N(\mu,\sigma^2), \mu \in R, \sigma^2 > 0\right\}$ is a two-parameter exponential family.

   b) Let $X_1, X_2, \ldots X_n$ be iid Poisson (λ). Obtain CAN estimator of $\lambda e^{-\lambda}$. Discuss its asymptotic distribution at $\lambda = 1$. **(6+8)**

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIII)
Planning and Analysis of Industrial Experiments

Day and Date : Monday, 20-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) In the field experimentation, when experimental material is heterogeneous, we use ________________

a) CRD b) RBD c) LSD d) None of these

2) The aliased defining relation of 2^{k-1} design is I = ABCD, then alias of A is ________________

a) BCD b) ACD c) ABD d) ABC

3) The selection of p generator of 2^{k-p} fractional factorial design is in such a way that ____________

a) it has lowest possible resolution
b) it has minimum aberration
c) it has highest possible resolution
d) it has maximum aberration

4) If all effects of same order are confounded with incomplete block difference, then it is said to be ________________

a) complete confounding b) partial confounding
c) balanced confounding d) none of these

5) The objects which are to be compared in comparative experiment are called ______________

a) blocks b) treatment
c) randomization d) all the above

P.T.O.

B) Fill in the blanks : **5**

1) In one-half fraction with I = + ABC is called ______________

2) Replication reflects source of variability both ________________ runs and ____________ runs.

3) Residual can be check by using formula e_{ij} = _____________

4) Each contrast among k treatments has _____________ degrees of freedom.

5) The shortest word length is called _______________

C) State whether the following statements are **True** and **False** : **4**

1) Factorial design is necessary when interaction may be present to avoid the misleading result.

2) Fractional Factorial design reduces the number of levels of size.

3) The procedure for moving sequentially along in the direction of maximization then it is said to be steepest descent.

4) In randomization the treatments are allocated to the experimental units has with equal probability.

2. a) Answer the following : **6**

i) Define orthogonal array. Give its example.

ii) What is location and dispersion modeling ?

b) Write short notes on the following : **8**

i) Total confounding.

ii) Control composite design.

3. a) Obtain a complete replicate with block size 8 for 2^5 factorial design having ABC and ADE interactions confounded simultaneously.

b) Describe a 3^2 factorial experiment with factors A and B. Give complete procedure of analysis along with the ANOVA table. **(7+7)**

4. a) Explain Taguchi in design of experiment in terms of mode and layout of the experiment.

   b) Find 2_{III}^{6-2} fraction having different aberrations. Use these fractions to state advantages of a fraction with less aberration over the other fractions. **(7+7)**

5. a) Write a short note on Response Surface Methodology (RSM). What are response surface designs ? Write a note on design for fitting the first order modes.

   b) Explain main effects and interactions in a factorial experiment with special reference to a 2^2 design. Give graphical representation to interaction in it. **(7+7)**

6. a) Explain the one quarter fraction of 2^k experiment.

   b) Describe the random effect model of one way classification. **(7+7)**

7. a) Define resolution of a design. What is resolution III, IV and V design ?

   b) Explain in brief about design of experiments. Also state the advantages and disadvantages of the same. **(7+7)**

____

SLR-BP – 490

Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
STATISTICS (Paper – XIV)
Modeling and Simulation

Day and Date : Wednesday, 22-4-2015 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

***Instructions** : 1) Question No. **1** and **2** are **compulsory**.*
*2) Attempt any **three** questions from Q. **3** to Q. **7**.*
*3) Figures to **right** indicate **full** marks.*

1. A) Select the correct alternative. **10**

i) The slack for an activity in network is equal to

a) LS-ES b) LF-LS c) EF-ES d) EF-LS

ii) If small orders are placed frequently, then total inventory cost is

a) Reduced b) Increased

c) Either reduced nor increased d) Minimized

iii) Simulation is

a) Descriptive in nature

b) Useful to analyze problem where analytical solution is difficult

c) A statistical experiments as such as its results are subject to statistical errors

d) All of the above

iv) Repetition of n independent Bernoulli trails reduced to

a) Poisson distribution b) Binomial distribution

c) Geometric distribution d) Hypergeometric distribution

v) Simulation of system in which the state changes smoothly with time are called _____________

a) Continuous system b) Discrete system

c) Deterministic system d) Probabilistic system

P.T.O.

vi) The activity which can be delayed without affecting the execution of immediate succeeding activity is determined by

a) Total float b) Free float
c) Independent float d) None of these

vii) In M/M/1 : ∞/FCFS queue model if λ is mean customer arrival rate and μ is the mean service rate then the probability of server being busy is equal to

a) $\frac{\lambda}{\mu}$ b) $\frac{\lambda}{\mu-\lambda}$ c) $\frac{\mu}{\mu-\lambda}$ d) $\frac{\mu}{\lambda}$

viii) Markov chain said to be ergodic chain if ________ of whose states are ergodic.

a) One b) Some c) All d) None

ix) In queue model completely specified in the symbolic form (a/b/c/) : (d/e), the last symbol 'e' specifies

a) The queue discipline b) The number of servers
c) The distribution of arrival d) The distribution of departure

x) If customer, on arriving at the service system stays in the system until served, no matter how he has to wait for service is called _______ customer.

a) a regular b) an irregular c) a patient d) an impatient

B) Fill in the blanks. **4**

i) In EOQ problem, minimum total cost occurs at a point where the ordering cost and _______ cost are equal.

ii) The long form of CPM is _____________.

iii) Chapman-Kolmogorov equation is $P_{ij}(t+T) =$ ___________.

iv) In inventory model, the number of unit required per period is called _______.

2. A) i) A customer arrive in a certain store according to Poisson process with rate $\lambda = 4$ per hour, given that the store opens at 9.00 am, then what is probability that exactly one customer has arrive by 9.30 am ? **3**

ii) What do you mean by movement inventories ? **3**

B) i) State and prove the Chapman-Kolmogorov equation. **4**

ii) Write note on simulation. **4**

3. A) Differentiate between PERT and CPM. **7**

B) Explain the generation of random sample from continuous uniform distribution. **7**

4 A) Explain the concept of inventory control. Write any four reasons for carrying inventories. **7**

B) The demand rate for a particular item is 12000 units/ year. The ordering cost of Rs. 1,000 per order and the holding cost is Rs. 0.80 per month. If no shortage are allowed and the replacement is instantaneous the determine **7**

i) Economic order quantity

ii) Number of order per year.

5. A) For various activity in the particular project the expected time (in days) of completions are as follow. **7**

Activity	0 –1	1 – 3	1 – 2	2 – 3	1 – 4	3 – 4	4 – 5
Duration	3	16	6	8	10	5	3

Draw a network diagram and identify the critical path.

B) Write steps in of Monte-Carlo simulation technique. **7**

6. A) Generate the five successive random number X_i, i = 1, 2, 3, 4, 5 by using $X_{i+1} = X_i{*}a$ (modulo m), starting with seed $X_0 = 3$ and parameters a = 7 and m = 15 (where m means that the number $\{X_i{*}a\}$ is divided by m repeatedly till the reminder is less than m). **7**

B) Define project duration, earliest event time, earliest start time, latest start time, and earliest finish time in critical path computation. **7**

7. A) Define simulation. Write the advantages and limitations of simulation. **7**

B) Explain pure birth process. **7**

____

SLR-BP – 492

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XVI)
Discrete Data Analysis

Day and Date : Thursday, 16-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate full marks.*

1. A) Select the correct alternative :

I) In rural notations, the logit transformation is given for

A) $\frac{\pi(x)}{1-\pi(x)}$ B) $ln\left(\frac{\pi(x)}{1-\pi(x)}\right)$ C) $ln\left(\frac{1-\pi(x)}{\pi(x)}\right)$ D) $\frac{1-\pi(x)}{\pi(x)}$

II) If the response variable in GLM follows Poisson distribution, then following link function is suitable

A) $\log(\theta)$ B) $\log\left(\frac{1}{\theta}\right)$ C) $\log\left(\frac{\theta}{1-\theta}\right)$ D) $\log\left(\frac{1-\theta}{\theta}\right)$

III) When the two categorical variables are independent the cross product ratio for a 2×2 table is equal to

A) 1 B) 0 C) $1/2$ D) $3/4$

IV) In regression analysis when the outcome variable is dichotomous, E [Y/X] must lie in

A) [0, 1] B) $(-\infty, \infty)$ C) $[0, \infty]$ D) {0, 1}

V) In log-linear model U_{12} is a higher order relative of

A) U_1 only B) U_2 only C) U_1 and U_2 D) U_{123}

P.T.O.

B) Fill in the blanks :

i) The odds ratio of 2×2 table is defined as ________________

ii) The logistic regression Y on X is ________________

iii) The number of independent U_{12} terms in an $I \times J \times K$ table are ________________

iv) A G^2 - statistic is distributed as ________________

v) The Kernal of the log-likelihood function based on a sample from Poisson distribution is ________________

C) State whether the following statements are **true** or **false** :

i) Brich results are used to assess the goodness of fit of a log-linear model.

ii) A GLM with log-link function is the classical linear model.

iii) In any log-linear model, the mle of expected cell frequencies always closed form expression.

iv) Logistic regression model is appropriate when the response variable is normal. **(5+5+4)**

2. a) Explain the following terms :

i) Brich's results

ii) Link function in GLM.

b) Write short notes on the following :

i) Polytomous logistic regression model.

ii) Multinomial sampling scheme. **(6+8)**

3. a) For an $I \times J$ table, write a log-linear model and obtain the relationship between the interaction term U_{12} and cross product ratio when $I = 2$.

b) With reference to an $I \times J \times K$ table with $U_{123} = U_{12} = 0$, obtain m.l.e. of elementary cell frequencies. **(7+7)**

4. a) Explain the following terms :
   i) Hierarchical family of models
   ii) Cross product ratio for 2×2 table
   iii) Relative risk.

   b) State and establish a condition for the existence of 'direct estimates' of elementary cell frequencies in an $I \times J \times K$ table. **(6+8)**

5. a) Explain the following terms :
   i) Deviance
   ii) One parameter exponential family of distribution
   iii) Generalized linear model.

   b) Derive Nelder and Wedderburn's weighted least squares estimator of the parameters of a GLM. **(6+8)**

6. a) Define the logistic regression model. Discuss the Pearson residual and deviance residual in the context of logistic regression.

   b) Consider logistic regression model with single dichotomous independent variable. Derive the log-odds ratio. Give a computational procedure for obtaining estimates of odds ratio. **(7+7)**

7. a) Explain the non-parametric regression and smoothing splines.

   b) Define the Poisson regression. Give a situation where poisson regression is an appropriate. Derive the score equation for the same. **(7+7)**

———————

SLR-BP – 493

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XVII)
Industrial Statistics

Day and Date : Saturday, 18-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select the correct alternative :

1) Quality is inversely proportional to _______________
a) Variability b) Cost c) Method d) Time

2) _______________ variability is unavoidable.
a) Chance-cause
b) Assignable cause
c) Both chance and assignable cause
d) None of chance and assignable cause

3) C_p _______________ C_{pk}.
a) $\leq$ b) $\geq$ c) < d) >

4) _______________ is not the dimension of quality.
a) Aesthetics b) Features c) Durability d) Cost

5) The probability of false alarm for $\overline{X}$ chart with 3σ limits is _______________
a) 0.027 b) 0.27 c) 0.0027 d) 0.0027% **(1×5)**

B) Fill in the blanks :

1) CUSUM and EWMA control charts are the better alternatives to ___________ charts for detecting small shifts in process parameters.

2) The formula for C_{pk} is ___________

3) S chart is preferred over R chart if the sample size is ___________

4) The process capability index ___________does not have interpretation in terms of the probability of nonconformance.

5) The SPC tool ___________visualizes the most significant problem to be worked out first. **(1×5)**

P.T.O.

C) State **true** or **false** :

1) Product control relies on inspectors.
2) False alarm is the indication of in-control state of a process when it is really out-of-control.
3) The process capability index C_{pk} does not take into account location of the process mean.
4) PDCA cycle may require several iterations for solving a quality problem. **(1×4)**

2. a) i) Describe types of variability.
   ii) Describe curtailed and semi-curtailed sampling plans. **(3+3)**
   b) Write short notes on the following :
   i) Cause and effect diagram.
   ii) Process capability index C_{pm}. **(4+4)**

3. a) Define statistical quality control. Describe product control and process control.
   b) Define the process capability index C_{pk}. State and prove the relation between C_{pk} and the probability of nonconformance associated with it. **(7+7)**

4. a) Describe construction, operation and the underlying statistical principle of p chart.
   b) Describe construction and operation of EWMA control chart for monitoring process mean. **(7+7)**

5. a) Define process capability index. Define index C_p with the necessary underlaying assumptions. What is its interpretation ?
   b) Describe the DIMAC cycle. **(7+7)**

6. a) Describe single attribute sampling inspection plan based on hypergeometric distribution.
   b) Describe sampling inspection plan by variables when both lower and upper specification limits are given and the standard deviation is known. **(7+7)**

7. a) Describe construction, operation and the underlying statistical principle of Hotelling's T^2 chart.
   b) Describe six-sigma methodology. **(7+7)**

———————

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XVIII)
Reliability and Survival Analysis

Day and Date : Tuesday, 21-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **(1)** and Q. No. **(2)** are **compulsory**.*
*3) Attempt **any three** from Q. No. **(3)** to Q. No. **(7)**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternative : **5**

1) For which of the following family each member has non-monotone failure rate ?

a) Exponential b) Weibull
c) Gamma d) Lognormal

2) For Weibull distribution ___________ parameter decides whether distribution belongs to the IFR class or the DFR class.

a) Location b) Shape
c) Scale d) All the above

3) In censored data actuarial estimator of survival function is ___________

a) $\sum_{i=1}^{k} \hat{P}_i$ b) $\frac{1}{k}\sum_{i=1}^{k} \hat{P}_i$

c) $\prod_{i=1}^{k} \hat{p}_i$ d) $\prod_{i=1}^{k} (\hat{p}_i)^{\frac{1}{2}}$

4) The i^{th} component of a system is relevant if ___________

a) $\phi(1_i, \underline{x}) = 1$ and $\phi(0_i, \underline{x}) = 1$

b) $\phi(1_i, \underline{x}) = 1$ and $\phi(0_i, \underline{x}) = 0$

c) $\phi(1_i, \underline{x}) = 0$ and $\phi(0_i, \underline{x}) = 0$

d) $\phi(1_i, \underline{x}) = 0$ and $\phi(0_i, \underline{x}) = 1$

5) A function g(x) defined on $[0, \infty)$ is star shaped function if for $0 \le \alpha \le 1$, ____________

a) $g(\alpha x) \le [g(x)]^{\alpha}$ b) $g(\alpha x) \ge [g(x)]^{\alpha}$

c) $g(\alpha x) \le \alpha g(x)$ d) $g(\alpha x) \ge \alpha g(x)$

B) Fill in the blanks : **5**

1) IFR property is preserved under ____________
2) $F \in$ IFRA if and only if – log R(t) is ____________
3) A sequence of (2×2) contingency tables is used in ____________
4) In type II censoring ____________ is fixed.
5) The minimal path sets for a structure φ are ____________ for its dual.

C) State whether the following statements are **True** or **False** : **4**

1) The dual of a parallel system is not a parallel.
2) For exponential distribution failure rate is constant.
3) Weibull distribution is IFR for all parameter values.
4) IFRA property is preserved under convolution.

2. a) Answer the following : **6**

1) Define minimal path sets and minimal cut sets. Illustrate the same by example.
2) Define k out of n system. Obtain the reliability function of this system.

b) Write short notes on the following : **8**

1) Empirical survival function and its properties.
2) Log-rank test.

3. a) Define NBU and NBUE classes of distributions. Prove that $F \in \text{IFRA} \Rightarrow F \in \text{NBU}$.

b) Give two definitions of star shaped function and prove their equivalence. **(7+7)**

4. a) Describe various censoring schemes.

b) Obtain MLE of the mean (θ) of an exponential distribution based on type I and type II censoring. **(7+7)**

5. a) Describe situations where random censoring occurs naturally. Obtain actuarial estimate of survival function and derive Greenwood's formula for the estimate of variance of the estimator.

   b) Define associated random variables. If $X_1, X_2, \ldots, X_n$ are binary associated random variables then prove that $P\left[\coprod_{j=1}^{n} X_j = 1\right] \leq \coprod_{j=1}^{n} P(X_j = 1)$. **(7+7)**

6. a) Let $F(x) = \int_{\alpha} F_{\alpha}(x) d\,G(\alpha)$ be a mixture of $\{F_{\alpha}\}$ with mixing distribution $G(\alpha)$. Prove that if each F_{α} is DFR then F is DFR.

   b) Obtain the actuarial estimator of the survival function. Clearly state the assumption that you need to make. State Greenwood's formula for the variance of the estimator. **(7+7)**

7. a) Show that Kaplan-Meier estimator of survival function is the generalized likelihood estimator of the survival function.

   b) Define structure function of a system. Obtain structure function of a system in terms of minimal path sets. **(7+7)**

____

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XIX)
Operations Research (Elective – I)

Day and Date : Thursday, 23-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) The given payoff matrix of a game is transposed. Which of the following is not true ?
a) value of the game changes
b) saddle point of a game if exist, changes
c) player B has as many strategies as A had, and A has as many strategies as B
d) optimum strategies of both players does not change

2) Quadratic programming is concerned with the NLPP of optimizing the quadratic objective function subject to ___________
a) linear inequality constraints
b) non-linear inequality constraints
c) non-linear equality constraints
d) no constraint

3) Which of the following methods of solving a Quadratic programming problem is based on modified simplex method ?
a) Wolfe's method
b) Beale's method
c) Frank-Wolfe method
d) Fletcher's method

4) Given an LPP to Maximize $Z = -5x_2$ subject to $x_1 + x_2 \leq 1$, $0.5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$. Then we have ______________

a) no feasible solution

b) unbounded solution

c) unique optimum solution

d) multiple optimum solution

5) Consider the LPP

Minimize $Z = 3x_1 + 5x_2$

Subject to the constraints,

$x_1 + 2x_2 \leq 4$, $2x_1 + x_2 \geq 6$ and $x_1, x_2 \geq 0$

The problem represents a :

a) Zero-one IPP
b) Pure IPP
c) Mixed IPP
d) Non-IPP

B) Fill in the blanks : **5**

1) The basic solution to the system is called ____________ if one or more of the basic variables vanish.

2) If all variables of an IPP are either 1 or 0, then problem is called __________

3) If either the primal or the dual problem has an has unbounded objective function value then the other has _____________

4) Dual simplex method is applicable to those LPPs that start with infeasible but otherwise _________________

5) A game in said to be determinable if ____________

C) State whether the following statements are **True** or **False** : **4**

1) A slack variable cannot be present in the optimum basis of an LPP.

2) If an LPP has unbounded solution, the objective function will always be unbounded.

3) The dual LPP must always be of minimization type.

4) For a bounded primal problem, the dual would be infeasible.

2. a) Answer the following : **6**

   1) Explain the use of artificial variables in linear programming .

   2) Define :

      i) Convex polyhedron

      ii) Convex function.

   b) Write short notes on the following : **8**

   i) Two phase method of solving LPP.

   ii) Primal-dual relationship.

3. a) State and prove basic duality theorem. **8**

   b) Use dual simplex method to solve the following LPP : **6**

   Minimize $Z = 10x_1 + 6x_2 + 2x_3$

   Subject to the constraints

   $-x_1 + x_2 + x_3 \geq 1, \; 3x_1 + x_2 - x_3 \geq 2$

   and $x_1, x_2, x_3 \geq 0$.

4. a) Describe Gomory's method of solving an all integer LPP. **7**

   b) Use Branch and Bound method to solve the following IPP : **7**

   Maximize $Z = 7x_1 + 9x_2$

   subject to the constraints,

   $-x_1 + 3x_2 \leq 6, \; 7x_1 + x_2 \leq 35, x_2 \leq 7$

   and $x_1, x_2 \geq 0$ and are integers.

5. a) Derive the K-T conditions for an optimal solution to a QPP. **6**

   b) Solve the following LPP using Beale's method : **8**

   Maximize $Z = 2x_1 + 3x_2 - 2x_2^2$

   Subject to the constraints,

   $x_1 + 4x_2 \leq 4, x_1 + x_2 \leq 2$

   and $x_1, x_2 \geq 0$.

6. a) Explain Maximin and Minimax principle used in game theory. **6**

b) Solve the following game by LP technique. **8**

$$\begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix}$$

7. a) Explain the theory of dominance in the solution of rectangular game. Illustrate with the following example. **8**

Player B

		I	II	III	IV
Player A	I	3	2	4	0
	II	3	4	2	4
	III	4	2	4	0
	IV	0	4	0	8

b) Define the following : **6**

i) Saddle point

ii) Basic feasible solution

iii) Two person zero sum game.

____

SLR-BP – 496

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
STATISTICS (Paper – XX) (Elective – II)
Clinical Trials

Day and Date : Saturday, 25-4-2015 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions *: 1) Attempt* ***five*** *questions.*
2) Q. No. ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
3) Attempt ***any three*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
4) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Select most correct alternative : **5**

1) In clinical trials, 'Experimental Unit' means ____________
   a) Patient
   b) Any healthy individual
   c) Any subject from target population
   d) All of the above

2) ___________ is the type of stratified randomization.
   a) Complete randomization
   b) Permuted block randomization
   c) Treatment adaptive randomization
   d) Response adaptive randomization

3) Suppose that we wish to have a 95% assurance that the error in the estimated mean is less than 10% of the standard deviation. What will be the optimum sample size for comparison of two treatment means ?
   a) 385 b) 150 c) 540 d) 300

4) The period between administration of reference drug and test drug is called ___________
   a) Washout Period b) Run-in Period
   c) Therapeutic Window d) None of the above

P.T.O.

5) For a ___________ design, each patient receives one and only one treatment in random fashion, whereas for a ____________ design each patient receives more than one treatment at different dosing periods.

a) parallel, crossover b) crossover, parallel

c) parallel, parallel d) crossover, crossover

B) Fill in the blanks : **5**

1) For n subjects and 2 treatments, the probability of imbalance in complete randomization is ____________

2) SOP stands for _____________

3) ___________ concurrent control can only be used when the placebo effect is negligible.

4) If the objective of the intended clinical trial is to show that the test treatment is better than a concurrent control in terms of its primary clinical endpoints, then it is referred to as the ___________ trial.

5) The number of patients/volunteers involves in phase III __________

C) State whether the following statements are **true** or **false** : **4**

1) An inadequate duration of treatment may provide an biased response of the study drug.

2) We can estimate intra-subject variability in 'Parallel Design'.

3) Statistical significance implies clinical significance.

4) The Kolmogorov-Simrov test can be used to verify whether the distributions of both the test and reference active control are larger than those of the placebo.

2. a) What is the difference between Investigational New Drug application (IND) and New Drug Application (NDA) ? What are the types of INDs ? Who sponsors for it ?

b) Explain the following terms related with Clinical Trials : **(8+6)**

i) Subject

ii) Clinical Endpoints

iii) Placebo.

3. a) What is population model and invoke population model ? State the difference between them.

   b) Explain the role of Bio-statistician in the planning and execution of clinical trials. Also state some sources of bias in clinical trials. **(7+7)**

4. a) Describe all phases of clinical development in clinical trials.

   b) What is randomization ? Why randomization is needed in clinical trials ? Give the types of randomization used in clinical trials. **(7+7)**

5. a) For selecting an appropriate design for clinical trials, which issues must be considered ?

   b) Explain : Dose response concurrent controls. **(6+8)**

6. a) i) What are the roots of administration of the drug ?

      ii) Discuss the objectives of repeated measures in clinical trials.

   b) What is cross over design ? Explain the different types of cross over design. **(6+8)**

7. a) What is bio-equivalence ? Explain the difference between average and variance bio-equivalence.

   b) i) Explain the paired t-test. In which situation of analysis of clinical trials paired t-test is useful ?

      ii) Explain 'Confidence Interval' useful in analysis of clinical trials with an example. **(8+6)**
