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Set **P**

**M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination:  
March/April – 2026  
Group and Ring Theory (2317101)**

Day & Date: Friday, 17-04-2026  
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ)****08**

- 1) If  $G$  is cyclic, then its commutator subgroup  $G' = \underline{\hspace{2cm}}$ .
  - a)  $\emptyset$
  - b)  $\{e\}$
  - c) non-trivial proper subgroup
  - d)  $G$
  
- 2) If  $G$  is a group and  $N$  is its normal subgroup such that  $N$  and  $\frac{G}{N}$  are nilpotent, then  $\underline{\hspace{2cm}}$ .
  - a)  $G$  is nilpotent
  - b)  $G$  is not solvable
  - c)  $G$  need not be nilpotent
  - d) None of these
  
- 3) Every group of order  $p^2$  is  $\underline{\hspace{2cm}}$ .
 

a) abelian	b) cyclic
c) non-abelian	d) non-cyclic
  
- 4) If  $G$  is a group of order 15, then by Cauchy's theorem,  $G$  has cyclic subgroups of order  $\underline{\hspace{2cm}}$ .
 

a) 2	b) 3
c) 5	d) both 3 & 5
  
- 5) If  $G$  is a group and  $a, b \in G$  such that  $a$  is conjugate of  $b$ , then  $\underline{\hspace{2cm}}$ .
 

a) $b$ is conjugate of $e$	b) $b$ is conjugate of $a$
c) $a$ is conjugate of $e$	d) $b$ is conjugate of $ab$
  
- 6)  $\mathbb{Z}$  is  $\underline{\hspace{2cm}}$ .
 

a) ED	b) PID
c) UFD	d) All of these
  
- 7) If an integral domain  $D$  not PID, then it is  $\underline{\hspace{2cm}}$ .
 

a) an ED	b) not an ED
c) an UFD	d) not an UFD

- 8) The polynomial  $f(x) = x^2 + 1$  is \_\_\_\_\_.
- a) reducible over  $\mathbb{Q}$       b) reducible over  $\mathbb{R}$   
 c) irreducible over  $\mathbb{Q}$       d) reducible over  $\mathbb{Z}$

**B) State whether True or False. 04**

- a) If  $G$  is a group, then  $\frac{G}{G'}$  is abelian.  
 b) Every normal series is subnormal.  
 c) Any two subnormal series of a group  $G$  have isomorphic refinements.  
 d)  $\mathbb{Z}[x]$  is field.

**Q.2 Answer the following. (Any Six) 12**

- a) Define: Principal series  
 b) Define: Composition series  
 c) Define: Sylow-p-subgroup  
 d) State Sylow's second theorem.  
 e) Define: Principal Ideal domain  
 f) Define: Primitive polynomial  
 g) By using Eisenstein's criteria, show that  $f(x) = x^2 - 2$  in  $\mathbb{Z}[x]$  is irreducible over  $\mathbb{Q}$ .  
 h) Define: Submodule

**Q.3 Answer the following. (Any Three) 12**

- a) If  $G$  is a group of order  $p^2$ , then prove that  $G$  is abelian.  
 b) Prove that every principal series is a composition series.  
 c) If  $p_1(x), p_2(x), \dots, p_n(x)$  are  $n$  primitive polynomials in a ring  $D[x]$ , then prove that  $p_1(x), p_2(x), \dots, p_n(x)$  is also a primitive polynomial.  
 d) Show that  $\frac{\mathbb{Z}_2[x]}{\langle x^2+x+1 \rangle}$  is a field.

**Q.4 Answer the following. (Any Two) 12**

- a) State and prove Cauchy's theorem.  
 b) Show that no group of order 96 is simple.  
 c) If  $\mathbb{F}$  is a field, then prove that an ideal  $\langle p(x) \rangle \neq \{0\}$  of  $\mathbb{F}[x]$  is maximal iff  $p(x)$  is irreducible over  $\mathbb{F}$ .

**Q.5 Answer the following. (Any Two) 12**

- a) An ideal  $\langle p \rangle$  in a PID is maximal iff  $p$  is an irreducible.  
 b) If  $f(x) = x^4 - 3x^2 + 2x^2 + 4x - 1, g(x) = x^2 - 2x + 3$  are two polynomials in  $\mathbb{Z}_5[x]$ , then using division algorithm find  $q(x), r(x)$  in  $\mathbb{Z}_5[x]$  such that  $f(x) = g(x)q(x) + r(x)$  with  $\deg r(x) < 2$ .  
 c) If  $G$  is a group and  $X$  is a  $G$ -set, then prove that  $\sum_{g \in G} |X_g| = r|G|$ , where  $r$  is the number of orbits in  $X$  under  $G$ .

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**M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination:  
March/April – 2026  
Real Analysis (2317102)**

Day & Date: Monday, 20-04-2026  
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ)**

**08**

- 1) The supremum of set of all lower sums is called \_\_\_\_\_.  
 a) Upper integral                      b) Lower integral  
 c) Both a and b                        d) None of these
- 2) The value of integral always lies between \_\_\_\_\_ and \_\_\_\_\_ of function corresponding to any partition of  $[a, b]$   
 a)  $M(b - a), m(a - b)$             b)  $m(b - a), M(a - b)$   
 c)  $m(b - a), M(b - a)$             d)  $M(b + a), m(b + a)$
- 3) Riemann - Stieltje's integral reduces to Riemann integral if  $a(x) =$  \_\_\_\_\_.  
 a) 1    b)  $x$   
 c)  $x^2$                                         d) 0
- 4) Oscillatory sum of  $f$  is given by \_\_\_\_\_.  
 a)  $M_i - m_i$                               b)  $M - m$   
 c)  $W(P, f)$                                 d)  $W(f, P)$
- 5) If  $f$  have local extrema at  $C$  then,  $f'(C) =$   
 a) 1    b)  $-1$   
 c) 0    d) constant
- 6) If  $f : R \rightarrow R$  then Total derivative is \_\_\_\_\_.  
 a) Real number                              b) Gradient vector  
 c) Real matrix                                d) None of these
- 7) If  $S$  is convex set then \_\_\_\_\_ for all  $x, y \in S$ .  
 a)  $L(x, y) \subseteq S$                         b)  $L(x, y) \supseteq S$   
 c)  $L(x, y) = S$                             d) None of these

- 8) Consider the following statements \_\_\_\_\_.
- I) Function having only one point discontinuity is integrable.
  - II) Function having finite no. of points of discontinuity is integrable.
- a) only I is true
  - b) only II is true
  - c) both are true
  - d) both are false

**B) Fill in the blanks.****04**

- 1) Every integrable function is continuous.
- 2) By first mean value theorem, if a function  $f$  is continuous on  $[a, b]$  then there exist a number  $\xi$  in  $[a, b]$  such that
 
$$\int_a^b f(x)dx = f(\xi)(b - a)$$
- 3) If  $P_1$  and  $P_2$  are two partitions of  $[a, b]$  then their common refinement is given by  $P^* = P_1 \cup P_2$
- 4) The partial derivatives of a function describe the rate of change of a function in the direction of co-ordinate axis.

**Q.2 Answer the following. (Any Six)****12**

- a) If a function  $f$  is continuous on  $[a, b]$  then prove that there exists a number  $\xi$  in  $[a, b]$  such that  $\int_a^b f(x)dx = f(\xi)(b - a)$
- b) Find the directional derivative of  $f(x, y) = x^2 + y^2$  at point  $(1, 2)$  in the direction  $(2, 3)$ .
- c) Define: Upper sum, Lower sum.
- d) If  $\int_{-1}^2 xdx = 3$  then find its mean value.
- e) Write Mean Value theorem for the functions  $f$  from  $R^n \rightarrow R$ .
- f) Prove that: A bounded function  $f$  is integrable on  $[a, b]$  if the set its point of discontinuity has only finite number of limit points.
- g) If  $f$  and  $g$  are integrable on  $[a, b]$  and  $g$  keeps the same sign over  $[a, b]$  and  $f$  is continuous on  $[a, b]$  then prove that there exists a number  $\zeta \in [a, b]$  such that,  $\int_a^b fgdx = f(\zeta) \cdot \int_a^b gdx$
- h) Prove that for any bounded function  $f$ ,  $\int_- f dx \leq \int^- f dx$

**Q.3 Answer the following. (Any Three)****12**

- a) Prove that every continuous function is integrable.
- b) Solve,  $\int_2^3 (2x + 3)dx$
- c) If  $f(x, y) = (xy, x^2 + y, x + y^2)$  then find  $Df(x, y)$
- d) Prove that: The oscillation of a bounded function  $f$  on an interval  $[a, b]$  is the supremum of the set  $\{|f(x_1) - f(x_2)|/x_1, x_2 \in [a, b]\}$  of numbers.

**Q.4 Answer the following. (Any Two)**

12

- a) If  $P^*$  is a refinement of a partition  $P$  then for a bounded function  $f$  then prove that
- $L(P^*, f, \alpha) \geq L(P, f, \alpha)$
  - $U(P^*, f, \alpha) \leq U(P, f, \alpha)$
- b) Prove that: A bounded function  $f$  is integrable on  $[a, b]$  iff for every  $\epsilon > 0$  there exists a partition  $P$  of  $[a, b]$  such that,  
 $U(P, f) - L(P, f) < \epsilon$
- c) If  $f$  have a continuous  $n^{\text{th}}$  (for some integer  $n \geq 1$ ) derivative in the open interval  $(a, b)$  and for some interior point  $c$  in  $(a, b)$  we have,  
 $f'(c) = f''(c) = \dots = f^{n-1}(c) = 0$  but  $f^n(c) \neq 0$  then prove that for  $n$  even,  $f$  has local minimum at  $c$  if  $f^n(c) > 0$  and  $f$  has local maximum at  $c$  if  $f^n(c) < 0$ . Also prove that if  $n$  is odd, there is neither a local maximum nor a local minimum at  $c$ .

**Q.5 Answer the following. (Any Two)**

12

- a) Find directional derivative of
- $$f(x, y) = \begin{cases} \frac{x^2 \cdot y}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$
- b) State and prove Abel's Lemma.
- c) Find extrema of a function  $f(x) = 3x^4 + 4x^3 - 84x^2 - 288x$

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**M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination:  
March/April - 2026  
Number Theory (2317107)**

Day & Date: Wednesday, 22-04-2026  
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ)****08**

- 1) Euler's theorem states that, for any integer 'a' satisfying  $\gcd(a, n) = 1$  \_\_\_\_\_.
  - a)  $a^n \equiv 1 \pmod{n}$
  - b)  $a^{\varphi(n)} \equiv 1 \pmod{n}$
  - c)  $a^n \equiv 1 \pmod{\varphi(n)}$
  - d)  $a^{\varphi(n)} \equiv 1 \pmod{\varphi(n)}$
- 2) For positive integers  $a$  and  $b$ ,  $\text{lcm}(a, b) = a \cdot b$  iff \_\_\_\_\_.
  - a)  $a \nmid b$
  - b)  $b \nmid a$
  - c)  $\gcd(a, b) = 1$
  - d)  $\gcd(a, b) = ab$
- 3) If  $ca \equiv cb \pmod{n}$  and  $\gcd(c, n) = d$  then \_\_\_\_\_.
  - a)  $a \equiv b \pmod{n}$
  - b)  $a \equiv b \pmod{d}$
  - c)  $a \equiv b \pmod{nd}$
  - d)  $a \equiv b \pmod{\frac{n}{d}}$
- 4) For  $n > 1$ , then sum of positive integers less than 'n' and relatively prime to 'n' is \_\_\_\_\_.
  - a)  $\frac{n \cdot \varphi(n)}{2}$
  - b)  $n \cdot \varphi(n)$
  - c)  $\frac{\varphi(n)}{2}$
  - d)  $\varphi(n)$
- 5) If  $a = bq + r$  then  $\gcd(a, b) =$  \_\_\_\_\_.
  - a)  $\gcd(b, r)$
  - b)  $\gcd(a, q)$
  - c)  $\gcd(q, r)$
  - d)  $\gcd(b, q)$
- 6) If  $p$  is an odd prime then there exists a primitive root  $r$  of  $p$  such that \_\_\_\_\_.
  - a)  $r^{p-1} \equiv 1 \pmod{p^2}$
  - b)  $r^{p-1} \not\equiv 1 \pmod{p}$
  - c)  $r^{p-1} \not\equiv 1 \pmod{p^2}$
  - d)  $r^{p-1} \equiv 1 \pmod{p}$
- 7) If the integer  $a$  has order  $k$  modulo  $n$ , then \_\_\_\_\_.
  - a)  $a^i \equiv a^j \pmod{n}$  if  $i \equiv j \pmod{n}$
  - b)  $a^i \equiv a^j \pmod{n}$  iff  $i \equiv j \pmod{n}$
  - c)  $a^i \equiv a^j \pmod{n}$  iff  $i \equiv j \pmod{k}$
  - d)  $i \equiv j \pmod{n}$  if  $a^i \equiv a^j \pmod{n}$

8) The congruence  $x \equiv a \pmod{n}$  and  $x \equiv b \pmod{m}$  admits a simultaneous solution iff \_\_\_\_\_.

- a)  $\gcd(n, m) | a - b$                       b)  $\gcd(n, m) \nmid a - b$   
 c)  $\gcd(n, m) = a - b$                       d)  $\gcd(n, m) = 2$

**B) Fill in the blanks.**

**04**

- 1) If  $\gcd(a, m) = 1$  then  $a$  is primitive root of  $m$  if and only if \_\_\_\_\_ for every prime divisor  $p$  of  $\varphi(m)$ .
- 2) If  $p$  is prime and  $p | a_1 \cdot a_2 \cdot a_3 \dots a_n$  then for  $1 \leq k \leq n$  \_\_\_\_\_.
- 3) If  $f(n) = n^2 + 2$  and  $n = 6$  then \_\_\_\_\_.
- 4) The number of integers of  $S = \{1, 2, 3, \dots, n\}$  divisible by a positive integer  $a$  is \_\_\_\_\_.

**Q.2 Answer the following. (Any Six)**

**12**

- a) If  $p$  and  $q$  are distinct primes then prove that  $p^{q-1} + q^{p-1} \equiv 1 \pmod{pq}$
- b) Find the remainder when  $72^{1001}$  is divided by 31
- c) If  $p$  and  $2p + 1$  are both primes and  $n = 4p$  then show that  $\varphi(n + 2) = \varphi(n) + 2$
- d) Factorize 2047 using Fermat factorization method.
- e) Show that one of every three consecutive integer is divisible by 3.
- f) If  $r$  is the smallest primitive root of  $n$  and  $r^h \equiv a \pmod{n}$  then show that  $h \equiv \text{ind } a \pmod{\varphi(n)}$ .
- g) Find  $\tau(20000)$  and  $\sigma(20000)$
- h) Show that square of an odd integer is of the form  $8q + 1$ .

**Q.3 Answer the following. (Any Three)**

**12**

- a) If  $f$  and  $F$  be two number theoretic functions related by the formula  $F(n) = \sum_{d|n} f(d)$  then show that,  $f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right)$
- b) Show that 1729 is an absolute pseudo prime.
- c) Find the general solution of the linear Diophantine equation  $11x + 5y = 79$
- d) If the orders of  $a_1$  and  $a_2$  modulo  $n$  be  $k_1$  and  $k_2$  respectively and  $\gcd(k_1, k_2) = 1$ . Then prove that the order of  $a_1 a_2 \pmod{n}$  is  $k_1 k_2$ .

**Q.4 Answer the following. (Any Two)**

**12**

- a) If  $a$  has order  $k \pmod{n}$  then show that  $a^h$  has order  $\frac{k}{d} \pmod{n}$  where  $d = \gcd(k, h)$
- b) If  $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$  is a prime factorization of  $n$  then prove that
  - i)  $\tau(n) = (k_1 + 1)(k_2 + 1) \dots (k_r + 1)$
  - ii)  $\sigma(n) = \left(\frac{p_1^{k_1+1} - 1}{p_1 - 1}\right) \left(\frac{p_2^{k_2+1} - 1}{p_2 - 1}\right) \dots \left(\frac{p_r^{k_r+1} - 1}{p_r - 1}\right)$

- c) Find the primes not exceeding 150 by using the method Sieve of Eratosthenes.

**Q.5 Answer the following. (Any Two)**

**12**

- a) State and prove Euler's theorem.
- b) Find an integer which leaves the remainder 5 when divided by 11 and 2 when divided by 19.
- c) Show that the integer  $2^n$  has no primitive root for  $n \geq 3$ .

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**M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination:  
March/April – 2026  
Research Methodology in Mathematics (2317103)**

Day & Date: Friday, 24-04-2026  
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ) 08**

- 1) The i-10 index indicates the number of academic publications an author has written that have been cited by \_\_\_\_\_ sources.
  - a) Exactly 10
  - b) more than 10
  - c) at least 10
  - d) less than 10
- 2) Who was the author of the book “Research Methodology- Methods and Techniques ”?
  - a) Redman and Mory
  - b) C. R. Kothari
  - c) Clifford Woody
  - d) D. Slesinger
- 3) The impact factor (IF) or journal impact factor (JIF) of an academic journal is a scientometric index calculated by \_\_\_\_\_.
  - a) Clarivate
  - b) UGC-CARE
  - c) Scopus
  - d) AMS
- 4) The research that aims at finding a solution for an immediate problem facing society or an industrial/business organization is called as \_\_\_\_\_ research.
  - a) Fundamental
  - b) Qualitative
  - c) Empirical
  - d) Applied
- 5) The method of sampling in which each and every item in the population has an equal chance of inclusion in the sample and in case of finite universe, has the same probability of being selected is called as \_\_\_\_\_ sampling.
  - a) Simple random
  - b) Deliberate
  - c) Cluster
  - d) area
- 6) The lack of a scientific training in the \_\_\_\_\_ is a great impediment for researchers in our country.
  - a) Laboratory
  - b) research methodology
  - c) Library
  - d) publication

- 7) The points that need to be avoided in the abstract is/are \_\_\_\_\_.  
 a) Mathematical equations    b) citing reference  
 c) Date of publication        d) all of the above
- 8) The AMS Mathematics subject Classification (1991) divide mathematics into \_\_\_\_\_ sections with numbers between 0 to 94 which are further divided into many subsections.  
 a) 61                                b) 10  
 c) finite                            d) 94

**B) State true or False.****04**

- 1) The number of keywords is usually ten or less.
- 2) The last part of research article is reference list.
- 3) Research provides the basis for nearly all government policies in our economic system.
- 4) Formulation of research problem does not require primary data collection.

**Q.2 Answer the following. (Any Six)****12**

- a) Define citation index.
- b) Write longform of SCI and AMS.
- c) Write two definitions of Research.
- d) Explain Stratified sampling.
- e) Give information about Math Sci Net
- f) Write about Mathematics Subject classification.
- g) Explain the need of UGC CARE
- h) What is research methodology?

**Q.3 Answer the following. (Any Three)****12**

- a) Explain Do's and Don'ts of Mathematical writing.
- b) Give the difference between Research methods and Research Methodology.
- c) Explain the Research Process.
- d) Write note on Development of Working Hypothesis.

**Q.4 Answer the following. (Any Two)****12**

- a) Explain Quantitative vs Qualitative research
- b) Give details of how to write abstract and conclusion in research article.
- c) Write detail information about different types of sampling.

**Q.5 Answer the following. (Any Two)****12**

- a) Write in detail about Criteria of Good Research.
- b) Write the file format of Research article.
- c) Explain the preparation of the report or the thesis.

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**M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination:  
March/April - 2026  
Field Extension Theory (2317201)**

Day & Date: Thursday, 16-04-2026  
Time: 11:00 AM To 01:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ) 08**

- 1) If  $F$  is a finite field of  $q$  elements and  $F \subseteq K$  is also a finite field such that  $[K:F] = n$  then  $K$  has \_\_\_\_\_ elements.
  - a)  $q^2$
  - b)  $q^n$
  - c)  $n^q$
  - d)  $n^2$
- 2) The splitting field of  $x^2 - 1$  over  $Q$  is \_\_\_\_\_.
  - a)  $Q(i)$
  - b)  $R$
  - c)  $Q$
  - d)  $C$
- 3) If  $a$  and  $b$  are algebraic over  $F$  of degrees  $m$  and  $n$  respectively then  $\frac{a}{b}$  ( $b \neq 0$ ) is algebraic of degree \_\_\_\_\_ over  $F$ .
  - a) atmost  $mn$
  - b) equal to  $mn$
  - c) atmost  $m/n$
  - d) equal to  $m/n$
- 4) The number  $\pi$  is algebraic over \_\_\_\_\_.
  - a)  $R$
  - b)  $Q$
  - c)  $Q(i)$
  - d)  $Q(\sqrt{2})$
- 5)  $O(G(C, R)) =$  \_\_\_\_\_.
  - a) 1
  - b) 0
  - c) 3
  - d) 2
- 6) Every automorphism of every field  $E$  leaves \_\_\_\_\_ elements of  $E$  fixed.
  - a) at least one
  - b) at least two
  - c) two
  - d) one
- 7) Two finite field of same order are \_\_\_\_\_.
  - a) homomorphic
  - b) isomorphic
  - c) not isomorphic
  - d) None of these
- 8) The set of all \_\_\_\_\_ forms a field.
  - a) integers
  - b) transcendental numbers
  - c) irrational
  - d) algebraic numbers

**B) State whether the following statements are True or False. 04**

- 1) There exists a finite field with 25 elements.
- 2) It is impossible to square any circle of constructible radius by straight edge and compass.
- 3)  $\sqrt{2} \in R$  is algebraic degree 1 over  $R$ .
- 4) A field of complex numbers is a normal extension of field of real numbers.

**Q.2 Answer the following. (Any Six) 12**

- a) Define Algebraic element and its degree.
- b) Check whether  $7 + \sqrt{5}$  is algebraic over  $Q$  or not.
- c) Write short note on finite field.
- d) Find degree and basis of  $Q(2^{1/3})$  over  $Q$ .
- e) Find all automorphisms on  $Q(\sqrt{5})$ .
- f) Define Perfect field and give an example.
- g) Construct a field with 9 elements.
- h) Define Field extension and Algebraic extension.

**Q.3 Answer the following. (Any Three) 12**

- a) Find all possible automorphisms on a field of rational numbers  $Q$ .
- b) Find Splitting field of:
  - i)  $x^2 + 5x - 6$  over  $Q$
  - ii)  $x^2 - 2$  over  $Q$
- c) If  $K$  is an extension of field  $F$  then prove that the set of all algebraic elements of  $K$  over  $F$  forms a subfield of  $K$ .
- d) With usual notations Prove or disprove that:  
 $Q(\sqrt{3} + \sqrt{7}) = Q(\sqrt{3}, \sqrt{7})$

**Q.4 Answer the following. (Any Two) 12**

- a) if  $\psi$  be an isomorphism of a field  $F$  onto a field  $F'$  such that  $\psi(\alpha) = \alpha'$  for all  $\alpha \in F$  then prove that there is an isomorphism  $\psi^*$  of  $F[x]$  onto  $F'[t]$  such that  $\psi^*(\alpha) = \psi(\alpha) = \alpha'$  for all  $\alpha \in F$ .
- b) Prove that: A field  $K$  is normal extension of field  $F$  of characteristic 0 iff  $K$  is a splitting field of some polynomial over  $F$ .
- c) If  $K$  is an extension of a field  $F$  and  $a \in K$  be algebraic of degree  $n$  over  $F$  then prove that  

$$F(a) = \{\beta_0 + \beta_1 a + \beta_2 a^2 + \dots + \beta_{n-1} a^{n-1} / \beta_0, \beta_1, \beta_2, \dots, \beta_{n-1} \in F\}$$

**Q.5 Answer the following. (Any Two) 12**

- a) If  $F$  is a field of characteristic zero and if  $a, b$  are algebraic numbers over  $F$  then prove that  $F(a, b)$  is a simple extension.
- b) If  $a \in K$  be algebraic over  $F$  then prove that any two minimal monic polynomials for  $a$  over  $F$  are equal.
- c) If a real number  $\alpha$  and  $\beta$  are constructible then prove that  $\alpha \cdot \beta$  and  $\alpha/\beta$  ( $\beta \neq 0$ ) are also constructible.

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Set **P**

**M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS)**  
**Examination: March/April - 2026**  
**General Topology (2317202)**

Day & Date: Saturday, 18-04-2026  
 Time: 11:00 AM To 01:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative.****08**

- 1) In discrete topological space  $\langle X, \mathfrak{T} \rangle$ , each subset of  $X$  is \_\_\_\_\_.
  - a) open but not closed
  - b) closed but not open
  - c) both open and closed
  - d) neither open nor closed
- 2) If  $X$  is a finite set, then the co-finite topology on  $X$  reduces to \_\_\_\_\_.
  - a) co-countable topology
  - b) discrete topology
  - c) indiscrete topology
  - d) None of these
- 3) Consider the two statements:  
 P: Every second axiom space is a first axiom space.  
 Q: Every  $T_1$  space is  $T_2$  Then \_\_\_\_\_.
  - a) only P is true
  - b) only Q is true
  - c) both P and Q are true
  - d) both P and Q are false
- 4) If  $\langle X, \mathfrak{T} \rangle$  and  $\langle X^*, \mathfrak{T}^* \rangle$  are two topological spaces and  $f: X \rightarrow X^*$  is a function then the function  $f$  is continuous on  $X$  if  $\langle X, \mathfrak{T} \rangle$  is \_\_\_\_\_.
  - a) discrete topological space
  - b) indiscrete topological space
  - c) co-finite topological space
  - d) co-countable topological space
- 5) A set  $A$  in a topological space is closed if \_\_\_\_\_.
  - a)  $A \subset \bar{A}$
  - b)  $\bar{A} \subset A$
  - c)  $i(A) = \emptyset$
  - d)  $\bar{A} = X$
- 6) In discrete T-space  $\langle X, \mathfrak{T} \rangle$ , the only connected sets are \_\_\_\_\_.
  - a) empty set
  - b) all open sets
  - c) all closed sets
  - d) all singleton sets
- 7) In any T-space  $\langle X, \mathfrak{T} \rangle$ , if  $A \subset X$  then  $i(A)$  is \_\_\_\_\_.
  - a) the smallest closed set containing  $A$
  - b) the smallest closed set contained in  $A$
  - c) the largest open set contained in  $A$
  - d) the largest open set containing  $A$



**Q.5 Answer the following. (Any Two)**

12

- a) If  $X$  is an uncountable set and  $\mathfrak{T} = \{\emptyset\} \cup \{A \subset X \mid X - A \text{ is countable}\}$ , then prove that  $\langle X, \mathfrak{T} \rangle$  is a topology on  $X$ .
- b) If  $A, B$  are separated sets in  $\langle X, \mathfrak{T} \rangle$ , then prove that  $A$  and  $B$  are both open and closed in  $A \cup B$  and conversely.
- c) Define homeomorphism. If  $\langle X, \mathfrak{T} \rangle$  and  $\langle X^*, \mathfrak{T}^* \rangle$  are any two spaces and  $f: X \rightarrow X^*$  is a function then prove that  $f: X \rightarrow X^*$  is continuous iff  $f^{-1}[i^*(E^*)] \subseteq i^*[f^{-1}(E^*)]$ , for any  $E^* \subseteq X^*$

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**M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination:  
March/April – 2026  
Complex Analysis (2317207)**

Day & Date: Tuesday, 21-04-2026  
Time: 11:00 AM To 01:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ) 08**

- 1) If  $f$  have an isolated singularity at  $z = a$  and  $f(z) = \sum_{n=-\infty}^{\infty} a_n(z - a)^n$  is its Laurent expansion about  $z = a$ . Then residue of  $f$  at  $z = a$  is \_\_\_\_\_.
  - a)  $a_{-1}$
  - b)  $a_0$
  - c)  $a_{-2}$
  - d)  $a_1$
- 2) Number of zeros of  $f(z) = e^z$  in a finite complex plane is \_\_\_\_\_.
  - a) zero
  - b) one
  - c) finite
  - d) countably infinite
- 3) Which of the following is not an entire function?
  - a)  $f(z) = z^2 + 2z + 1$
  - b)  $f(z) = x - iy$
  - c)  $f(z) = \sin z$
  - d)  $f(z) = e^z$
- 4) If  $f$  is an entire function then \_\_\_\_\_.
  - a)  $f$  has power series expansion
  - b)  $f$  has not a power series expansion
  - c)  $f$  is constant
  - d)  $f$  is polynomial
- 5)  $\int_C \frac{f(z)}{z-a} dz$  is equal to \_\_\_\_\_.
  - a)  $2\pi i f(a)$
  - b)  $2\pi i \operatorname{Im} f(a)$
  - c)  $2\pi i \operatorname{Res} f(a)$
  - d)  $-2\pi i \operatorname{Res} f(a)$
- 6) A non-constant analytic function maps open set to a \_\_\_\_\_.
  - a) Circle
  - b) Straight line
  - c) Open set
  - d) Closed set
- 7) The radius of convergence of the power series  $\sum_{n=0}^{\infty} (n + 2i)^n z^n$  is \_\_\_\_\_.
  - a) 0
  - b) 1
  - c)  $\infty$
  - d)  $n^2 + 4$

- 8) The simple pole of the function  $f(z) = \frac{z^2}{(z-1)^2(z+2)}$  is \_\_\_\_\_.
- a) at  $z = -2$                       b) at  $z = 1$   
 c) at  $z = 0$                          d) at  $z = 2$

**B) Fill in the blanks.****04**

- 1) If  $f$  has an isolated singularity at  $a$  and  $\lim_{z \rightarrow a} (z - a)f(z) = 0$  then the point  $z = a$  is \_\_\_\_\_.
- 2) The value of integral  $\int_{|z|=\frac{1}{2}} \frac{e^z}{(z-2)(z-4)} dz =$  \_\_\_\_\_.
- 3) A function which has poles as its only singularities in the finite part of the plane is said to be \_\_\_\_\_.
- 4) If  $f(z)$  is analytic in a simply connected domain  $D$ , then for every cloth path  $C$  in  $D$ ,  $\oint_C f(z) dz =$  \_\_\_\_\_.

**Q.2 Answer the following. (Any Six)****12**

- a) Define Magnification and Inversion mapping.
- b) Evaluate:  $\int_{\gamma} \frac{\cos 2z - e^z}{(z+1)^2(z+2)^2} dz$  over  $\gamma: |z| = 1.5$
- c) Show that the polynomial of degree  $n$  has no singularities in the finite part of the complex plane.
- d) State Cauchy Residue theorem.
- e) Find  $\text{Res}(f; 1)$  for  $f(z) = \frac{z^2}{(z-1)^2(z-2)^2}$
- f) What is triangular path?
- g) Define isolated and Non-isolated singularities.
- h) Evaluate:  $\int_{|z|=\frac{3}{2}} \cot \pi z dz$

**Q.3 Answer the following. (Any Three)****12**

- a) If  $f$  is analytic in  $B(a, R)$  and suppose that  $|f(z)| \leq M$  for all  $z$  in  $B(a, R)$  then prove that  $|f^n(a)| \leq \frac{n!M}{R^n}$
- b) Show that  $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z} ds = 2\pi$  where  $|z| < 1$
- c) Calculate residue of  $\frac{z^2-2z}{(z^2+4)(z+1)^2}$
- d) Find the Mobius transformation which maps the given points  $z_1 = -1, z_2 = 0$  and  $z_3 = 1$ , onto the points  $w_1 = i, w_2 = 0$  and  $w_3 = \infty$

**Q.4 Answer the following. (Any Two)**

12

- a) Show that  $\int_0^\pi \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = 0$
- b) Find Laurent series expansion of  $\frac{1}{z(z-1)(z-2)}$  in
- 1)  $ann(0; 0,1)$
  - 2)  $ann(0; 1,2)$
- c) If  $p(z)$  is a non-constant polynomial then there is a complex number  $a$  with  $p(a) = 0$ .

**Q.5 Answer the following. (Any Two)**

12

- a) Evaluate  $\int_{|z|=\frac{3}{2}} \frac{3z^2-z+1}{2z^3-z^2+2z+1} dz$
- b) If  $f$  is analytic in  $B(a, R)$  then  $f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n; |z-a| < R$   
Where,  $a_n = \frac{1}{n!} f^n(a)$  and the series has radius of convergence  $\geq R$
- c) Prove that all the roots of  $z^7 - 5z^3 + 12 = 0$  lie between the circles  $|z| = 1$  and  $|z| = 2$

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**M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS)**  
**Examination: March/April - 2026**  
**Functional Analysis (2317301)**

Day & Date: Friday, 17-04-2026  
 Time: 11:00 AM To 01:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ)**

**08**

- 1) The set of bounded linear transformation  $B(X, Y)$  is complete if \_\_\_\_\_.
  - a)  $X$  is open
  - b)  $Y$  is closed
  - c)  $X$  is complete
  - d)  $Y$  is complete
- 2) If  $H$  is a Hilbert space then the conjugate space  $H^*$  is also a Hilbert space with the inner product defined by,  $\langle F_x, F_y \rangle =$  \_\_\_\_\_.
  - a)  $\langle x, y \rangle$
  - b)  $\langle x, 0 \rangle$
  - c)  $\langle y, x \rangle$
  - d)  $\langle 0, y \rangle$
- 3) By Zorn's lemma, Every non-empty partially ordered set in which each chain has an upper bound has a \_\_\_\_\_.
  - a) bound
  - b) supremum
  - c) infimum
  - d) maximal element
- 4) If  $H$  is a Hilbert space,  $x \in H$  and  $x \perp x$ , then  $x$  must be \_\_\_\_\_.
  - a) unit
  - b)  $x^2$
  - c)  $\sqrt{x}$
  - d) zero
- 5) The Minkowski's inequality for scalar sequences  $a = (a_i)$  and  $b = (b_i)$  states that \_\_\_\_\_.
  - a)  $\| a + b \| \leq \| a \| + \| b \|$
  - b)  $\| ab \| \leq \| a \| \cdot \| b \|$
  - c)  $\| ab \| \geq \| a \| \cdot \| b \|$
  - d)  $\| a + b \| \geq \| a \| + \| b \|$
- 6) If  $E$  is a normed space and  $d$  be the metric induced by the norm, then for  $x, y \in E$ ,  $d(x - y, 0)$  equals \_\_\_\_\_.
  - a)  $d(x, 0) - d(y, 0)$
  - b)  $d(x, x - y)$
  - c)  $d(x, y)$
  - d) None of these

- 7) By closed graph theorem, if  $B$  and  $B'$  are Banach spaces and  $T$  is a linear transformation of  $B$  into  $B'$  then  $T$  is continuous mapping iff \_\_\_\_\_.
- a) its graph is open set      b) its graph is closed set  
 c) its graph is finite set      d) its graph is countable set
- 8) If  $N$  is a normed linear space then the space  $N$  is a Banach space iff the set  $\{x \in N / \|x\| \leq 1\}$  is \_\_\_\_\_.
- a) complete      b) dense  
 c) bounded      d) finite

**B) True or False.**

**04**

- 1) An operator  $N$  is said to be normal operator if it commutes with its adjoint operator.
- 2) Every Banach space is finite dimensional.
- 3) The mapping  $\phi : H \rightarrow H^*$  is linear.
- 4) Any two finite dimensional normed linear spaces over same scalar field are topologically isomorphic.

**Q.2 Answer the following. (Any Six)**

**12**

- a) State and prove Pythagorean theorem.
- b) Define: Inner Product and Norm
- c) Define Graph of  $T$  and Hilbert space.
- d) With usual notation prove that :  $\{0\}^\perp = H$
- e) If  $T: X \rightarrow Y$  is a bounded linear operator then prove that the null space  $N(T)$  of  $T$  is subspace of  $X$ .
- f) With usual notation prove that :  $s(x; r) = x + s(0; r)$ .
- g) Prove that: Every closed subspace of Banach space is complete.
- h) If  $X$  is a complex IPS then Prove that  
 $\langle x, ay + bz \rangle \geq \bar{a} \langle x, y \rangle + \bar{b} \langle x, z \rangle$

**Q.3 Answer the following. (Any Three)**

**12**

- a) If  $T: X \rightarrow Y$  is a linear transformation and  $T$  is continuous at any point of  $X$  then prove that  $T$  is continuous on  $X$ .
- b) If  $B$  and  $B'$  are Banach spaces,  $T$  is linear transformation of  $B$  into  $B'$  and  $T$  is continuous mapping then prove that its graph  $T_G$  is closed.
- c) If  $H$  is a Hilbert space and  $f$  is an arbitrary functional in  $H^*$  then prove that there exists a unique vector  $y \in H$  such that  $f(x) = \langle x, y \rangle$  for every  $x \in H$  and  $\|f\| = \|y\|$
- d) State and prove Riesz Lemma.

**Q.4 Answer the following. (Any Two) 12**

- a) If  $M$  be a closed linear subspace of a Hilbert space  $H$  then prove that  $H = M \oplus M^\perp$ .
- b) If  $M$  is a linear subspace of normed linear space  $N$  and  $f$  is a linear functional defined on  $M$ ,  $x_0 \notin M$  and  $M_0$  is linear space spanned by  $M$  and  $x_0$  then prove that  $f$  can be extended to a functional  $f_0$  on  $M_0$  such that  $\|f_0\| = \|f\|$  (only for the real scalar field)
- c) State and Prove Schwarz's inequality

**Q.5 Answer the following. (Any Two) 12**

- a) If  $M$  be a linear subspace of a Hilbert space  $H$  then prove that  $M$  is closed if and only if  $M = M^{\perp\perp}$ .
- b) If  $M$  is a closed linear subspace of a Hilbert space  $H$  and  $x$  be a vector not in  $M$  and  $d = d(x, M)$  then prove that there exists a unique vector  $y_0$  in  $M$  such that  $\|x - y_0\| = d$ .
- c) State and prove Open Mapping Theorem.

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**M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination:  
March/April – 2026  
Linear Algebra (2317302)**

Day & Date: Monday, 20-04-2026  
Time: 11:00 AM To 01:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative.**

**08**

- 1) The trace of a square matrix is equal to the \_\_\_\_\_.
  - a) Determinant of the matrix
  - b) Sum of the matrix's eigen values
  - c) Product of the matrix's eigen values
  - d) Sum of the matrix's eigen vectors
  
- 2) If  $u$  and  $v$  are two vectors in  $R^n$ , they are orthogonal if their inner product is \_\_\_\_\_.
  - a) 1
  - b) 0
  - c) -1
  - d) Sum of the matrix's eigen vectors
  
- 3) Which of the following is a defining property of a normal operator  $N$  on a Hilbert space  $H$ ?
  - a)  $N = N^{**}$
  - b)  $N^*N = I$  (where  $I$  is the identity)
  - c)  $NN^{**} = N^*N$
  - d)  $N(N^{**}) = N^{**}(N) = 0$
  
- 4) What are the conditions for an  $n \times n$  matrix to be a Jordan block matrix?
  - a) All entries equal, diagonal entries 1, other entries 0
  - b) All diagonal entries equal, each entry above diagonal is 1, all other entries 0
  - c) Equal diagonal and above diagonal entries, other entries 1
  - d) Diagonal entries 1, all other entries 0
  
- 5) If the characteristic polynomial of a matrix  $A$  is  $p(\lambda) = (\lambda - 1)^2(\lambda - 2)^3$  and its minimal polynomial is  $m(\lambda) = (\lambda - 1)(\lambda - 2)^2$ , what can you conclude about matrix  $A$ ?
  - a)  $A$  is diagonalizable over the field of complex numbers.
  - b)  $A$  is not diagonalizable over the field of complex numbers.
  - c) The smallest Jordan block for the eigenvalue 2 has size 3.
  - d) The characteristic polynomial and minimal polynomial are equal.

- 6) What is the fundamental statement of the Cayley-Hamilton Theorem?
- The determinant of a matrix is always zero
  - Every matrix satisfies its characteristic equation
  - Every square matrix is invertible
  - The trace of a matrix is equal to its determinant
- 7) If  $A$  be a  $3 \times 3$  real matrix whose characteristic polynomial  $p(T)$  is divisible by  $T^2$ . Which of the following statements is true?
- The eigenspace of  $A$  for the eigenvalue 0 is two-dimensional
  - $A^3 = 0$
  - All the eigenvalues of  $A$  are real
  - $A$  is diagonalizable
- 8) If  $A$  is an  $n \times n$  square nilpotent matrix of index  $k$ , then its minimal polynomial is \_\_\_\_\_.
- |              |          |
|--------------|----------|
| a) $x^{k-1}$ | b) $x^k$ |
| c) $x^{k+1}$ | d) 0     |

**B) State True or False:**

**04**

- A sufficient condition for orthogonality of the pair  $\{u, v\}$  is  $\|u + v\| = \|v - u\|$
- If  $V$  be a vector space over the field  $F$  then a linear transformation  $T: V \rightarrow V$  is called linear functional on  $V$ .
- If  $V$  be the vector space over the field  $F$  and  $S = V$  then  $S^\perp = 0$ .
- A matrix is diagonalizable if and only if its determinant is non-zero.

**Q.2 Answer the following. (Any Six)**

**12**

- a) Find the characteristic values of a matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 0 \\ 0 & 5 & 6 \end{bmatrix}$
- b) Prove that every finite dimensional inner product space has an orthonormal basis.
- c) Define Dual Space of vector space.
- d) State Schwarz's Inequality.
- e) If  $V$  be an  $n$  dimensional vector space over the field  $F$  and  $\alpha$  is a nonzero vector in  $V$  then prove that there exists a linear functional  $f$  on  $V$  such that  $f(\alpha) \neq 0$ .
- f) Define orthogonal complement.
- g) Prove that the matrix  $A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 4 \\ 9 & 8 & 7 \end{bmatrix}$  is triangularisable.
- h) If  $S$  be any subset of vector space  $V$  then prove that  $S^0$  is a subspace of  $V^*$

**Q.3 Answer the following. (Any Three)**

12

- a) Find the minimal polynomial of matrix  $A = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$
- b) Show that the minimal polynomial of a matrix or of a linear operator is unique.
- c) Show that the mapping  $T: R^3 \rightarrow R^3$  defined by  $T(x, y, z) = (x - y + 2z, 2x + y - z, -x - 2y)$  is a Linear Transformation.
- d) Prove that a form is Hermitian iff the corresponding linear operator  $T$  is self adjoint.

**Q.4 Answer the following. (Any Two)**

12

- a) If  $V$  be an inner product space over the field  $F$  then for  $\alpha, \beta \in V$  and  $c \in F$  then prove that
- $\langle \alpha, \beta + \gamma \rangle = \langle \alpha, \beta \rangle + \langle \alpha, \gamma \rangle$
  - $\langle \alpha, c\beta \rangle = \bar{c} \langle \alpha, \beta \rangle$
  - $\langle \alpha, 0 \rangle = \langle 0, \alpha \rangle = 0$
  - $\langle \alpha, \alpha \rangle = 0 \Leftrightarrow \alpha = 0$

- b) Find the Jordan canonical form of the matrix  $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{bmatrix}$

- c) If  $B = \{(1, -1, 1), (1, 0, 1), (1, 1, 2)\}$  is a basis of  $V_3(R)$  then find the dual basis of  $B$ .

**Q.5 Answer the following. (Any Two)**

12

- a) If  $V$  be a finite dimensional vector space over the field  $F$  and  $T$  be a linear operator on  $V$  then prove that  $T$  is triangularisable iff the minimal polynomial for  $T$  is a product of linear factors over  $F$ .
- b) If  $V$  be a finite dimensional inner product space and  $T$  be normal operator on  $V$  then prove that  $\alpha$  is a characteristic vector of  $T$  with characteristic value  $c$  iff  $\alpha$  is characteristic vector for  $T^*$  with characteristic value  $\bar{c}$ .
- c) If  $B = \{(1, -1, 1), (1, 0, 1), (1, 1, 2)\}$  is a basis of  $V_3(R)$  then find the dual basis of  $B$ .

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**M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination:  
March/April - 2026**

**Advanced Discrete Mathematics (2317306)**

Day & Date: Wednesday, 22-04-2026  
Time: 11:00 AM To 01:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ)**

**08**

- 1) In a lattice  $L$ , if  $a, b \in L$  and if  $a \wedge b = a$  then \_\_\_\_\_.
  - a)  $a \lesssim b$
  - b)  $a \gtrsim b$
  - c)  $a > b$
  - d)  $a \neq b$
- 2) The complete graph  $K_5$  has \_\_\_\_\_ different spanning trees.
  - a) 130
  - b) 110
  - c) 120
  - d) 125
- 3) Every tree has \_\_\_\_\_ centres.
  - a) one
  - b) exactly one
  - c) at most two
  - d) two
- 4) Four persons enter into car, where there are 5 seats. In how many ways can they take up their seats?
  - a) 50
  - b) 15
  - c) 60
  - d) 120
- 5) The minimum number of edges in a connected graph with 'n' vertices is \_\_\_\_\_.
  - a)  $n + 1$
  - b)  $n - 1$
  - c)  $n$
  - d)  $2n$
- 6) Which of the following pair of elements are not comparable in the poset  $(Z^+, /)$ ?
  - a) 3, 6
  - b) 8, 49
  - c) 4, 12
  - d) 9, 36
- 7) An expression for geometric series  $\frac{1}{(1-x)^n}$  is \_\_\_\_\_.
  - a)  $\sum_{r=0}^{\infty} n - r_{c_r} x^r$
  - b)  $\sum_{r=0}^{\infty} n - 1 + r_{c_r} x^r$
  - c)  $\sum_{r=0}^{\infty} n + r_{c_r} x^r$
  - d)  $\sum_{r=0}^{\infty} n + 1 - r_{c_r} x^r$

- 8) If  $A$  and  $B$  are any two finite disjoint sets then  $|A \cup B| = \underline{\hspace{2cm}}$ .
- a)  $|A| + |B| - |A \cap B|$       b)  $|A| + |B| + |A \cap B|$   
 c)  $|A| + |B|$                       d) All of these

**B) State True or False.****04**

- 1) The union of two sublattices is again a sublattice.
- 2) An element ' $b$ ' in a lattice  $L$  is called a complement of an element ' $a$ ' in  $L$  if  $a \wedge b = 0$  and  $a \vee b = 1$ .
- 3) If ' $e$ ' be an edge of the graph  $G$  and let  $G - e$  be the subgraph obtained by deleting  $e$  then  $w(G - e) \leq w(G) + 1$
- 4) The length of a largest cycle in the graph is called trail of a graph.

**Q.2 Answer the following. (Any Six)****12**

- a) Show that the pentagonal lattice is not distributive.
- b) Prove that in any graph  $G$  there are even numbers of odd degree vertices.
- c) What is sub lattice? Show that  $D_{10}$  is a sub lattice of  $D_{50}$ .
- d) Define Tree and Spanning Tree.
- e) Find the value of combination of  $n$  different things taken  $r (\leq n)$  at a time?
- f) If  $G$  be a  $k$ -regular graph, where  $k$  is an odd number then prove that the number of edges in  $G$  is a multiple of  $k$ .
- g) If  $G$  be a connected graph with  $n$  vertices and  $n - 1$  edges then prove that  $G$  is tree.
- h) State and prove pigeonhole principle.

**Q.3 Answer the following. (Any Three)****12**

- a) In how many ways 5 boys and 4 girls can be seated at a round table if
  - i) all the four girls seat together
  - ii) all the four girls do not seat together
- b) Show that every chain is a distributive lattice.
- c) If  $2n + 1_{P_{n-1}} : 2n - 1_{P_n} = 3 : 5$  then find the value of  $n$ ?
- d) If  $T$  is a tree with  $n$  vertices then prove that  $T$  has precisely  $n - 1$  edges.

**Q.4 Answer the following. (Any Two)****12**

- a) Write a note on Finite Boolean algebra and show that  $D_{30}$  is a Finite Boolean algebra under partial order of divisibility.
- b) If  $G$  be a graph with  $n$  vertices  $v_1, v_2, v_3, \dots, v_n$  &  $A$  denote the adjacency matrix of  $G$  with respect to this listing of vertices. Let  $B = [b_{i,j}]$  be the matrix  $B = A + A^2 + A^3 + \dots + A^{n-1}$ . Then show that  $G$  is connected graph iff for every pair of distinct indices  $i, j$  we have  $b_{i,j} \neq 0$ .
- c) Find the number of primes less than 100 using principle of Inclusion-Exclusion?

**Q.5 Answer the following. (Any Two)**

- a)** Describe matrix representation of graph with example.  
**b)** Determine the generating function of the following sequences.

i)  $a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$

ii)  $a_r = (r + 1)3^r$

- c)** If  $(L, \lesssim)$  be a lattice then for any  $a, b, c, d \in L$  Show that,

i)  $a \lesssim b \Rightarrow a \vee c \lesssim b \vee c$

ii)  $a \lesssim b \Rightarrow a \wedge c \lesssim b \wedge c$

iii)  $a \lesssim b \text{ and } c \lesssim d \Rightarrow a \vee c \lesssim b \vee d$

iv)  $a \lesssim b \text{ and } c \lesssim d \Rightarrow a \wedge c \lesssim b \wedge d$

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**M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination:  
March/April – 2026  
Partial Differential Equations (2317401)**

Day & Date: Thursday, 16-04-2026  
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ)**

**08**

- 1) For any constant  $a$  a parametric equation  $x = a \sin u \cos v$ ,  $y = a \sin u \sin v$ ,  $z = a \cos u$  represent the surface \_\_\_\_\_.
  - a) sphere
  - b) cone
  - c) circle
  - d) None of these
  
- 2) The partial differential equation formed by eliminating arbitrary functions from the relation  $z = f(x + at) + g(x - at)$  is \_\_\_\_\_.
  - a)  $\frac{\partial^2 z}{\partial t^2} - \frac{\partial^2 z}{\partial x^2} = 0$
  - b)  $\frac{\partial^2 z}{\partial t^2} + a \frac{\partial^2 z}{\partial x^2} = 0$
  - c)  $\frac{\partial^2 z}{\partial t^2} - a^2 \frac{\partial^2 z}{\partial x^2} = 0$
  - d)  $a \frac{\partial^2 z}{\partial t^2} + \frac{\partial^2 z}{\partial x^2} = 0$
  
- 3) A necessary & sufficient condition for the Pfaffian differential equation  $\vec{x} \cdot d\vec{r} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$  is integrable is that \_\_\_\_\_.
  - a)  $\vec{x} \cdot \text{curl } \vec{x} = 0$
  - b)  $\vec{x} \cdot \text{curl } \vec{x} \neq 0$
  - c)  $\vec{x} \cdot \text{curl } \vec{x} = 1$
  - d) None of these
  
- 4) The equation  $x^2(y - 1)z_{xx} - x(y^2 - 1)z_{xy} + y(y^2 - 1)z_{yy} + z_x = 0$  is hyperbolic in the entire  $xy$ -plane except along \_\_\_\_\_.
  - a)  $x$ -axis
  - b)  $y$ -axis
  - c) a line parallel to  $y$ -axis
  - d) a line parallel to  $x$ -axis
  
- 5) D'Alembert's solution of the wave equation is \_\_\_\_\_.
  - a)  $u(x, t) = \varphi(x + ct) - \Psi(x - ct)$
  - b)  $u(x, t) = \varphi'(x + ct) + \Psi'(x - ct)$
  - c)  $u(x, t) = \varphi(x + ct) + \Psi(x - ct)$
  - d)  $u(x, t) = \varphi'(x - ct) - \Psi'(x - ct)$



- Q.3 Answer the following. (Any Three) 12**
- Find a partial differential equation by eliminating arbitrary constant from  $2z = (ax + y)^2 + b$ .
  - Show that the solution of Neumann problem is either unique or it differs from one another by constant.
  - Find the general solution of  $2x(y + z^2)p + y(2y + z^2)q = z^3$
  - Describe Charpit's method of solving a first order partial differential equation  $f(x, y, z, p, q) = 0$
- Q.4 Answer the following. (Any Two) 12**
- Reduce the equation  $4u_{xx} - 4u_{xy} + 5u_{yy} = 0$  to a canonical form.
  - Discuss how a general solution is used to determine the integral surface which passes through a given curve.
  - Prove that the singular integral is obtained by eliminating  $p$  and  $q$  from the equations  
 $f(x, y, z, p, q) = 0, f_p(x, y, z, p, q) = 0, f_q(x, y, z, p, q) = 0.$
- Q.5 Answer the following. (Any Two) 12**
- If  $\vec{X} = (P, Q, R)$  is a vector such that  $\vec{X} \cdot \text{curl } \vec{X} = 0$  and  $\mu$  is an arbitrary differentiable function of  $x, y$  and  $z$  then prove that  $\mu\vec{X} \cdot \text{curl}(\mu\vec{X}) = 0$ .
  - Find the complete integral of  $z^2 - pqxy = 0$  by Charpit's method.
  - Find the condition that a one parameter family of surfaces forms a family of equipotential surfaces.

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**M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS)**  
**Examination: March/April - 2026**  
**Integral Equations (2317402)**

Day & Date: Saturday, 18-04-2026  
 Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
 2) Figures to the right indicate full marks.

**Q.1 A) Choose the correct alternative. (MCQ)**

**08**

- 1) The second iterated kernel  $K_2(x, t)$  for the integral equation  $y(x) = f(x) + \int_0^1 e^{x-t} y(t) dt$  is \_\_\_\_\_.
  - a) 0
  - b) 1
  - c)  $e^{x-t}(x-t)$
  - d)  $e^{x-t}$
  
- 2) An integral equation  $g(x)u(x) = f(x) + \int_a^x K(x, t)u(t)dt$  is said to be of the second kind if \_\_\_\_\_.
  - a)  $g(x) = 0$
  - b)  $g(x) = 1$
  - c)  $f(x) = 0$
  - d)  $f(x) = 1$
  
- 3) If  $R(x, t; \lambda)$  is the resolvent kernel of a Fredholm integral equation  $g(x)u(x) = f(x) + \lambda \int_a^x K(x, t) u(t)dt$  then the resolvent kernel is a solution of \_\_\_\_\_.
  - a)  $R(x, t; \lambda) = K(x, t) + \lambda \int_t^x K(x, z) R(z, t; \lambda) dt$
  - b)  $R(x, t; \lambda) = K(x, t) + \lambda \int_a^x K(x, z) R(z, t; \lambda) dt$
  - c)  $R(x, t; \lambda) = K(x, t) + \lambda \int_a^b K(x, z) R(z, t; \lambda) dt$
  - d) All of above
  
- 4) Which of the following is a homogeneous Fredholm integral equation?
  - a)  $y(x) = \int_0^2 y(t) dt$
  - b)  $x = \int_0^2 \sin t y(t) dt$
  - c)  $y(x) = e^t + \int_0^1 \sin(x+t)y(t) dt$
  - d) All of these



**Q.3 Answer the following. (Any Three)****12**

- a) Find the eigenvalues and eigenfunctions for  $y(x) = \lambda \int_0^{\frac{\pi}{4}} \sin^2 x y(t) dt$
- b) Show that  $y(x) = 1 - x$  is solution of  $\int_0^x e^{x-t} y(t) dt = x$
- c) Convert into an integral equation  $y'' + y = 0, y(0) = y'(0) = 0$
- d) Solve:  $\int_0^\infty F(x) \cos px dx = \begin{cases} 1 - p, & 0 \leq p \leq 1 \\ 0, & p > 1 \end{cases}$

**Q.4 Answer the following. (Any Two)****12**

- a) Convert the following integral equation into initial value problem:

$$y(x) = 1 - x - 4 \sin x + \int_0^x [3 - 2(x - t)] y(t) dt$$

- b) Determine the eigenvalues and eigenfunctions of the integral equation:

$$y(x) = \lambda \int_0^1 k(x, t) y(t) dt \text{ where } k(x, t) = \begin{cases} x(t - 1), & 0 \leq x \leq t \\ t(x - 1), & t \leq x \leq 1 \end{cases}$$

- c) Solve:  $y(x) = e^x + \lambda \int_0^1 2e^x e^t y(t) dt$

**Q.5 Answer the following. (Any Two)****12**

- a) Prove that if a kernel is symmetric, then all its iterated kernel are also symmetric.

- b) Solve:  $Y(x) = 1 + \int_0^t Y(x) \sin(t - x) dx.$

- c) Solve by using the resolvent kernel:  $y(x) = \sin x + 2 \int_0^x e^{x-t} y(t) dt$

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**M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination:  
March/April – 2026  
Measure and Integration (2317405)**

Day & Date: Tuesday, 21-04-2026  
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

**Instructions:** 1) All questions are compulsory.  
2) Figures to the right indicate full marks.

**Q.1 A) Choose correct alternative. (MCQ) 08**

- 1) A measure space  $(X, \mathcal{B}, \mu)$  said to be \_\_\_\_\_ if  $\mathcal{B}$  contains all subsets of set of measure zero.
  - a) saturated
  - b) finite
  - c) complete
  - d)  $\sigma$ -finite
  
- 2) Two measures  $\nu_1$  and  $\nu_2$  on a measurable space  $(X, \mathcal{B})$  are said to be mutually singular if there exist sets  $A$  and  $B$  with  $X = A \cup B$  such that \_\_\_\_\_.
  - a)  $\nu_1(A) = 0, \nu_1(B) = 0$
  - b)  $\nu_1(B) = 0, \nu_1(A) = 0$
  - c) both a and b
  - d) None of the above
  
- 3) If  $f$  be a non-negative measurable function and  $\int f = 0$  then \_\_\_\_\_.
  - a)  $f = 0$
  - b)  $f = 0$  almost everywhere
  - c)  $f \geq 0$
  - d)  $f \geq 0$  almost everywhere
  
- 4) Consider the following statements:
  - I) Every  $\sigma$ -finite measure is saturated.
  - II) Every measurable set is locally measurable.
  - a) only I is true
  - b) only II is true
  - c) Both are false
  - d) Both are true
  
- 5) If  $E$  and  $F$  are disjoint sets then which of the following relation/relations holds?
  - a)  $\mu_*(E) + \mu_*(F) \leq \mu_*(E \cup F)$
  - b)  $\mu_*(E) + \mu_*(F) \leq \mu_*(E) + \mu^*(F)$
  - c)  $\mu_*(E \cup F) \leq \mu_*(E) + \mu^*(F)$
  - d) All of the above
  
- 6) In  $(X, \mathcal{B}, \mu)$  measure space, the measure  $\mu$  is called \_\_\_\_\_ if every locally measurable set is measurable set.
  - a) finite
  - b) saturated
  - c)  $\sigma$ -finite
  - d) complete

7) In  $(X, \mathcal{B}, \mu)$  measure space,  $E$  is a measurable set and  $\phi$  is a non-negative simple function then with usual notation.

$$\int_E \phi d\mu = \underline{\hspace{2cm}} \text{ for } \phi = \sum_{i=1}^n C_i \cdot \chi_{E_i}$$

- |   |  |
|---|--|
| a) $= \sum_{i=1}^n C_i \cdot \mu(E_i \cap E)$ | b) $= \sum_{i=1}^n C_i \cdot \mu(E_i)$           |
| c) $= \sum_{i=1}^n \phi_i \cdot \mu(E_i)$     | d) $= \sum_{i=1}^n \phi_i \cdot \mu(E_i \cap E)$ |

8) Every signed measure is a \_\_\_\_\_ of two measures.

- |            |               |
|------------|---------------|
| a) sum     | b) difference |
| c) product | d) reciprocal |

**B) State True or False:**

**04**

- 1) Every measurable set is locally measurable set.
- 2) Every saturated measure is  $\sigma$  finite measure.
- 3) The collection  $\mathcal{R}$  of measurable rectangles is an algebra.
- 4) The condition of  $\sigma$  finiteness is not necessary in Randon Nikodym theorem.

**Q.2 Answer the following. (Any Six)**

**12**

- a) Define Finite measure and  $\sigma$ -finite measure.
- b) If  $f$  and  $g$  are non negative extended real valued measurable functions on  $(X, \mathcal{B}, \mu)$  and  $E \in \mathcal{B}$  then prove that

$$f \leq g \text{ a. e on } E \implies \int_E f d\mu \leq \int_E g d\mu$$

- c) Define mutually singular measure and absolutely continuous measure.
- d) If  $c$  is a constant and  $f$  is a measurable function defined on  $X$  then prove that  $f + c$  is a measurable function.
- e) Define x-cross section and y-cross section of set  $E$ .
- f) Define Semi algebra and Product measure.
- g) If  $\nu_1 \ll \mu$  and  $\nu_2 \ll \mu$  where  $\nu_1, \nu_2, \mu$  are measures then prove that  $c_1 \nu_1 + c_2 \nu_2 \ll \mu$  where  $c_1, c_2$  are constants.
- h) If  $E$  is negative set then prove that  $\nu^+(E) = 0$

**Q.3 Answer the following. (Any Three)**

**12**

- a) If  $\nu$  is a signed measure on measurable space  $(X, \mathcal{B})$  then there is a positive set  $A$  and negative set  $B$  such that  $X = A \cup B, A \cap B = \emptyset$
- b) Prove that: The collection  $\mathcal{R}$  of measurable rectangles is a semi algebra.
- c) If  $E_i \in \mathcal{B}, \mu(E_1) < \infty$  and  $E_i \supseteq E_{i+1}, \forall_i$  then prove that  $\mu(\cap_{i=1}^{\infty} E_i) = \lim_{n \rightarrow \infty} \mu(E_n)$
- d) If  $E \subseteq F$  then with usual notations prove that  $\mu_*(E) \leq \mu_*(F)$

**Q.4 Answer the following. (Any Two)**

12

- a) If  $(X, \mathcal{B}, \mu)$  is a measure space and  $\mathcal{C}$  be the  $\sigma$ -algebra of locally measurable sets, for any  $E \in \mathcal{C}$  define  $\bar{\mu}(E) = \mu(E)$  if  $E \in \mathcal{B}$  and  $\bar{\mu}(E) = \infty$  if  $E \notin \mathcal{B}$  then prove that  $(X, \mathcal{C}, \bar{\mu})$  is a measure space.
- b) If  $\mu_1$  and  $\mu_2$  are measures on a measurable space  $(X, \mathcal{B})$  such that at least one of them is finite and  $\nu(E) = \mu_1(E) - \mu_2(E)$  for all  $E \in \mathcal{B}$  then prove that  $\nu$  is a signed measure.
- c) State and prove Jordan Decomposition theorem.

**Q.5 Answer the following. (Any Two)**

12

- a) If  $g$  is integrable over  $E \in \mathcal{B}$  and  $\{f_n\}$  is sequence of measurable functions such that on  $E, |f_n(x)| \leq g(x)$  and  $f_n \rightarrow f$  a.e. on  $E$  then prove that  $\int_E f = \lim \int_E f_n$
- b) Show that the condition of  $\sigma$ -finiteness is essential in Radon Nikodym theorem.
- c) If  $E$  is measurable set such that  $0 < \nu(E) < \infty$  then prove that there is a positive set  $A \subseteq E$  such that  $\nu(A) > 0$ .