SLR-AP – 397

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (New)
Object Oriented Programming Using C++ (Paper – I)

Day and Date : Wednesday, 15-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose correct alternatives : **10**

1) Which of the following header file includes definition of cin and cout ?

A) istream.h B) ostream.h
C) iomanip.h D) iostream.h

2) Which of the following provides a reuse mechanism ?

A) Abstraction B) Inheritance
C) Dynamic binding D) Encapsulation

3) A constructor that accepts ______________ parameters is called the default constructor.

A) one B) two
C) three D) no

4) Destructor has the same name as the constructor and it is preceded by ________________

A) ~ B) |
C) ? D) $

5) Array indexing always starts with the number

A) 1 B) 0
C) 2 D) –1

P.T.O.

6) Setprecision requires the header file ________________

A) stdlib.h
B) iomanip.h
C) console.h
D) conio.h

7) Which of the following is user defined data type ?

A) Public
B) Private
C) Class
D) Both A) and B)

8) The static member variable is initialized to ________________

A) 0
B) 1
C) 2
D) 3

9) The mechanism that binds code and data together and keeps them secure from outside world is known as ________________

A) Abstraction
B) Encapsulation
C) Inheritance
D) Polymorphism

10) What does your class can hold ?

A) data
B) function
C) both A) and B)
D) None of these

B) State whether following statements are **True** or **False** : **4**

1) An inline function is a function that is expanded in line when it is invoked.
2) We can not have virtual constructors, but we can have virtual destructors.
3) In operator overloading, we can change the basic meaning of an operator.
4) A destructor is used to destroy the object that have been created by constructor.

2. A) Write a short note on following : **8**

i) Function overloading
ii) Inline function.

B) Answer the following : 6

i) Define Flowchart. Explain different symbols used in flowchart.

ii) Explain primary data types used in C++.

3. Answer the following :

A) What is manipulator ? Explain the use of width(), precision and fill() manipulators. 7

B) What is Arrays of objects ? Explain with example. 7

4. Answer the following :

A) Write a program in C++ to study parameterized constructor. 7

B) What is Template ? Explain class template. 7

5. Answer the following :

A) Write a C++ program to implement single inheritance in which take 'STUDENT' as base class and derive the class named 'MARKS'. (Assume your own data/variables). 7

B) What do you mean by operator overloading ? Explain with suitable example. 7

6. Answer the following :

A) What is virtual function ? Explain characteristics of virtual functions. 7

B) Explain various concepts of Object Oriented Programming. 7

7. Answer the following :

A) Define a File. Explain I/O commands in file handling. 7

B) Explain call by reference and return by reference with example. 7

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SLR-AP – 398

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (New)
Algebra – I (Paper – II)

Day and Date : Friday, 17-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct answer (**one** mark **each**) : **5**

1) If G is a group and N normal subgroup of G then which of the following is false ?

a) $gng^{-1} \in N, \forall g \in G$ and $n \in N$

b) $gN = Ng \; \forall g \in G$

c) $Ng_1g_2 = Ng_1 \, Ng_2$

d) $N = G$

2) A normal subgroup M of G is a maximal subgroup if and only if

a) G/M is a simple group

b) G/M is an abelian group

c) $M = G$

d) None of these

3) Consider the two statements

I) S_3 is solvable

II) S_3 is nilpotent

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

P.T.O.

4) Let F be a field, then F(x) is always

I) a field

II) an Euclidean domain

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

5) If G is a group such that $O(G) = p^n$, where p is a prime number and n is a positive integer then

a) $z(G) = \{e\}$

b) $z(G) \neq \{e\}$

c) $z(G) = G$

d) None of these, where z(G) is a center of G

B) State whether the following statements are **true** or **false** (**one** mark **each**) : **5**

1) If $f(x) \in R(x)$ is of degree one, then it is irreducible over R.

2) The group S_3 is not solvable.

3) Any finite group of prime order is cyclic.

4) If N and M are normal subgroups of G, then NM is also normal subgroup of G, then NM is also normal subgroup of G.

5) If G be a finite group and $|G| = 98$ then G not contain a subgroup of order 7.

C) Fill in the blanks (**one** mark **each**) : **4**

1) The ideal $N \neq R$ in a commutative ring R is a prime ideal if ________

2) An integral domain D is a PID if ________

3) In a ring z_n, the divisors of zero are precisely those elements that are not relatively ________

4) A group G is said to be solvable, if _________

2. a) State zassenhaus lemma

b) Show that the product of two nilpotent groups is a nilpotent group.

c) If $O(G) = p^2$, show that the group G is abelian.

d) Prove that every PID is a UFD. **(4+3+3+4)**

3. a) State and prove sylow second theorem.

   b) Prove that homomorphic image of a solvable group is solvable. **(7+7)**

4. a) Let G be a non-abelian group such that $|G| = p^3$, where p is any prime number show that $G' = z(G)$.

   b) Show that no group of order 30 is a simple. **(7+7)**

5. a) Define a normal series of a group G. Show that the group z has no composition series.

   b) If f(x) and g(x) $\neq$ ° are two elements of F(x), then show that there exists two polynomials t(x) and r(x) in F(x) such that f(x) = t(x) g(x). + r(x) , where r(x) = o or deg r(x) < deg g(x). **(6+8)**

6. a) Prove that if D be UFD, then D(x) is UFD.

   b) Prove that in a PID every nonzero and non unit element can be factored into finite product of irreducibles. **(7+7)**

7. a) Prove that every Euclidean domain is a PTD.

   b) Let R be a commutative ring with unity. Then prove that M is a maximal ideal of R if and only if R/M is a field. **(6+8)**

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SLR-AP – 399

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (New)
Real Analysis – I (Paper No. – III)

Day and Date : Monday, 20-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

*N.B. : 1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) $f'(c)(v)$ is a linear combination of ________ of f.

2) The Riemann Stieltjes integral reduces to Riemann integral when

3) The Riemann sum for the bounded function f on [a, b], is given by S (p, f) = ________

4) The ________ of a partition P is the length of largest subinterval.

5) A function can have a finite directional derivative $f'(c;u)$ for every u but may fail to be ________ at c.

6) The directional derivative of f at c in the direction u denoted by $f'(c,u)$ is defined by

7) Riemann integration is defined for ________ function.

B) State **true** or **false** (**one** mark **each**) :

1) Every continuous function possesses a primitive.

2) No upper sum can ever be less than any lower sum.

3) Every closed interval is connected.

4) The oscillatory sum U (p, f) – L (p, f) is always non-negative.

P.T.O.

C) Choose correct alternative (**one** mark **each**) :

1) The non-vanishing $J_f(a)$ guarantees that f is _______ on a neighbourhood of a

a) onto
b) into
c) one-one
d) none of these

2) The existence of directional derivative at point fails to imply

a) boundedness
b) differentiability
c) uniform continuity
d) continuity

3) For any two partitions P_1 and P_2

I) $L(P_2, f) \leq \cup (P_1, f)$

II) $L(P_1, f) \leq \cup (P_2, f)$

a) only I is true
b) only II is true
c) both are true
d) both are false

2. a) If function f is itegrable over [a, b] and there is a number I between L (p, f) and U (p, f) then prove that for any $\in > 0$ there exists a partition P of [a, b] such that $|U(p, f) - I| < \in$ and $|I - L(p, f)| < \in$. **4**

b) Write short note on total derivative of a function. **3**

c) Define :

i) Partition

ii) Norm of partition

iii) Refinement of partition. **3**

d) Let $f : R^2 \to R^3$ be defined by f (x, y) = (sinx cosy, sinx siny, cosx cosy). Determine Df (x, y). **4**

3. a) Let $f : S \to R^m$ be differentiable at an interior point C of S where $S \subseteq R^n$. If $v = v_1u_1 + v_2u_2 + \ldots + V_n u_n$ where $u_1, u_2, u_3 \ldots u_n$ are unit co-ordinate vectors is R^n then prove that $f'(c)(v) = \sum_{k=1}^{n} V_k \cdot D_k f(c)$. In particular if f is real valued function then prove $f'(c)(v) = \nabla f(c) \cdot V$. **6**

b) Assume that $f = (f_1, f_2 \dots f_n)$ has continuous partial derivatives Djfi on an open set S in R^n and that the Jacobian determinant $J_f(a) \neq 0$ for some point a in S. Then prove that there is an n-ball B (a) on which f is one-to-one. **8**

4. a) If f_1 and f_2 are two bounded and integrable function on [a, b] then prove that their product $f_1 f_2$ is also bounded and integrable on [a, b]. **7**

b) Evaluate $\int_1^2 (5x+3)\,dx$. **7**

5. a) If f and g are integrable on [a, b] and g keeps same sign over [a, b] then prove that there exists a number μ lying between the bounds of f such that,

$$\int_a^b fg\ dx = \mu \cdot \int_a^b g\, dx.$$ **8**

b) Find the directional derivative of $f(x, y) = x^2 y$ at the point (3, 2) in the direction of (2, 1). **6**

6. a) If f = u + iv is a complex valued function with the derivative at point Z in C then prove that $J_f(z) = |f'(z)|^2$. **7**

b) Justify the statement by an example : If $f'(c\,;u)$ exists in every direction u then in particular all the partial derivatives $D_1 f(c), D_2 f(c) \dots D_n f(c)$ exists. But converse is not true. **7**

7. a) If a function f is bounded and integrable on [a, b] the prove that the function F defined as $F(x) = \int_a^x f(t)\,dt\ ;\ a \le x \le b$ is continuous on [a, b]. Furthermore if f is continuous at a point c of [a, b] then prove that F is derivable at c and $F'(c) = f(c)$. **7**

b) State and prove Abel's lemma. **7**

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SLR-AP – 400

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (New)
Differential Equations (Paper – IV)

Day and Date : Wednesday, 22-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

N. B. : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*

2) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*

3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**One** mark **each**) : **7**

1) If Wronskian of f and g is $3e^{4x}$ and $f(x) = e^{2x}$ then the differential equation for g(x) is ______________

2) J_0 is a Bessel function of zero order of first kind then the value of $J_0(x)$ at $x = 0$ is ______________

3) The expression for Legendre's polynomial is ______________

4) The two solutions of $3y'' + 2y' = 0$ are $\phi_1(x) =$ ______________ and $\phi_2(x) =$ ______________

5) Generating function of Legendre's polynomial is ______________

6) The first approximation of $y' = x^2 + y^2$; $y(0) = 0$ is given by $\phi_0(x) =$ ______________

7) The solution of the non-homogeneous equation can be generated by using solutions of corresponding homogeneous equation, together with an integration involving the function _____________

B) Choose correct alternative (**one** mark **each**) : **5**

1) If $\phi_1, \phi_2 .... \phi_n$ are n solutions of n^{th} order linear differential equation with constant co-efficients which forms a basis for solution space the dimension of this space is _____________

a) finite b) n c) n + 1 d) n^2

2) For a linear differential equation $a_0(x)y^{(n)} + a_1(x)y^{(n-1)} + ... + a_n(x).y = 0$. A singular point is any point $x = x_0$ for which $a_0(x_0)$ = ____________

a) finite b) infinite c) zero d) none of these

3) With usual notations $\frac{d}{dx}\left[x^n J_n(x)\right]$ = ____________

a) $x^n \ J_{n+1}(x)$ b) $x^n \ J_{n-1}(x)$

c) $x^{n-1} \ J_n(x)$ d) $x^{n+1} \ J_n(x)$

4) If r_1 is a root of multiplicity m of characteristic polynomial p(r) of n^{th} order LDE with constant co-efficients then,
$p(r_1) = 0,\ p'(r_1) = 0\ ....\ p^{(m-1)}(r_1)$ = ____________

a) 0 b) n c) n! d) 1

5) The Wronskian of functions $\phi_1(x) = x^2$ and $\phi_2(x) = x^2 \log x$ is __________

a) x^2 b) x^3 c) $x^2 \log x$ d) $x^3 \log x$

C) State **True** or **False** (**one** mark **each**) : **2**

1) If $\phi_1,\ \phi_2$ are linearly independent functions on an interval I then they are linearly independent on any interval J contained inside I.

2) $||\phi(x)||$ is just a magnitude or length of vector with components $\phi(x)$ and $\phi'(x)$.

2. a) Consider the equation $L(y) = y'' + a_1 y' + a_2 y = 0$. If r_1, r_2 are distinct roots of the characteristic polynomial then prove that the functions $\phi_1(x) = e^{r_1 x}$ and $\phi_2(x) = e^{r_2 x}$ are solutions of differential equation. **3**

b) Find general solution of $y'' + 4k y' - 12k^2 y = 0$. **3**

c) For the differential equation $y'' + \alpha(x)\, y = 0$ where α is continuous functions on $-\infty < x < \infty$. Let $\phi_1,\ \phi_2$ be basis for solutions satisfying $\phi_1(0) = 1$, $\phi_2(0) = 0,\ \phi_1'(0) = 0,\ \phi_2'(0) = 1$. Show that $w(\phi_1,\ \phi_2)(\,(x) = 1$ for all x. **4**

d) Solve $y'' + \frac{1}{x}y' - \frac{1}{x^2}y = 0$ for $x > 0$. **4**

3. a) Consider the equation $y'' + a_1 y' + a_2 y = 0$. If $\alpha \pm i\beta$ are roots of characteristic polynomial where the real parts of the roots are negative then prove that every solution of differential equation tends to zero as $x \to \infty$. **7**

   b) If ϕ_1, ϕ_2 are linearly independent solutions of differential equation $y'' + a_1 y' + a_2 y = 0$ and let $W(\phi_1, \phi_2)$ be abbreviated as W then show that W is constant iff $a_1 = 0$. **7**

4. a) Let ϕ_1 be a solution of $L(y) = 0$ on an interval I and suppose $\phi_1(x) \neq 0$ on I then prove that we can reduce the order of equation $L(y) = 0$ by one. If $V_2, V_3 \ldots V_n$ are LI solutions of the reduced differential equation of order $n - 1$ and if $V_k = U_k'$, $k = 2, 3 \ldots n$ then prove that $\phi_1, u_2\phi_1 \ldots u_n\phi_1$ are linearly independent solutions of $L(y) = 0$ on I where $L(y) = y^{(n)} + a_1(x).y^{(n-1)} + \ldots + a_n(x)y^{(n)}$. **8**

   b) For the equation $L(y) = y'' + a_1(x).y' + a_2(x)y = 0$ where a_1, a_2 are continuous on some interval I. Let ϕ_1, ϕ_2 and ψ_1, ψ_2 be two bases for the solution $L(y) = 0$. Show that there is a non-zero constant k such that $W(\phi_1, \phi_2)(x) = k.$ $W(\psi_1, \psi_2)(x)$. **6**

5. a) Find the solution of $4y'' - y = e^x$. **7**

   b) Prove that $\phi_1(x) = x$ $(x > 0)$ is one solution of $y'' - 2xy' + 2y = 0$ and find the second independent solution. **7**

6. a) Compute the first four approximations of $y' = y^2$; $y(0) = 0$. **7**

   b) State and prove the uniqueness theorem for n^{th} order linear differential equation with constant co-efficient. **7**

7. a) Define Lipschitz constant and find Lipschitz constant of $f(x, y) = a(x)y + b(x)$ on $S : |x| \leq 1, |y| < \infty$. **7**

   b) Let b(x) be continuous on an interval I. P.T. Every solution ψ of $L(y) = y'' + a_1 y' + a_2 y = b(x)$ on I can be written as $\psi = \psi_p + C_1\phi_1 + C_2\phi_2$ where ψ_p is a particular solution, ϕ_1, ϕ_2 are two linearly independent solutions of $L(y) = 0$ and C_1, C_2 are constants. Prove that A particular solution ψ_p is given by, $\psi_p(x) = \int_{x_0}^{x} \frac{[\phi_1(t)\phi_2(x) - \phi_1(x)\phi_2(t)]\, b(t)}{w(\phi_1, \phi_2)(t)} dt$. **7**

SLR-AP – 401

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (New)
Classical Mechanics (Paper No. – V)

Day and Date : Friday, 24-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions *: 1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any two*** *from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) : **7**

1) In δ-variation _________ is not conserved.

2) If one of the point of rigid body is fixed ________ motion is absent.

3) The constraint involved in case of simple pendulum is given by _________ (l is length of pendulum).

4) The least possible number of linearly independent co-ordinates which are used to describe the motion of system of particles by taking into account the constrains is called _____________.

5) If the Lagrangian function L do not contain time t explicitly then the ________ of the conservative system is conserved.

6) ___________ is a curve of shortest distance between two points on given surface.

7) The determinant of orthogonal matrix is equal to ___________.

B) Choose correct alternative **(one** mark **each)** : **3**

1) The particle moving in a plane has _________ degrees of freedom.

a) 1 b) 2 c) 3 d) finite

P.T.O.

2) If N particles are moving in space with holonomic constraints expressed in k equations then independent co-ordinates are

a) N + K b) N – K c) 3N + K d) 3N – K

3) If the total force $\bar{F}$ is zero then _________ is conserved.

a) linear momentum b) angular momentum

c) generalized momentum d) total energy

C) State **true** or **false** (**one** mark **each**) : **4**

1) Frictional force is conservative.

2) Hamiltonian H always represent total energy.

3) In case of rigid body consisting of N particles free from constraints can have 3N degrees of freedom.

4) For non-cyclic co-ordinates Routhian function acts as a Lagrangian.

2. a) Write short note on virtual work. **3**

b) Prove that six generalised co-ordinates require to specify configuration of rigid body in the space. **4**

c) Define constraints and explain its four types. **4**

d) State modified Hamilton's principle. **3**

3. a) Explain physical significance of Hamiltonian H. **7**

b) Obtain Newton's equation of motion from Lagranges equation. **7**

4. a) If the transformation equation do not contain time t explicitly then show that kinetic energy is a homogeneous quadratic function of generalised velocity. **7**

b) Show that the curve is a catenary for which the area of the surface of revolution is minimum when revolved about Y-axis. **7**

5. a) Find the extremal of the functional $\int_a^b y'^2 dx$ subject to another function $\int_a^b y\,dx = c$. **7**

   b) Derive Lagranges equation of motion from Hamilton's principle. **7**

6. a) Prove with usual notations $\left(\frac{d}{dt}\right)_{fix} = \left(\frac{d}{dt}\right)_{rot} + \bar{\omega} X$. **7**

   b) State and prove principle of least action **7**

7. a) If the Lagrangian function L do not contain time t explicitely then prove that the total energy of the conservative system is conserved. **7**

   b) Obtain the angular momentum of a rigid body about a fixed point of the body when the body rotates instantaneously with angular velocity ω in terms of inertia tensor. **7**

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (Old)
Object Oriented Programming Using C++ (Paper – I)

Day and Date : Wednesday, 15-4-2015 Total Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternatives : **10**

1) An object is ____________
   A) A variable of class data type
   B) Same as a class
   C) Just like a global variable
   D) Collection of data-members and member functions

2) Wrapping up of data and functions together in a class is known as ________
   A) Overloading B) Data Abstraction
   C) Polymorphism D) Encapsulation

3) Which of the following is not a type of constructor ?
   A) Copy constructor B) Friend constructor
   C) Default constructor D) Parameterized constructor

4) The mechanism of deriving a new class from base class is known as __________
   A) Polymorphism B) Encapsulation
   C) Overloading D) Inheritance

5) Which of the following can replace a simple if-else construct ?
   A) Ternary operator B) While loop
   C) Do-while loop D) For loop

P.T.O.

6) Which of following concepts means waiting until runtime to determine which function to call ?

A) Dynamic casting B) Data hiding

C) Data binding D) Dynamic loading

7) Which of the following operator is overloaded for object cout ?

A) >> B) << C) ?: D) +

8) Which of the following cannot be used with the keyword virtual ?

A) Constructor B) Member function

C) Class D) Destructor

9) Which of the following operators cannot be overloaded ?

A) [ ] B) -> C) ?: D) *

10) The ability to take more than one form is known as

A) Polymorphism B) Encapsulation

C) Constructor D) Inheritance

B) State whether following statements are **True** or **False** : **4**

1) A static class function can be invoked by simply using the name of the function alone.

2) Members declared as private in a class are accessible to all member functions of that class.

3) Inheritance provides the idea of reusability.

4) The mechanism of deriving class from another derived class is known as multiple inheritance.

2. A) Write a short note on following : **8**

i) Flowchart

ii) Default arguments.

B) Answer the following : **6**

i) Explain the use of scope resolution operator with example.

ii) What do you mean by user defined data type ? Explain in short.

3. Answer the following :
   A) What is friend function ? Explain with example. **7**
   B) What is constructor ? Explain parameterized constructor with example. **7**

4. Answer the following :
   A) Write a program in C++ to study Inline function. **7**
   B) What is function overloading ? Explain with suitable example. **7**

5. Answer the following :
   A) Write a C++ program to implement single inheritance. **7**
   B) Explain the importance of virtual function with its characteristics. **7**

6. Answer the following :
   A) What is Template ? Explain function template. **7**
   B) What is manipulator ? Explain the use of width(), precision() and fill() manipulators. **7**

7. Answer the following :
   A) What is File ? Explain the procedure for opening the file. **7**
   B) Write a program to implement Arrays of objects. **7**

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Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS
Algebra – I (Paper – II) (Old)

Day and Date : Friday, 17-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

N.B. : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Choose correct alternatives (**one** mark **each**) : **5**

1) G is group and H is subgroup of G such that [G : H] = 2 then __________
   a) H = G
   b) $G \subseteq H$
   c) $H \trianglelefteq G$
   d) None of these

2) I) $|G| = P$ ('P' is prime) $\Rightarrow G' = \{e\}$
   II) $|G| = P^2$ ('P' is prime) $\Rightarrow G' = \{e\}$
   a) Both statements are true
   b) Both statements are false
   c) Statement I is true
   d) Statement II is true

3) Every finite group G has at least __________ composition series.
   a) one
   b) two
   c) three
   d) four

4) $f(x) = 2x^5 - 5x^4 + 5$ is __________ over Q.

a) Reducible

b) Irreducible

c) Constant

d) Zero

5) M be R-module, for any submodules N_1 and N_2 of M $N_1 + N_2$ is submodules such that ____________

a) $N_1 + N_2$ contains N_1 only

b) $N_1 + N_2$ contains N_2 only

c) $N_1 + N_2$ contains N_1 and N_2 both

d) $N_1 + N_2$ does not contains N_1 and N_2

B) Fill in the blanks : **5**

1) A group G is said to be Nilpotent if ___________

2) If H be a P-subgroup of a group if p\(G : H) Then ____________

3) Let F be field and let $f(x) \in F(x)$ be a non zero polynomial of degree n then f(x) has __________ roots.

4) Primitive polynomials $f(x) \in R[x]$ may be __________ over R.

5) Let R be ring such that $1 \in R$. An R-module M is cyclic if and only if ___________

C) Define the term : **4**

1) Solvable group

2) Nilpotent group

3) Irreducible polynomial

4) Isotropy subgroup of G.

2. a) Show that for every finite group G of order 15 is not simple. **3**

b) State first and second Sylow theorem. **4**

c) $f(x) = x^4 + x^3 + x^2 + x + 1 \in Z[x]$ then show that f(x) is irreducible over Q. **4**

d) Define the term Unique Factorization Domain (UFD). **3**

3. a) State and prove Zassenhaus lemma. **8**

b) Prove that a group G is solvable if and only if G has normal series with abelian factor. **6**

4. a) Let G be a finite group with $|G| = p.q$ where p and q are distinct prime and $p < q$ then show that
   i) G contain normal subgroup of order q
   ii) G is not simple
   iii) If $p \times (q-1)$ then G is cyclic. **7**

   b) If 'F' be a field then prove that F[x] is an Euclidian domain. **7**

5. a) Prove that two subnormal series of a group G have isomorphic refinements. **7**

   b) Let M be any R-module then prove that
   i) $0.m = 0 \ \forall \ m \in M$
   ii) $r.0 = 0 \ \forall \ r \in R$
   iii) $(-r)m = (-rm) = r(-m) \ \forall \ r \in R, m \in M$ **7**

6. a) State and prove Eisenstein's criterion for irreducibility of polynomial. **7**

   b) Show that the polynomial $x^4 + 3x^3 + 2x + 4 \in z_5[x]$ can be factored into linear factors in $z_5[x]$. **7**

7. a) If A and B be R-submodules of an R-module then prove that $\frac{A+B}{A} \cong \frac{B}{A \cap B}$. **7**

   b) If M and N be R-modules and $f : M \longrightarrow N$ be R-homomorphism then prove that f is one-one if and only if $\mathrm{Ker} f = \{0\}$. **7**

Seat No.	

M.Sc. I (Semester – I) Examination, 2015
MATHEMATICS (Paper – III) (Old)
Real Analysis – I

Day and Date : Monday, 20-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Note : *1) Q.No. **1** and Q. No. **2** are **compulsory**.*
*2) Attempt **any three** from Q. No. **3** to Q. No. **7**.*

1. A) Fill in the blanks. **Each** question carries **1** mark.

1) The ______________ of the set of lower sums is called lower integral of F over [a, b].

2) The point C is said to be ______________ point of F if $F'(C) = 0$.

3) The Cauchy Riemann equations along with ______________ of u and v imply existence of $f'(C)$.

4) If ____________ for any $y \in [\alpha, \beta]$, then ϕ is strictly monotonic in $[\alpha, \beta]$.

B) State whether the following are **true** or **false**. **Each** question carries **1** mark.

1) The Riemann stieltjes integral reduces to Riemann integral.

2) The upper and lower integrals always exist for bounded functions.

3) If P^* is a refinement of a partition P, then for a bounded function f, $L(P^*, f) \geq L(P, f)$.

4) Every continuous function is R-integrable.

C) Define the following. **2** marks **each** :

1) Upper sum.
2) Primitive of f.
3) Jacobian determinant.

P.T.O.

2. a) If f = u + iv is a complex valued function with a derivative at a point Z in $\mathbb{C}$, then show that $J_f(z) = |f'(z)|^2$.

b) For any bounded function f, show that $\underline{\int} f dx \leq \overline{\int} f dx$.

c) If f is differentiable at C, then show that f is continuous at C.

d) Find extreme values of the function $(x - 3)^2 (x + 1)$. **(4+3+4+3)**

3. a) State and prove implicit function theorem.

b) Let $f : \mathbb{R}^2 \to \mathbb{R}^3$ be defined by f (x, y) = (sinx cosy, sinx siny, cosx cosy). Determine the Jacobian matrix Df (x, y). **(10+4)**

4. a) If a bounded function f is integrable on [a, b] then prove that it is also integrable on [a, c] and [c, b] where C is a point of [a, b].

b) Let $\bar{F} : S \to \mathbb{R}^m$ be differentiable at an interior point $\bar{C}$ of S. Where $S \subseteq \mathbb{R}^n$. If $\bar{V} = v_1 \bar{u}_1 + v_2 \bar{u}_2 + \ldots\ldots + v_n \bar{u}_n$, where $\bar{u}_1, \bar{u}_2, \ldots\ldots, \bar{u}_n$ are the unit coordinators in $\mathbb{R}^n$, then prove that $\bar{f}'(\bar{C})(\bar{V}) = \sum_{k=1}^{n} v_k D_k \bar{f}(\bar{C})$. **(7+7)**

5. a) Assume that one of the partial derivatives $D_1 \bar{f}, D_2 \bar{f}, \ldots\ldots, D_n \bar{f}$ exist at $\bar{C}$ and that the remaining n – 1 partial derivatives exist in some n – ball B $(\bar{C})$ and are continuous at $\bar{C}$, then prove that $\bar{f}$ is differentiable at $\bar{C}$.

b) Let S be an open connected subset of $\mathbb{R}^n$ and let $\bar{f} : S \to \mathbb{R}^m$ be differentiable at each point of S. If $f'(c) = 0$ for each C in S, then show that f is constant on S. **(9+5)**

6. a) State and prove fundamental theorem of calculus.

b) Find the extreme values of $f(x, y) = y^2 - x^3$. **(10+4)**

7. a) State and prove necessary and sufficient condition for the integrability of a bounded function.

b) Prove that a bounded function having finite number of points of discontinuity on [a, b] is integrable on [a, b]. **(9+5)**

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SLR-AP – 405

Seat No.	

M.Sc. – I (Semester – I) Examination, 2015
MATHEMATICS (Old)
Differential Equations (Paper – IV)

Day and Date : Wednesday, 22-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

N.B. : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Choose the correct alternative (**one** mark **each**) : **5**

1) With usual notations, $\frac{d}{dx}\left[x^n J_n(x)\right] =$ ______________

a) $x^{n+1} J_n(x)$ b) $x^{n-1} \cdot J_n(x)$ c) $x^n \cdot J_{n-1}(x)$ d) $x^n J_{n+1}(x)$

2) Generating function for Legendre's polynomial is ______________

a) $\left(1+2xh+h^2\right)^{1/2}$ b) $\left(1-2xh+h^2\right)^{1/2}$

c) $\left(1+2xh+h^2\right)^{-1/2}$ d) $\left(1-2xh+h^2\right)^{-1/2}$

3) The solutions of $y'' - 2y' + 4y = 0$ are $\phi_1(x) =$ ______________,

$\phi_2(x) =$ ______________

a) e^{2x}, e^{-2x} b) e^{2x}, e^{2x} c) $e^{2x}, x\,e^{2x}$ d) $e^{2x}, \log e^{2x}$

4) The regular singular points of $x(x-1)^2(x+2)\,y'' + x^2y^1 - (x^3 + 2x - 1) = 0$ are ______________

a) 0, 1 b) 0, 2 c) 0, –2 d) 1, –2

5) The solution of the equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ have convergent power series expansion on ______________

a) $|x| < 0$ b) $|x| > 0$ c) $|x| < 1$ d) $|x| > 1$

B) Fill in the blanks (**one** mark **each**) : **5**

1) $\| \phi (x) \|$ is just the magnitude or length of the vector with components ______________ and ______________

2) If r_1 is root of multiplicity m_1 of characteristic polynomial p(r) of n^{th} order LDE with constant co-efficients then $p(r_1) = 0$, $p^1(r_1) = 0$ ______________ $p^{(m_1 - 1)}(r_1) =$ ______________

3) The second order Euler's equation is ______________

4) The expression for Legendre's polynomial is Pn(x) = ______________

5) The two solutions of $3y'' + 2y' = 0$ are, $\phi_1(x) =$ ______________, $\phi_2 (x) =$ ______________

C) State whether the following statements are **true** or **false** (**one** mark **each**) : **4**

1) The function f(x, y) = $x^2(y)$ then $\frac{\partial f}{\partial y}$ exist at (x, 0) if $x \neq 0$.

2) L is a differential operator which operates on functions which have n derivatives on I and transforms such a function ϕ into a function L (ϕ).

3) The functions cosx and sinx are linearly dependent.

4) x = 1 is regular singular point of Eulers differential equation.

2. a) Find all solutions of $y'' + 2iy' + y = 0$. **4**

b) Give the geometrical interpretation of the inequality,
$\| \phi (x_0)\| e^{-k|x-x_0|} \leq \| \phi (x)\| \leq \| \phi (x_0)\| e^{k|x-x_0|}$. **3**

c) Determine whether the functions given below defined on $-\infty < x < \infty$ are linearly independent or linearly dependent, $\phi_1(x) = \sin x$, $\phi_2(x) = e^{ix}$. **4**

d) Prove that $\phi_1(x) = x$ is a solution of differential equation,
$(1 - x^2)y'' - 2xy' + 2y = 0$. **3**

3. a) Using the fact that $p_0(x) = 1$ is a solution of, $(1 - x^2)y'' - 2xy' = 0$. Find the second independent solution. **8**

b) Prove that two solutions ϕ_1, ϕ_2 of $y'' + a_1y' + a_2y = 0$ are linearly independent on an interval I iff $w(\phi_1, \phi_2)(x) \neq 0$ for all x in I. **6**

4. a) Find all solutions of the following equation for x > 0 $x^2y'' + xy' - 4y = x$. **7**

b) If $P_n(x)$ is Legendre's function of first kind then prove that,

$$\int_{-1}^{1} P_n(x)\,P_m(x)\,dx = 0 \text{ if } m \neq n.$$ **7**

5. a) Define n^{th} Legendre polynomial and derive an expression for it. **7**

b) Show that the first solution of Bessel's equation for n = 0 is given by,

$$J_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2}\left(\frac{x}{2}\right)^{2k}.$$ **7**

6. a) State and prove uniqueness theorem for the n^{th} order linear differential equation with constant co-efficients. **7**

b) Compute first four approximations to the following problem,
$y^1 = y^2$; $y(0) = 0$. **7**

7. a) A function ϕ is a solution of initial value problem $y' = f(x, y)$; $y(x_0) = y_0$ on an interval I iff it is a solution of integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\,dt$ on I. **7**

b) Suppose ϕ_1, ϕ_2 are linearly independent solutions of equation $y'' + a_1y' + a_2y = 0$ S.T. $W(\phi_1, \phi_2)(x)$ is constant iff $a_1 = 0$. **7**

Seat No.	

M.Sc. I (Semester – I) Examination, 2015
MATHEMATICS
Classical Mechanics (Paper No. – V) (Old)

Day and Date : Friday, 24-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q.No. **1** and Q. No. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q.No. **3** to Q.No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Choose the correct alternatives (**1** mark **each**) :

1) If the particle of mass M moving along Y-axis then its Lagrangian (L) is

a) $\frac{1}{2}M\left(\dot{x}^2+\dot{y}^2\right)-mgh$ b) $\frac{1}{2}M\dot{x}^2$

c) $\frac{1}{2}M\dot{y}^2-mgh$ d) $\frac{1}{2}M\dot{y}^2$

2) If the system has 5 generalized co-ordinates with 2 constraints then number of its Hamilton's canonical equations are

a) 5 b) 6 c) 3 d) 10

3) Number of Routh's equations of motion of any dynamical system is equal to

a) Number of generalized co-ordinates

b) Number of degrees of freedom

c) Both (a) and (b)

d) Neither (a) nor (b)

4) Number of Cayley-Klein parameters are

a) 1 b) 2 c) 3 d) 4

5) If the constraint relation is expressed as an equation then they are called as

a) Scleronomic b) Rheonomic

c) Holonomic d) Non-holonomic

6) The problem in calculus of variation such that one functional is restricted to another functional called as

a) Brachistochrone problem b) Isoperimetric problem

c) Area problem d) None of the above

7) Total number of generalized co-ordinates required to describe Atwood's machine are

a) 1 b) 2 c) 3 d) 4

B) State **true** or **false** (**1** mark **each**) :

8) Hamiltonian formulation of any dynamical system is simpler than Lagrangian formulation.

9) Lagrange's equations of motion can not determined from D'Alembert's principle.

10) Hamiltonian always represents total energy.

11) Lagrange's equations of motion of Lagrangian L and (AL + B), where A, B are constants are same.

12) Shortest distance between two points on the sphere is an arc of great circle.

13) Rigid body motion with one point fixed has only rotational motion.

14) Rigid body motion is an example of non-holonomic constraints.

2. a) Find kinetic energy of a particle with mass M in polar plane. **4**

b) If virtual work done vanishes then derive D'Alembert's principle. **4**

c) Define constraints and generalized co-ordinates. **3**

d) State fundamental lemma of calculus of variations. **3**

3. a) Find equations of motion of one dimensional harmonic oscillator using Hamilton's principle. **7**

   b) Derive Hamilton's canonical equations of motion. **7**

4. a) Derive Lagrange's equations of motion from D'Alembert's principle. **8**

   b) A particle of mass m moves in one dimension such that it has the Lagrangian.

   $$L = \frac{m^2\dot{x}^4}{12} + m\dot{x}^2 V(x) - V^2(x),$$ where V is some differentiable function of x

   then find Lagrange's equations of motion. **6**

5. a) Obtain the matrix of transformation from space axes to body axes in terms of Eulerian angles. **10**

   b) Discuss generalized co-ordinates of rigid body motion. **4**

6. a) State and prove principle of least action. **8**

   b) Obtain equation of motion for simple pendulum. **6**

7. a) Derive Euler's-Lagrange's differential equation to extremize the functional

   $$\int_a^b F(x, y, y')dx.$$ **7**

   b) Find plane curve with fixed perimeter and maximum area. **7**

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Seat No.	

M.Sc. – I (Sem. – II) Examination, 2015
MATHEMATICS (New)
Algebra – II (Paper No. – VI)

Day and Date : Thursday, 16-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks (**one** mark **each**) :

1) The derivative of a non-constant polynomial can be zero if the field is a field of ____________ characteristic.

2) If $f(x) \in F[x]$ is irreducible where the field F is of characteristic zero then f(x) has ___________

3) For a prime number p, the splitting field over the field of rational numbers of the polynomial $x^p - 1$ is of degree __________

4) The characteristic of integral domain is either _________ or __________

5) A unit is an element of F[x] which has _____________

6) A field is a commutative ring in which we can divide by any _________

7) If k is an extension of a field of real numbers then every rational number is left fixed by every ______________ of k.

8) The degree of a splitting field of $x^3 - 2$ over Q is __________

B) Define **each** term (**two** marks **each**) :

1) Normal extension

2) Galois extension

3) Separate extension.

2. a) Construct a field with 9 elements. **3**

   b) Show that it is impossible by straight edge and compass alone to trisect 60°. **4**

   c) Find the basis elements of a vector space. $K = F\left(2^{1/3}, \omega\right)$ over F.

   $\left(\omega = \frac{-1+i\sqrt{3}}{2}\right)$. **3**

   d) Show that $\sqrt{\sqrt[3]{2-i}}$ is algebraic over Q. **4**

3. a) Find the splitting field k in C of the polynomial $x^4 - 4x^2 - 1 \in Q[x]$. **6**

   b) If F is a field of characteristic $P \neq 0$ then prove that the polynomial

   $x^{p^n} - x \in F(x)$ for $n \geq 1$ has distinct roots. **8**

4. a) If L is a finite extension of k and if k is finite extension of F then prove that L is finite extension of F and [L : F] = [L : K] [K : F]. **7**

   b) If ψ be an automorphism of field F onto a field F' such that $\phi(\alpha) = \alpha' \ \forall \alpha \in F$ then prove that there is an isomorphism ψ^* of F[x] onto $F'[t]$ such that,

   $\psi^*(\alpha) = \psi(\alpha) = \alpha' \ \forall \alpha \in F$. **7**

5. a) Find the Galois group of $x^2 + 1$ over the field of real numbers. **6**

   b) If k is a finite extension of a field F of characteristic O and H is a subgroup of C(k, F), k_H be fixed field of H then prove that, [k ; k_H] = O(H). **8**

6. a) If k is a field and $\sigma_1, \sigma_2 \ldots \sigma_n$ are distinct automorphisms of k then show that they are linearly independent. **7**

   b) Prove that : The polynomial f(x) $\in$ F(x) has a multiple root iff f(x) and $f'(x)$ have a non trivial common factor. **7**

7. a) Show that the polynomials $x^2 + 3$ and $x^2 - x + 1$ have the same splitting field over F, the field of rational numbers. **7**

   b) Show that a finite field of p^n elements has exactly one subfield of p^m elements for each divisor m of n. **7**

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Seat No.	

M.Sc. I (Semester – II) Examination, 2015
MATHEMATICS (New)
Real Analysis – II (Paper – VII)

Day and Date : Saturday, 18-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *i) Q. No. **1** and **2** are **compulsory**.*
*ii) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*iii) Figures to the **right** indicate **full** marks.*

1. a) Fill in the blanks : **7**

i) If $m^*(A) = 0$ then $m^*(A \cup B) =$ ____________

ii) If χ_A is characteristic function of the set A then $\chi_{\tilde{A}} =$ ____________

iii) If f and g are two measurable real-valued function defined on same domain then f g is ___________

iv) A function φ is concave means that $-\varphi$ is ___________

v) The sum of two absolutely continuous functions is _____________

vi) If the function f is of bounded variation on [a, b] then $T_a^b(f) =$ ___________

vii) If $D^+ f(x) = D_+ f(x)$ then their common value is denoted by _____________

b) State whether following is **True** or **False** : **7**

i) If $A \subseteq B$ then $m^*(A) \leq m^*(B)$.

ii) The set [0, 1] is uncountable.

iii) A non-negative measurable function f is integrable over the measurable set E if $\int_E f = \infty$.

iv) Every absolutely continuous function is the indefinite integral of its derivative.

P.T.O.

v) Fatou's Lemma remains valid if "convergence a.e." is replaced by "convergence in measure".

vi) If $f_n(x) \to f(x)$ for each $x \in E$ then the sequence $\langle f_n \rangle$ convergence pointwise to f on E.

vii) The upper right-hand derivative of the function f at x is given by

$$D^+ f(x) = \overline{\lim_{h \to 0^+}} \frac{f(x+h) - f(x)}{h}.$$

2. a) If A is countable set then prove that $m^*(A) = 0$. **3**

b) If f and g are two non-negative measurable functions. If f is integrable over E and $g(x) < f(x)$ on E then prove that g is also integrable on E and **3**

$\int_E f - g = \int_E f - \int_E g$.

c) If $g(x) = f(-x)$, then prove that $D^+g(x) = -D\_f(-x)$. **4**

d) If f is integrable over E, and c is constant then prove that cf is integrable over E and $\int_E cf = c\int_E f$ **4**

3. a) If $\{A_n\}$ is a countable collection of sets of real numbers then prove that **7**

$$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \le \sum_{n=1}^{\infty} m^*(A_n)$$

b) If f is a nonnegative function which is integrable over a set E, then prove that for given $\in > 0$ there is a $\delta > 0$ such that for every $A \subset E$ with $m(A) < \delta$ we have **7**

$\int_A f < \in$

4. a) If E is a given set, then prove the following statements are equivalent : **7**

i) E is measurable.

ii) Given $\in > 0$, there is an open set $O \supset E$ with $m^*(O \sim E) < \in$.

iii) Given $\in > 0$, there is a closed set $F \subset E$ with $m^*(E \sim F) < \in$.

b) Prove that a function F is an indefinite integral if and only if it is absolutely continuous. **7**

5. a) If φ is a continuous function on (a, b) and if $D^{+}\varphi$ is nondecreasing, then prove that φ is convex function. **7**

   b) If f is bounded and measurable function on [a, b] and **7**

   $$F(x) = \int_a^x f(t)dt + F(a),$$

   then prove that $F'(x) = f(x)$ for almost all $x \in [a, b]$.

6. a) If f is absolutely continuous function on [a, b] and if $f'(x) = 0$ a.e., then prove that f is constant. **7**

   b) If $\langle E_n \rangle$ is an infinite decreasing sequence of measurable sets and if $m(E_1)$ is finite then prove that **7**

   $$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} m(E_n).$$

7. a) Prove that a function f is of bounded variation on [a, b] if and only if the function f is the difference of two monotone real-valued functions on [a, b]. **7**

   b) If a function f is integrable on [a, b] and if **7**

   $$\int_a^x f(t)dt = 0$$

   for all $x \in [a, b]$, then prove that f(t) = 0 a.e. in [a, b].

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Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS
General Topology (Paper – VIII) (New)

Day and Date : Tuesday, 21-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) State the followings either **true** or **false** : **10**

1) Every family of subsets of a set X will form a base for some topology on X.
2) Co-countable topology on a countable set is discrete.
3) Metric space is completely regular.
4) Second countability is not a topological property.
5) The intersection of any collection of compact subsets of a Hausdorff space is compact.
6) Cofinite topological space is disconnected if X is infinite.
7) Every discrete topological space $\langle X, D \rangle$, where X contains more than one point is connected.
8) Usual topological space $\langle \mathbb{R}, U \rangle$ and indiscrete topological space $\langle \mathbb{R}, I \rangle$ are not homeomorphic.
9) A mapping $F : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is not continuous.
10) Any mapping $F : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is open.

B) Fill in the blanks : **4**

11) $\overline{A \cup B}$ = _________
12) A topological space X is said to be T_0 if _________
13) Two non-empty sets A, B of topological space $\langle X, \mathcal{J} \rangle$ are said to be $\mathcal{J}$-separated if and only if _________
14) A mapping $F : X \to Y$, where X and Y are topological spaces is said to be continuous if _________

P.T.O.

2. a) Prove that every metric space is first countable. **4**

   b) Prove that a discrete topological space X is separable if and only if X is countable. **3**

   c) Show that usual topological space is T_1. **4**

   d) If X = {a, b, c} and $J = \{\phi, \{a\}, \{b, c\}, X\}$ then show that (X, J) is normal space. **3**

3. a) Prove that two disjoint sets A and B are separated in a topological space (X, J) if and only if they are both open and closed in the subspace $A \cup B$. **7**

   b) Prove that a subset E of IR is connected if and only if it is an interval. **7**

4. a) Prove that every compact subset A of a Hausdorff space is closed. **8**

   b) Prove that closed subset of compact set is compact. **6**

5. a) Prove that every open continuous image of a second countable space is second countable. **7**

   b) Prove that every second countable space is separable. **7**

6. a) Prove that every T_2-space is a T_1-space. Is the converse true ? Explain. **7**

   b) Prove that every compact Hausdorff space is a T_3-space. **7**

7. a) Prove that every regular Lindelof space is normal. **9**

   b) Prove that every completely regular space is a regular space. **5**

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Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (New)
Complex Analysis (Paper – IX)

Day and Date : Thursday, 23-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) Solve ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct answer (**one** mark **each**). **7**

1) The cross ratio of (i – 1, ∞, 1 + i, 0) =

a) $\frac{2}{1+i}$ b) $\frac{-2}{1+i}$ c) (1 + i) d) – (1 + i)

2) If f has an isolated singularity at a and $\lim\limits_{z\to a}(z-a)\,f(z)=0$ then the point z = a is

a) Removable singularity of f
b) Pole of f
c) Essential singularity of f
d) None of these

3) The radius of convergence of the series $\sum\limits_{n=0}^{\infty} k^n z^n$, for k is an integer not equal to zero is

a) k b) $\frac{1}{|k|}$ c) |k| d) k^2

4) The value of integral $\int\limits_{|z|=\frac{1}{2}} \frac{e^2}{(z-2)(z-4)}\,dz =$

a) $2\pi i$ b) $-2\pi i$
c) 2π d) 0

P.T.O.

5) If $f(z) = \frac{e^2}{z^2+1}$ then Res (f; i) is

a) $\frac{e^{-i}}{-2i}$

b) $\frac{e^{i}}{2}$

c) $\frac{e^{i}}{2i}$

d) $\frac{e^{-i}}{2i}$

6) Let G be an open set and H(G) the space of all analytic functions in G. Consider the statements

I. H (G) is a closed subspace of $\mathbb{C}$ (G; $\mathbb{C}$)

II. H (G) is a complete metric space then

a) Both I and II are true

b) Both I and II are not true

c) Only I is not true

d) Only II is not true

7) $\int_C \frac{(2z+2i-5)}{(z-5)(z+2i)} dz =$ _________; where c is a circle with center origin and radius 3.

a) one b) zero c) $-2\pi i$ d) $2\pi i$

B) Fill in the blanks (**one** mark **each**) : **7**

1) If f is an analytic function on region G, $a \in G$, f(a) = 0 then z = a is a zero of f of multiplicity two if __________

2) A function f is called a mesomorphic function on G if f is analytic on G except for _____________ where G is a region.

3) A bilinear transformation $s(z) = \frac{az+b}{cz+d}$ with _________ is called a Mobius transformation.

4) Let $f : G \to \mathbb{C}$ is an analytic function then zero's of f, z_f =___________

5) The point z = a is called a pole of f if $\lim_{z \to a} f(z) =$ _________

6) A Mobius transformation takes circles on to ___________

7) If $r : (0, 1) \to \mathbb{C}$ is closed rectifiable curve and $a \notin \{r\}$ then $\frac{1}{2\pi i}\int_r \frac{dz}{z-a}$ is __________

2. a) Show that cross is invariant under Mobius transformation. **3**

   b) State and prove Liouville's theorem. **4**

   c) Evaluate $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z}$ ds if $|z| < 1$. **3**

   d) Prove or disprove that $z = 0$ is an essential singularity of $f(z) = z \cdot \sin \frac{1}{z}$. **4**

3. a) Suppose f is analytic in B(a; R) and let $\alpha = f(z)$. If $f(a) - \alpha$ has a zero of order m at $z = a$ then show that there is an $\in > 0$ and $\delta > 0$ such that for $|\xi - \alpha| < \delta$, the equation $f(z) = \xi$ has exactly m simple roots in $B(a; \in)$. Hence deduce the open mapping theorem. **8**

   b) State and prove Goursat's theorem. **6**

4. a) State and prove the laurent series development theorem of analytic function defined on an annulus. **9**

   b) State the Residue theorem and use it to evaluate $\int_0^{\pi} \frac{d\theta}{a + \cos\theta}$. **5**

5. a) State and prove Cauchy's integral formula of first version. **8**

   b) State and prove Cauchy's estimate. **6**

6. a) State and prove Schwarz's lemma. **7**

   b) A family f in H(G) is normal iff f is locally bounded. **7**

7. a) Let $z = a$ be an isolated singularity of f. If $\lim_{z \to a} (z - a) f(z) = 0$, then prove that $z = a$ is a removable singularity. **7**

   b) State and prove Rouche's theorem. **7**

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS
Relativistic Mechanics (New) (Paper – X)

Day and Date : Saturday, 25-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

*N.B. : 1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) Fill in the blanks (**one** mark **each**) : **10**

1) The total electric charge of an isolated system is relativistically ________
2) The length of rod remains ________ in a direction perpendicular to the direction of motion.
3) The net transfer of electric charge per unit time is called ________
4) Any index which is repeated in a given term so that the summation convention implies is called.
5) No material particle can have the speed greater than the ________
6) The comparison between the direction of propagation of wave motion as observed by the observers in S and S′ frames is called ________
7) A body is said to be relatively at rest if it does not change its position with respect its ________
8) In inelastic collision, the two bodies coalesce and move with ________
9) The relation between relativistic energy and relativistic momentum is given by ________
10) The four momentum is defined as the product of the rest mass of the particle and its ________

B) State **true** or **false** (**one** mark **each**) : **4**

1) Maxwell's field equations are invariant under Lorentz transformations.
2) The scalar product of two four vectors is invariant quantity.
3) The velocity of fluid is a covariant vector.
4) $\Box^2$ is invariant under G.T.

2. a) Write a short note on time dilation. **3**

b) Determine metric tensor in spherical co-ordinates. **4**

c) At what speed should a clock be moved so that it may appear to lose 1 minute in each out ? **4**

d) Define :

i) Simultaneous events

ii) Co-local events

iii) Co-incident events. **3**

3. a) State the principles on which special theory of relativity is based. Derive an their basis Lorentz transformations for two frames in relative uniform motion. **7**

b) Prove that three dimensional volume element dx dy dz is not invariant but four dimensional volume element dx dy dz dt is invariant under Lorentz transformation. **7**

4 a) Deduce Einsteins law of addition of velocities and show that the addition of any velocity to the velocity of light results again in velocity of light. **7**

b) Define four momentum and show that fourth component of four momentum is energy and is conserved. **7**

5. a) Derive the transformation equations for electric field. **7**

b) A particle moves with velocity represented by a vector $4' = 3i + 4j + 12k$ m/s in frame S'. Find the velocity of the particle in a frame S if S' moves with velocity 0.8c relative to S along x' – axis. **7**

6. a) Show that Lorentz transformations forms a group. **7**

b) Derive the relativistic aberration formula. **7**

7. a) P.T the charge is not altered by motion relative to an observer. **7**

b) P.T. There is no distinction between contravariant and co-variant components of Cartesian tensor. **7**

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (Old)
Algebra – II (Paper – VI)

Day and Date : Thursday, 16-4-2015 Max Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

N.B. : *1) Q.* ***1*** *and* ***2*** *are* ***compulsory.***
2) Attempt ***any three*** *from Q. No.* ***3*** *to* ***7.***
3) Figures to the ***right*** *indicates* ***full*** *marks.*

1. A) Fill in the blanks : **(One mark each)**

1) Degree of $Q\left(\sqrt{2}+\sqrt{3}\right)$ over Q is ________________

2) For every prime number P and every positive integer m there is a unique field having ________________ elements.

3) The necessary and sufficient condition for a non-empty subset K of a field F to be a subfield of F are ________________ & ________________.

4) Every finite extension of a field is ________________

5) The splitting field of $x^2 + 1 \in R[x]$ over R is ________________

6) If f(x) is irreducible over the field F and characteristics of F is O then f(x) has no ________________

7) The extension K of F is called an algebraic extension of F if ________________

B) Choose the correct alternative. **(One mark each)**

1) A polynomial of degree n over a field can have ________________ roots in any extension field.

a) At least n
b) Atmost n
c) Less than n
d) Greater than n

2) For a field of rational numbers Q, $\left[Q(\sqrt{2}) : Q\right]$ = ____________

a) 1 b) 2 c) 3 d) None of these

3) If the order of 1 is zero then the field F is of characteristic ___________

a) 0 b) 1 c) Finite d) P.

C) State whether the following are **true** or **false :** **(one mark each)**

1) Splitting field exists for every f(x) $\in$ F[x].

2) There is a field with 10 elements.

3) A regular hexagon is constructible.

4) Every algebraic extension is finite.

2. a) Show that 8 x^3 – 6x – 1 is irreducible over Q. **3**

b) Construct a field with g elements. **3**

c) If α, β are constructible real numbers then prove that $\alpha+\beta$ & $\alpha-\beta$ are also constructible. **4**

d) Prove that : Every finite extension of a field is algebraic. **4**

3. a) Prove that : A field K is a normal extension of a field F of characteristic 'O' iff K is a splitting field of some polynomial over F. **7**

b) If f(x) $\in$ F[x] be a polynomial of degree n$\geq$1 then prove that there is a finite extension E of F of degree at most n ! in which f(x) has n roots. **7**

4. a) Show that : It is impossible by straight edge and compass alone to trisect 60°. **7**

b) Prove that the polynomial f(x) $\in$ F[x] has a multiple root iff f(x) & $f'(x)$ have a non-trivial common factor. **7**

5. a) If K is a finite extension of a field F then prove that G(K, F) is a finite group and its order O(G(K, F)) satisfies the relation O(G(K,F)) $\leq$[K:F]. **7**

b) If A(K) be the collection of all automorphisms of a field K then prove that A(K) is a group w.r.t. Composite of two functions. **7**

6. a) Show that the field of complex numbers is normal extension of field of real numbers. **7**

b) Show that any two finite fields having same number of elements are isomorphic. **7**

7. a) Determine the splitting field of x^3 – 2 over Q. **7**

b) Show that $Q\left(\sqrt{2}, \sqrt{3}\right) = Q\left(\sqrt{2}+\sqrt{3}\right)$. **7**

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SLR-AP – 413

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (Old)
Real Analysis – II (Paper – VII)

Day and Date : Saturday, 18-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

***Instructions**: 1) Q. No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. No. **3** to Q. No. **7**.*
*3) Figures to the **right** indicate **full** marks.*

1. A) Fill in the blanks : **7**

1) If ϕ is empty set them $m^*\phi$ = ________________

2) If A and B are any two sets then $\chi_{A \cap B}$ = ________________

3) If $< u_n >$ is a sequence of nonnegative measurable functions and if $f = \sum_{n=1}^{\infty} U_n$ then $\int f$ = ________________

4) If E_1 and E_2 are measurable sets then $m(E_1 \cup E_2) + m(E_1 \cap E_2) =$ ______________

5) If T is a total variation of f over [a, b] and if $T < \infty$ then f is of __________ over [a, b].

6) If $D^- f(x) = D\_f(x)$ then their common value is denoted by ____________

7) Sum and difference of two absolutely continuous functions is ___________

P.T.O.

B) State whether **true** or **false :** **7**

1) If $A \subseteq B$ then $m^*B \leq m^*A$.

2) If $E \subseteq [0, 1)$ is measurable set then for each $y \in [0, 1)$ the set $E \dotplus y$ is measurable and $(E \dotplus y) = mE$.

3) If f and g are bounded measurable functions defined on a set E of finite measure. If $f \leq g$ a.e. then $\int_E g \leq \int_E f$.

4) If $< f_n >$ is a sequence of measurable functions defined on a measurable set E of finite measure and $f_n \to f$ a.e. then $< f_n >$ converges to f in measure.

5) If φ is continuous function on (a, b) and if $D^+\varphi$ is nondecreasing then φ is convex.

6) Every absolutely continuous function is the indefinite integral of its derivative.

7) If $f_n(x) \to f(x)$ for each $x \in E$ then the sequence $< f_n >$ converges absolutely to f on E.

2. a) State Egoroff's Theorem. **3**

b) If f and g are integrable over E and if $f \leq g$ a.e then prove that $\int_E f \leq \int_E g$. **4**

c) If g(x) = f(–x) then show that $D^+g(x) = -D_-f(-x)$. **4**

d) If f is absolutely continuous on [a, b] then prove that f is of bounded variation on [a, b]. **3**

3. a) Prove that the interval (a, ∞) is measurable. **7**

b) If $< f_n >$ is a sequence of measurable functions that converges in measure to f. Prove that there is a subsequence $< f_{nk} >$ that converges to f almost everywhere. **7**

4. a) Prove that a function F is an indefinite integral if and only if it is absolutely continuous. **7**

   b) If $< f_n >$ is a sequence of nonnegative measurable functions and $f_n(x) \to f(x)$ a.e. on a set E, then prove that

   $\int_E f \le \frac{\lim}{n \to \infty} \int_E f_n$. **7**

5. a) If f is a bounded and measurable function on [a, b] and

   $F(x) = \int_a^x f(t)dt + F(a)$,

   then prove that $F'(x) = f(x)$ for almost all x in [a, b]. **7**

   b) Prove that a function f is of bounded variation on [a, b] if and only if f is the difference of two monotone real-valued functions on [a, b]. **7**

6. a) If φ is a convex function on $(-\infty, \infty)$ and f is an integrable function on [0, 1], then prove that

   $\int \varphi (f(t))dt \ge \varphi \left[\int f(t)\, dt \right]$. **7**

   b) Show that if E is a measurable set, then each translate E + y of E is also measurable. **7**

7. a) If φ and ψ are simple functions which vanish outside of a set of finite measure then prove that

   $\int (a\varphi + b\psi) = a\int \varphi + b\int \psi$

   Where a and b are constants. **7**

   b) If φ is a convex function on a finite interval [a, b). Then show that φ is bounded from below. **7**

SLR-AP – 414

Seat No.	

M.Sc. – I (Semester – II) Examination, 2015
MATHEMATICS (Old)
General Topology (Paper – VIII)

Day and Date : Tuesday, 21-4-2015 Max. Marks : 70
Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q. **1** and Q. **2** are **compulsory**.*
*2) Attempt **any three** questions from Q. **3** to Q. **7**.*
*3) Figures to the **right** indicates **full** marks.*

1. A) State whether the following statements are **true** or **false**. (**1** mark for **each**) :

1) If τ_1 and τ_2 are topologies on the set X then $\tau_1 \cup \tau_2$ is also topology on X.
2) If ϕ is an empty set then B = $\{\phi\}$ is a basis.
3) $\left\{ \{x\}^C \mid x \in X \right\}$ is a subbasis for cofinite topology on X.
4) In the indiscrete space X, X is the only neighbourhood of any point.
5) Intersection of finitely many neighbourhoods of x cannot be neighbourhood of x.
6) Any function f : X $\to$ Y is continuous, if X has the discrete topology.
7) If f is one-one and open function and $g \circ f$ is continuous then g is continuous.
8) If X, Y are cofinite topological spaces and f : X $\to$ Y then f is an open function.
9) Every infinite subset of [a, b] has an accumulation point.
10) If X is an infinite set then the countable topology on X is compact.

B) Read following two statements and choose correct alternatives : **4**

11) I : In any topological space, the empty set is connected.
II : In an indiscrete space, every subset is connected.
a) Only (I) is true b) Only (II) is true
c) Both (I), (II) are true d) Both (I), (II) are false

12) (I) : In a Hausdorff topological space, then intersection of any family of compact sets is compact.
(II) : If (X, τ) is a T_0-topological space then cardinality of τ is smaller than cardinality of X.
a) Only (I) is true b) Only (II) is true
c) Both (I), (II) are true d) Both (I), (II) are false

P.T.O.

13) (I) : Every discrete topological space is separable.
(II) : Every cofinite topological space is not separable.
a) Only (I) is true
b) Only (II) is true
c) Both (I), (II) are true
d) Both (I), (II) are false

14) (I) : Every indiscrete topological space is compact.
(II) : Every discrete topological space is first countable.
a) Only (I) is true
b) Only (II) is true
c) Both (I), (II) are true
d) Both (I), (II) are false

2. a) If Y = (0, 2] is the subspace of the real line $\mathbb{R}$, $A = \left(0,\ \frac{3}{2}\right)$ is a subset of Y then find closure of A in $\mathbb{R}$ and Y. **4**

b) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by f(x) = 3x + 1 is a homomorphism. **4**

c) If A is closed in Y, Y is closed in X then show that A is closed in X. **3**

d) Prove that : $\overline{A \cap B} = \overline{A} \cap \overline{B}$. **3**

3. a) Prove that the union of a collection of connected sets that have a point in common is connected. **7**

b) Prove that every compact subset of a Hausdorff space is closed. **7**

4. a) If X is a topological space such that one-point sets in X are closed then prove that X is normal if and only if given a closed set A and open set U containing A, there is an open set V containing A such that $\overline{V} \subset U$. **7**

b) Show that a closed subspace of a normal space is normal. **7**

5. a) If X has a countable basis then prove that every open covering of X contains a countable subcollection covering X. **7**

b) Prove that every compact Hausdorff space is a normal space. **7**

6. a) In a T_1-space, prove that a point x is a limit point of a set $E \subseteq X$ if and only if every open set containing x contains an infinite number of distinct points of E. **7**

b) Prove that a continuous and onto image of Lindelof space is a Lindelof space. **7**

7. a) Prove that every T_4-space is a Tychonov space. **7**

b) Prove that every second countable space is first countable. Is the converse true ? Explain. **7**

SLR-AP – 415

Seat No.	

M.Sc. I (Semester – II) Examination, 2015
MATHEMATICS
Complex Analysis (Paper – IX) (Old)

Day and Date : Thursday, 23-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *i) Figures in **right** indicate **full** marks.*

*ii) Q.No. **1** and Q. No. **2** are **compulsory**.*

*iii) Solve **any three** questions from Q.No. **3** to Q.No. **7**.*

1. a) Fill in the blanks : **4**

1) If f is analytic in G and $a \in G$ such that $|f(a)| \geq |f(z)|$ for every $z \in G$ then f is ________.

2) A family F in H (G) is normal iff F is _________.

3) Let f be analytic in B (a ; R) and suppose $|f(z)| \leq M \ \forall \ z \in B(a; R)$. Then $|f^2(a)| \leq$ __________.

4) Suppose f has a pole of order m at z = a and put g (z) = $(z – a)^m$ f(z), then Res (f ; a) = ___________.

b) State whether **true** or **false** : **4**

1) A Mobius transformation takes circles onto circles.

2) Every entire function is constant.

3) $f(z) = \frac{1}{z}$ has no removable singularity.

4) A set $F \subset H(G)$ is compact iff it is locally bounded.

P.T.O.

c) Define the following : **6**

1) Harmonic function

2) Isolated singularity

3) Locally bounded.

2. a) If z_2, z_3, z_4 are distinct points and T is any Mobius transformation then prove that

$(z_1, z_2, z_3, z_4) = (Tz_1, Tz_2, Tz_3, Tz_4)$

For any point z_1.

b) State and prove Hurwitz's theorem.

c) Evaluate the integral $\int_\gamma \frac{e^z - e^{-z}}{z^n} dz,$ where n is a positive integer and

$\gamma(t) = re^{it},\ 0 \le t \le 2\pi \cdot$

d) If p(z) is a non constant polynomial then prove that there is a complex number a with p (a) = 0. **(4+4+3+3)**

3. a) If γ is a piecewise smooth and f : [a, b] $\to \mathbb{C}$ is continuous then prove that

$\int_a^b fd\gamma = \int_a^b f(t)\gamma'(t)dt$.

b) Let f : [a, b] $\to \mathbb{C}$ be analytic and suppose $B(a; r) \subset G\ (r > 0)$. If $\gamma(t) = a + re^{it}$, $0 \le t \le 2\pi,$ then prove that

$$f(z) = \frac{1}{2\pi i}\int_\gamma \frac{f(w)}{w - z} dw$$

for |z – a | < r. **(7+7)**

4. a) State and prove Morera's theorem.

b) Show that for a > 1,

$$\int_0^\pi \frac{d\theta}{a + \cos\theta} = \frac{\pi}{\sqrt{a^2 - 1}}.$$ **(8+6)**

5. a) State and prove Rouche's theorem.

   b) Let G be an open set and let $f : G \to \mathbb{C}$ be a differentiable function then prove that f is analytic on G. **(8+6)**

6. a) Find the fixed points of dilation, translation and the inversion in C_∞.

   b) If F has an essential singularity at z = a then prove that for every

   $\delta > 0, \left\{ f\left[\text{ann}\left(a; 0, \delta \right) \right] \right\}^{-} = \mathbb{C}.$ **(7+7)**

7. a) Show that f(z) = tan z is analytic function in $\mathbb{C}$ except for simple poles at $z = \frac{\pi}{2} + n\pi,$ for each integer n. Determine the singular part of f at each of these poles.

   b) State and prove Montel's theorem. **(6+8)**

Seat No.	

M.Sc. I (Semester – II) Examination, 2015
MATHEMATICS
Relativistic Mechanics (Old) (Paper No. – X)

Day and Date : Saturday, 25-4-2015 Max. Marks : 70

Time : 11.00 a.m. to 2.00 p.m.

Instructions : *1) Q.No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q.No.* ***3*** *to* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks. (**one** mark **each**) :

1) The Einstein's time dilation equation is given by ______________
2) The relativistic transverse Doppler effect is given by γ = ______________
3) Relativistic expression for Hamiltonian is ______________
4) The transformation equation for mass is given by m = ______________
5) For a particle, the instantaneous rest force is ______________ the corresponding force in any other frame.
6) Minkowski's space time is flat but ______________
7) The value of co-efficient of restitution 'e' depends upon the ______________
8) If the charges are at rest then ______________ density is zero.
9) The time recorded by a clock moving with a given system is called ______________ for that system.
10) No material particle can have the speed greater than ______________

B) State **true** or **false** (**one** mark **each**) :

1) Lorentz transformations forms a group.
2) In the process of contraction, the rank of tensor is reduced by 3.
3) All the laws of physics are the same for all inertial observers.
4) Magnetic field and electric field have no separate existence.

2. a) Explain Doppler effect with examples. **3**

   b) Define :

      i) Current

      ii) Electric field

      iii) Magnetic field. **3**

   c) Write the formula of relativistic expression of mass and show that if u<<<c then m = mo. **4**

   d) Prove that the fourth component of four momentum is energy. **4**

3. a) Explain geometrical interpretation of Lorentz transformations. **7**

   b) Derive the relation between the mass of a particle as measured in two inertial system of co-ordinates moving with respect to each other. **7**

4. a) Derive the law of relativistic force. **7**

   b) Prove or disprove electromagnetic wave equation is invariant under Galilean transformations. **7**

5. a) Show that $ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2$ is invariant under Lorentz transformations. **7**

   b) An electron is moving with a speed of 0.85 c in a direction opposite to that of a moving photon. Calculate the relative velocity of electron and photon.

   (Velocity of photon = c) **7**

6. a) Derive an expression for charge density and current density. **7**

   b) A co-variant tensor Ai has components (2, 1, 3) in a rectangular Cartesian co-ordinates. Find its components in spherical polar co-ordinates. **7**

7. a) Derive the expression

$$K = M_0 c^2 \left\{ \frac{1}{\sqrt{1 - u^2 / c^2}} - 1 \right\}$$

   For the relativistic kinetic energy. **7**

   b) Show that simultaneity of event is not an absolute concept in special theory of relativity. **7**

Seat No.	

M.Sc. – II (Mathematics) (Semester – III) Examination, 2015
ADVANCED DISCRETE MATHEMATICS (Paper – XII)

Day and Date : Friday, 17-4-2015 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : 1) *Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*
2) *Solve* ***any three*** *questions from Q.No.* ***3*** *to Q. No.* ***7****.*
3) *Figures to the* ***right*** *indicates* ***full*** *marks.*

1. A) Select correct alternative : **5**

i) A bijective order preserving function need not be
a) Homomorphism b) Isomorphism
c) Both a) and b) d) None of these

ii) A vertex of degree ___________ is called pedant vertex.
a) Zero b) One c) At least one d) At most one

iii) The degree of any graph is always ___________
a) Even b) Odd c) May be even d) Zero

iv) Total number of vertices in km, n graph is ___________
a) $m \cdot n$ b) $m \cdot (n - 1)$ c) $(m - 1) \cdot n$ d) $(m + n)$

v) An expression of geometric series $1/(1 + ax)^n$ is ___________

a) $\sum_{r=0}^{\infty} {}^{(n-1+r)}C_r a^r x^r$ b) $\sum_{r=0}^{\infty} {}^{(n-1+r)}C_r (-1)^r a^r x^r$

c) $\sum_{r=0}^{\infty} {}^{n+r}C_r a^r x^r$ d) $\sum_{r=0}^{\infty} {}^{n+r}C_r (-1)^r a^r x^r$

B) Fill in the blanks : **5**

i) Any totally ordered set is ____________

ii) In any graph the number of vertices of odd degree is always _________

iii) A walk is said to be open if ____________

iv) If G is connected graph then W(G) = _________

v) An expression for geometric series $\frac{1}{1 + ax}$ is _________

P.T.O.

C) State **true** or **false** : **4**

i) Every modular lattice is semi-modular lattice.

ii) A connected graph with n vertices and (n – 1) edges is tree.

iii) The number of branches is called nullity of graph.

iv) If A and B are finite sets then $|A \cup B| = |A| + |B| - |A \cap B|$.

2. a) Write short note on Matrix representation of graph. **4**

b) Write note on spanning trees. **3**

c) Prove that every totally ordered set is lattice. **4**

d) Suppose 14 students in a class appear at university exam. Prove that there exist at least two among them whose seat number differ by multiply of 13. **3**

3. a) If $(L, \wedge, \vee)$ be Triplet with non-empty set L and '$\wedge$' and '$\vee$' are binary operation on L which satisfy associative, commutative idempotent and absorption low then prove that L is lattice. **7**

b) Prove that the lattice of normal subgroup of a group is modular lattice. **7**

4. a) Let a graph G be a nonempty graph with at least two vertices then prove that G is bipartite graph iff it has no odd cycle. **7**

b) Show that a graph G is connected iff given any pair of 'U' and 'V' of vertices in G there is a path from U to V in G. **7**

5. a) State and prove Bridge Theorem. **9**

b) Let G be a connected graph then prove that G is a tree iff every edge of G is bridge. **5**

6. a) Find solution of $a_r - 7a_{r-1} + 10a_{r-2} = 0$; $a_0 = 1$, $a_1 = 5$. **7**

b) Find general solution of $a_r - 5a_{r-1} + 6a_{r-2} = 8r + 5$. **7**

7. a) Find the number of integers between 1 to 2000 both inclusive which are divisible by 10, 11, 12. **7**

b) Let L and L′ be any two lattices. Then prove that a bijective function $f : L \rightarrow L'$ is lattice isomorphism iff both f and f^{-1} preserve order. **7**

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Seat No.	

M.Sc. II (Semester – III) Examination, 2015
MATHEMATICS
Linear Algebra Paper – XIII (Elective – I)

Day and Date : Monday, 20-4-2015 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *i) Figures in **right** indicate **full** marks.*
*ii) Q. No. **1** and Q. No. **2** are **compulsory**.*
*iii) Solve **any three** questions from Q. No. **3** to Q. No. **7**.*

1. a) Fill in the blanks :
   i) A form f on V is called positive if __________
   ii) A linear operator E on a vector space V is called projection if _________
   iii) A linear operator U on V is called unitary iff _________
   iv) If minimal polynomial of T has distinct roots then T is ________ **(1+1+1+1)**

   b) State whether **true** or **false**.
   i) Every finite dimensional inner product space has an orthonormal basis.
   ii) Every set of orthogonal vectors is linearly independent.
   iii) Row rank of a matrix is same as its column rank.
   iv) Any self adjoint operator is normal. **(1+1+1+1)**

   c) Define the following :
   i) Elementary Jordan matrix
   ii) Self adjoint
   iii) Invariant subspace. **(2+2+2)**

2. a) If f and g are linear functionals on a vector space V, then prove that g is a scalar multiple of f iff the null space of g contains the null space of f.

   b) Find the minimal polynomial of $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$.

   c) Prove that similar matrices have the same characteristic polynomial.

   d) Let $\alpha = (1, 2), \beta = (-1, 1)$. If γ is a vector such that $<\alpha, \gamma> = -1$ and $<\beta, \gamma> = 3$, then find γ. **(4+4+3+3)**

P.T.O.

3. a) Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. The null space of T^t is the annihilator of the range of T. If V and W are finite dimensional, then show that
   i) $\text{rank}(T^t) = \text{rank}(T)$
   ii) then range of T^t is the annihilator of the null space of T.
   b) State and prove Cayley-Hamilton theorem. **(7+7)**

4. a) Let V be a finite dimensional vector space over the field F and let W be a subspace of V. Then show that $\dim W + \dim W^0 = \dim V$.
   b) For any linear operator T on a finite dimensional inner product space V, show that there exists a unique linear operator T^* on V such that $(T\alpha \mid \beta) = (\alpha \mid T^*\beta)$ for all α, β in V. **(7+7)**

5. a) If f is a non zero linear functional on the vector space V, then show that the null space of f is a hyperspace in V. Conversely, prove that every hyperspace in V is the null space of a (not unique) non zero linear functional on V.
   b) Let V be a finite dimensional vector space. Let $W_1, \ldots, W_k$ be subspaces of V and let $W = W_1 + \ldots + W_k$. Then show that $W_1, \ldots, W_k$ are independent iff for each j, $2 \le j \le k$. We have $W_j \cap (W_1 + \ldots + W_{j-1}) = \{0\}$. **(7+7)**

6. a) Let T be a linear operator on an n-dimensional vector space V. Show that the characteristic and minimal polynomial for T have the same roots, except for multiplicities.
   b) Let T be a l linear operator on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix $\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that T is diagonalizable by exhibiting a basis for $\mathbb{R}^3$, each vector of which is a characteristic vector of T. **(7+7)**

7. a) Apply the Gram-Schmidt process to the vectors $\beta_1 = (3, 0, 4)$, $\beta_2 = (-1, 0, 7)$, $\beta_3 = (2, 9, 11)$, to obtain an orthonormal basis for $\mathbb{R}^3$ with the standard inner product.
   b) Let V be a complex vector space and f a form on V such that $f(\alpha, \alpha)$ is real for ever α. Then show that f is Hermitian. **(7+7)**

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Seat No.	

M.Sc. (Part – II) (Semester – III) Examination, 2015
MATHEMATICS (Paper – XIV)
Modeling and Simulation (Elective – II)

Day and Date : Wednesday, 22-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

***Instructions** : 1) Question No. **1** and **2** are **compulsory**.*
*2) Attempt any **three** questions from Q. **3** to Q. **7**.*
*3) Figures to **right** indicate **full** marks.*

1. A) Select the correct alternative. **10**

i) The slack for an activity in network is equal to

a) LS-ES b) LF-LS c) EF-ES d) EF-LS

ii) If small orders are placed frequently, then total inventory cost is

a) Reduced b) Increased

c) Either reduced nor increased d) Minimized

iii) Simulation is

a) Descriptive in nature

b) Useful to analyze problem where analytical solution is difficult

c) A statistical experiments as such as its results are subject to statistical errors

d) All of the above

iv) Repetition of n independent Bernoulli trails reduced to

a) Poisson distribution b) Binomial distribution

c) Geometric distribution d) Hypergeometric distribution

v) Simulation of system in which the state changes smoothly with time are called _____________

a) Continuous system b) Discrete system

c) Deterministic system d) Probabilistic system

vi) The activity which can be delayed without affecting the execution of immediate succeeding activity is determined by

a) Total float b) Free float

c) Independent float d) None of these

vii) In M/M/1 : ∞/FCFS queue model if λ is mean customer arrival rate and μ is the mean service rate then the probability of server being busy is equal to

a) $\frac{\lambda}{\mu}$ b) $\frac{\lambda}{\mu-\lambda}$ c) $\frac{\mu}{\mu-\lambda}$ d) $\frac{\mu}{\lambda}$

viii) Markov chain said to be ergodic chain if ________ of whose states are ergodic.

a) One b) Some c) All d) None

ix) In queue model completely specified in the symbolic form (a/b/c/) : (d/e), the last symbol 'e' specifies

a) The queue discipline b) The number of servers

c) The distribution of arrival d) The distribution of departure

x) If customer, on arriving at the service system stays in the system until served, no matter how he has to wait for service is called _______ customer.

a) a regular b) an irregular c) a patient d) an impatient

B) Fill in the blanks. **4**

i) In EOQ problem, minimum total cost occurs at a point where the ordering cost and _______ cost are equal.

ii) The long form of CPM is _____________.

iii) Chapman-Kolmogorov equation is $P_{ij}(t + T) =$ ___________.

iv) In inventory model, the number of unit required per period is called _______.

2. A) i) A customer arrive in a certain store according to Poisson process with rate $\lambda = 4$ per hour, given that the store opens at 9.00 am, then what is probability that exactly one customer has arrive by 9.30 am ? **3**

ii) What do you mean by movement inventories ? **3**

B) i) State and prove the Chapman-Kolmogorov equation. **4**

ii) Write note on simulation. **4**

3. A) Differentiate between PERT and CPM. **7**

   B) Explain the generation of random sample from continuous uniform distribution. **7**

4 A) Explain the concept of inventory control. Write any four reasons for carrying inventories. **7**

   B) The demand rate for a particular item is 12000 units/ year. The ordering cost of Rs. 1,000 per order and the holding cost is Rs. 0.80 per month. If no shortage are allowed and the replacement is instantaneous the determine **7**

   i) Economic order quantity

   ii) Number of order per year.

5. A) For various activity in the particular project the expected time (in days) of completions are as follow. **7**

Activity	0 –1	1 – 3	1 – 2	2 – 3	1 – 4	3 – 4	4 – 5
Duration	3	16	6	8	10	5	3

Draw a network diagram and identify the critical path.

   B) Write steps in of Monte-Carlo simulation technique. **7**

6. A) Generate the five successive random number X_i, i = 1, 2, 3, 4, 5 by using $X_{i+1} = X_i{}^*a$ (modulo m), starting with seed $X_0 = 3$ and parameters a = 7 and m = 15 (where m means that the number $\{X_i{}^*a\}$ is divided by m repeatedly till the reminder is less than m). **7**

   B) Define project duration, earliest event time, earliest start time, latest start time, and earliest finish time in critical path computation. **7**

7. A) Define simulation. Write the advantages and limitations of simulation. **7**

   B) Explain pure birth process. **7**

Seat No.	

M.Sc. – II (Semester – III) Examination, 2015
MATHEMATICS (Paper – XV) (Elective – III)
Numerical Analysis

Day and Date : Friday, 24-4-2015 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

N.B. : *1) Question No. **1** and **2** are **compulsory**.*
*2) Attempt **any three** questions from Question No. **3** to Question No. **7**.*
*3) Figures to the **right** indicate **full** marks.*
*4) **Use** of calculator is **allowed**.*

1. A) State whether **true** or **false** (**one** mark **each**). **4**

i) In Gauss-Elimination method the coefficient matrix is reduced to an upper triangular matrix.

ii) Householders method is used to obtain eigen values of symmetric matrices.

iii) $\nabla = E - 1$.

iv) Newton Raphson method is also called method of tangents.

B) Choose the correct alternative (**2** marks **each**) : **6**

a) Which of the following is not direct method

i) Gauss-Elimination

ii) LU decomposition

iii) Gauss-Seidal

iv) Gauss-Jordan

b) The real root of the equation $xe^x - 1 = 0$ lies between

i) 0 and 1 ii) 1 and 2

iii) 2 and 3 iv) 3 and 4

c) If f(0) = 1, f(1) = 3, f(3) = 55 then the divided f[0 1 3] is

i) 2 ii) 8 iii) 26 iv) $^{54}/_{3}$

C) Fill in the blanks (**one** mark **each**) : **4**

1) The backward difference operator is ____________
2) Power method is used to find ___________
3) In Newton-Raphson method the iterative formula to find $1/N$ is given by ________
4) If A is upper triangular matrix then A^{-1} is ___________

2. a) Derive general error formula. **4**

b) Show that $E^{-1} = 1 - \nabla$. **3**

c) Define eigen values and eigen vector. **3**

d) Using method of false position find a real root of the equation $x^3 - 2x - 5 = 0$. **4**

3. a) Find the root of the equation $2x = \cos x + 3$ correct to three decimal places. **6**

b) Given $\frac{dy}{dx} - 1 = xy$ and $y(0) = 1$. Obtain the Taylor series for $y(x)$ and compute $y(0.1)$ correct to four decimal places. **8**

4. a) Prove that Newton-Raphson method converges quadratically. **6**

b) Solve the system of equations

$2x + 3y + z = 9$

$x + 2y + 3z = 6$

$3x + y + 2z = 8$

by using factorization method. **8**

5. a) Show that by using the method of separation of symbols. **7**

$$\Delta^n u_{x-n} = u_x - n\, u_{x-1} + \frac{n(n-1)}{2} u_{x-2} + .. + (-1)^n u_{x-n}.$$

b) Determine y(0.02) using Euler's modified method. Given that

$\frac{dy}{dx} = x^2 + y, y(0) = 1$. **7**

6. a) Derive Lagranges interpolation formula. **7**

b) Using Trapezoidal rule find the area bounded by the curve and the x-axis from x = 7.47 to x = 7.52. **7**

x	7.47	7.48	7.49	7.50	7.51	7.52
f(x)	1.93	1.95	1.98	2.01	2.03	2.06

7. a) Derive Newton's general interpolation formula with divided differences. **6**

b) Using Householders transformation reduce the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into tridiagonal matrix. **8**

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
MATHEMATICS (Paper – XVI)
Measure and Integration

Day and Date : Thursday, 16-4-2015 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Q. No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct answer (**one** mark **each**) : **5**

1) Let $\langle X, B, \mu \rangle$ be a measure space and if <Ei> is a measurable set for all i then

a) $\mu\left(\bigcup_{i=1}^{\infty} Ei\right) \le \sum_{i=1}^{\infty} \mu(Ei)$ b) $\mu\left(\bigcup_{i=1}^{\infty} Ei\right) \ge \sum_{i=1}^{\infty} \mu(Ei)$

c) $\mu\left(\bigcup_{i=1}^{\infty} Ei\right) = \sum_{i=1}^{\infty} \mu(Ei)$ d) none of these

2) A nonnegative function f is called integrable on E if f is measurable and

a) $\int_E f d\mu = 0$ b) $\int_E f d\mu < \infty$ c) $\int_E f d\mu \ge 0$ d) $\int_E f d\mu = \infty$

3) In Tonelli's theorem the measures μ and γ are

a) σ-finites b) semifinites c) finites d) completes

4) Consider the statements

I) Every positive set is null

II) Every null set is positive

a) Only I is true b) Only II is true

c) Both I and II are true d) Both I and II are false

5) An outer measure μ^* is said to be regular if given any subset E of X and any $\in > 0$, there is a μ^*-measurable set A with $E \subset A$ and

a) $\mu^*(E) < 0$ b) $\mu^*(E) = 0$

c) $\mu^*(A) \le \mu^*(E) + \in$ d) $\mu^*(E) \le \mu^*(A) + \in$

B) State **true** or **false** (**one** mark **each**) : **5**

1) Lebesque measure on $(-\infty, \infty)$ is a σ-finite measure.
2) Collection of locally measurable sets is not a σ-algebra.
3) The union of a countable collection of positive set is positive.
4) It γ_1 and γ_2 are absolutely continuous with respect to μ then $2\gamma_1 + 3\gamma_2$ is also absolutely continuous with respect to μ.
5) If B be a μ^* – measurable set then $\mu_*(B) = \mu^*(B)$.

C) Fill in the blanks (**one** mark **each**) : **4**

1) The measure μ is called saturated if __________
2) A decomposition of X into two disjoint sets A and B such that A is ________ for γ and B is ___________ for γ is called a Hahn decomposition for γ.
3) Each measure space can be completed by the addition of subsets of ___________
4) If $f \in L^P(\mu)$, with $1 \le p < \infty$ then $\|f\|_p =$ ___________

2. a) If $E_i \in B$, $\mu(E_1) < \infty$ and $E_i \supseteq E_i + 1$, then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu(E_n)$.

b) If $\gamma_1 << \mu$ and $\gamma_2 << \mu$ and μ is a σ-finite measure then prove that

$$\left[\frac{d(\gamma_1 + \gamma_2)}{d\mu}\right] = \left[\frac{d\gamma_1}{d\mu}\right] + \left[\frac{d\gamma_2}{d\mu}\right]$$ almost everywhere with respect to μ.

c) For any two disjoint sets E and F then prove that

$$\mu_*(E) + \mu_*(F) \le \mu_*(EUF) \le \mu_*(E) + \mu^*(F) \le \mu^*(EUF) \le \mu^*(E) + \mu^*(F).$$

d) Let E be a set for which $\mu \times \gamma(E) = 0$. Then prove that for almost all x we have $\gamma(Ex) = 0$. **(4+3+4+3)**

3. a) State and prove Fatou's Lemma.

   b) Let g be integrable over E, and suppose that $\{f_n\}$ is a sequence of measurable functions such that on E, $|f_n(x)| \leq g(x)$ and almost everywhere on E, $f_n(x) \to f(x)$. Then show that $\int_E f = \lim_{n\to\infty} \int_E f_n$. **(8+6)**

4. a) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in B$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then show that there is a unique measurable extended real valued function f on x such that $f \leq \alpha$ on B_α and $f \geq \alpha$ on $X - B_\alpha$.

   b) State and prove Radon Nikodym theorem. **(6+8)**

5. a) Let E be a measurable set such that $0 < \gamma(E) < \infty$. Then show that there is a positive set A contained in E with $\gamma(A) \geq 0$.

   b) State and prove Hahn decomposition theorem. **(6+8)**

6. a) Let {Ai} be a disjoint sequence of sets in Q. Then show that

   $$\mu_*(E \cap \bigcup_{i=1}^{\infty} Ai) = \sum_{i=1}^{\infty} \mu_*(E \cap Ai).$$

   b) Show that the class B of μ^* – measurable sets is a σ– algebra. If $\bar{\mu}$ is μ^* – restricted to B, then also show that $\bar{\mu}$ is a complete measure on B. **(6+8)**

7. a) If $A \in a$, then show that A is measurable with respect to μ^*.

   b) State and prove Fubini's theorem. **(6+8)**

Seat No.	

M.Sc. – II (Semester – IV) Examination, 2015
MATHEMATICS (Paper – XVII)
Partial Differential Equations

Day and Date : Saturday, 18-4-2015 Max. Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Question No.* ***1*** *and* ***2*** *are* ***compulsory****.*
2) Attempt ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*
3) Figures to the ***right*** *indicate* ***full*** *marks.*

1. A) Choose the correct alternative **(1 mark each)** :

1) A function f(x, y) is said to be homogenious function of x and y of degree n if it satisfies

a) $f(\lambda x, \lambda y) = \lambda^n f(x, y)$ b) xfx + yfy = nf

c) Both a) and b) d) None of a) or b)

2) Suppose that u(x, y) is harmonic in a bounded domain D and is continuous on $\overline{D} = D \cup B$ where B is boundary of D. Then u(x, y) attains its maximum.

a) Inside D as well as on B b) Outside D as well as on B

c) Inside D but not on B d) On B

3) A second order p.d.e. $(\sin^2 x)\, u_{xx} + 2(\cos x)\, u_{xy} - u_{xy} = 0$ is

a) Parabolic type b) Hyperbolic type

c) Elliptic type d) None of a), b), c)

4) The general solution of the linear p.d.e. Pp + Qq = R is

a) $\phi(u, v) = 1$ b) $\phi(u, v) = -1$

c) $\phi(u, v) = 0$ d) None of a), b, c)

5) The two solutions of Neumann problem differ by

a) function of x b) function of y

c) function of x and y d) constant

6) f(x, y, z, p, q) = 0 and g (x, y, z, p, q) = 0 are compatible on D if

a) $\frac{\partial(f, g)}{\partial(p, q)} \neq 0$ and dz = pdx + qdy is not integrable

b) $\frac{\partial(f, g)}{\partial(p, q)} \neq 0$ and dz = pdx + qdy is integrable

c) $\frac{\partial(f, g)}{\partial(p, q)} = 0$ and dz = pdx + qdy is integrable

d) $\frac{\partial(f, g)}{\partial(p, q)} = 0$ and dz = pdx + qdy is not integrable

7) The equation (2x + 3y) p + 4xq – 8pq = x + y is

a) Linear b) Non-linear c) Quasi-linear d) Semi-linear

B) Fill in the blank **(1 mark each)** :

1) The Pfaffian differential equation for continuous functions Fi's is __________

2) A two parameter family of solutions z = f (x, y, a, b) is called complete integral of the p.d.e. f(x, y, z, p, q) = 0 if the rank of the matrix __________ is two.

3) Laplance eq^n is __________

4) A p.d.e. formed after eliminating a.b from z = (x + a) (y + b) is __________

5) The complete integral of the p.d.e. z = px + qy + log (pq) is __________

6) Write true or false : The solution of Dirichlef problem if it exists is unique.

7) Write true or false : $e^x p - yxq = xz^2$ is an example of quasi-linear p.d.e.

2. a) Let u(x, y) and v(x, y) be two functions of x and y such that $\frac{\partial v}{\partial y} \neq 0$. If further $\frac{\partial(u, v)}{\partial(x, y)} = 0$ then prove that there exists a relation f(u, v) = 0 between u and v not involving x and y explicitly. **4**

b) Find the general solution of $x(y^2 - z^2)\, p - y(z^2 + x^2)\, q = (x^2 + y^2)z$. **3**

c) Let D be bounded domain in $\mathbb{R}^2$, bounded by a smooth closed curve B. Let $\{u_n\}$ be a sequence of functions each of which is continuous on $\overline{D} = D \cup B$ and harmonic in D. If $\{u_n\}$ converges uniformly on B then prove that $\{u_n\}$ converges uniformly on $\overline{D}$. **4**

d) Find the envelope of one parameter family of surfaces $x^2 + y^2 + (z - a)^2 = 1$. **3**

3. a) Prove that necessary and sufficient condition that the Pfaffian differential equation

$\overline{X}.d\bar{r} = P(x, y, z)\, dx + Q(x, y, z)\, dy + R(x, y, z)\, dz = 0$

be integrable is that $\overline{X}$.curl $\overline{X}$=0.. **7**

b) Reduce the equation $u_{xx} - 4x^2u_{yy} = \frac{1}{x}u_x$ to a canonical form. **7**

4. a) If $h_1(x, y, z, u_x, u_y, u_z, a) = 0$ and $h_2(x, y, z, u_x, u_y, u_z, b) = 0$ are compatible with $f(x, y, z, u_x, u_y, u_z) = 0$ then prove that h_1 and h_2 satisfy.

$\frac{\partial(f, h)}{\partial(x, ux)} + \frac{\partial(f, h)}{\partial(y, uy)} + \frac{\partial(f, h)}{\partial(z, uz)} = 0$ where $h = h_i$ $(i = 1, 2)$. **7**

b) Find an integral surface of $p^2x + qy - z = 0$ containing the initial line $x_0(s) = s$. $y_0(s) = 1$ $z_0(s) = -5$. **7**

5. a) Derive d'Alembert's solution which describes the vibrations of an infinite string. **8**

b) Find complete integral of the p.d.e. $x^2p^2 + y^2q^2 - 4 = 0$. **6**

6. a) Show that solution for the Dirichlet problem for a circle of radius a is given by Poisson integral formula. **8**

b) Find the integral surface passing through the initial data curve c : $x_0 = -1$, $y_0 = s$, $z_0 = 5s$ of the equation $(x + 2)\, p + 2yq = 2z$. **6**

7. a) Find the solution of problem

$u_{xx} + u_{yy} = 0, -\infty < x < \infty, y > 0.$

$u(x, 0) = f(x), -\infty < x < \infty$ so that u is bounded as $y \to \infty$, u and ux vanish as $|x| \to \infty$ **7**

b) Show that the pfaffian differential equation $(y^2 + yz)\, dx + (xz + z^2)\, dy + (y^2 - xy)\, dz = 0$ is integrable and find its integral. **7**

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SLR-AP – 424

Seat No.	

M.Sc. II (Semester – IV) Examination, 2015
MATHEMATICS (Elective – I)
Integral Equations (Paper No – XVIII)

Day and Date :Tuesday, 21-4-2015 Max. Marks : 70

Time : 3.00p.m. to 6.00 p.m.

N.B. : *1) Q. No.* ***1*** *and Q. No.* ***2*** *are* ***compulsory****.*

2) Solve ***any three*** *questions from Q. No.* ***3*** *to Q. No.* ***7****.*

3) Figure to the ***right*** *indicate* ***full*** *marks.*

1. A) Fill in the blanks : **7**

i) A homogeneous Frednolm integral equation of the second kind is given by ____________

ii) A given function is said to be square–integrable if ___________

iii) Resolvent Kernel for integral equation $y(x) = f(x) + \int_a^x k(x,t)\, y(t)\, dt$ is given by __________

iv) If Kernel is symmetrical and not identical zero then it has _________ eigenvalues.

v) G (x, t) is continuous and has continuous derivative with respect to x upto order ___________ inclusive $a \le x \le b$ ·

vi) Abel's formula is given by ________

vii) Two functions F (x) and g (x) are said to be orthogonal on [a, b] if ________

P.T.O.

B) Select correct alternative : **7**

i) The value of parameter λ for which $y(x) = \lambda \int_a^b k(x,t)\, y(t)\, dt$ has ________ solution are known as an eigen value.

a) zero b) non-zero c) unique d) none of these

ii) A finite or infinite set $\{\phi_k(x)\}$ defined on an interval $a \le x \le b$ is said to be an orthogonal set if

a) $(\phi_i, \phi_j) = 0 \quad i \ne j$

b) $\int_a^b \phi_i(x)\, \phi_j(x)\, dx = 0 \quad i \ne j$

c) both (a) and (b)

d) none of these

iii) Fredholm operator K is defined as

a) $K\phi = \int_a^b K(x,t)\, \phi(t)\, dt$

b) $K\phi = \int_a^b K(t,x)\, \phi(t)\, dt$

c) $K\phi = \int_a^b K(x,t)\, \bar{\phi}(t)\, dt$

d) $K\phi = \int_a^b \bar{K}(x,t)\, \phi(t)\, dt$

iv) Every symmetric Kernel with norm not equal to zero has

a) zero eigen value

b) at least one eigen value

c) one eigen value

d) at most one eigen value

v) Green's function exists for B.V.P. if it has __________ solution.

a) non-zero b) trivial c) finite d) infinite

vi) If $F(x) = \int_a^b K(x,t)\, h(t)\, dt$ where K (x, t) is symmetric L_2- function and h (x) is L_2 function then fourier coefficient of f (x) related to fourier coefficient of h (x) by

a) $fn = hn / \lambda n$

b) $fn = hn - \lambda n$

c) $hn = fn / \lambda n$

d) $fn = \lambda n / hn$

vii) The eigenvalues of symmetric Kernel are

a) complex b) finite c) infinite d) real

2. a) Solve the volterra integral equation $y(t) = t^2 + \int_0^t y(t)\sin(t-u)\,du.$. **4**

b) Convert $y'' + y = 0$ with $y(0) = y'(0) = 0$ into integral equation. **4**

c) Give Leibnitz's rule of differentiation under integral sign. **3**

d) Show that $y(x) = (1 + x^2)^{(-3/2)}$ is solution of $y(x) = \frac{1}{1+x^2} - \int_o^x \frac{t}{1+x^2} y(t)\,dt$. **3**

3. a) Find the eigenvalues and eigen functions of $y(x) = \lambda \int_0^1 \sin \pi x \cos \pi t y(t)\,dt$. **7**

b) Solve $y(x) = x + \lambda \int_0^1 (xt^2 + x^2 t)\, y(t)\,dt$. **7**

4. a) Explain successive approximation method. **7**

b) Solve $y(x) = f(x) + \lambda \int_0^1 (x + t)\, y(t)\,dt$. **7**

5. a) Construct Green's function for $\frac{d^4y}{dx^4} = 0;\ y(0) = y'(0) = y(1) = y'(1) = 0$. **7**

b) Prove that the eigen functions of a symmetric Kernel, corresponding to different eigenvalues are orthogonal. **7**

6. a) Find the Resolvent Kernel of K (x, t) = (2 + cosx)/(2 + cost). **7**

b) Transform $\frac{d^2y}{dx^2} + xy = 1$; y (0) = 0 ; y (1) = 1. **7**

7. a) Convert $y'' - \sin x\, y' + e^x y = x$ with y (0) = 1, y′ (0) = – 1. **7**

b) Solve the integral equation $\int_0^\infty F(x) \cos px\,dx = \begin{cases} (1-p) & 0 \le p \le 1 \\ 0 & p > 1 \end{cases}$ **7**

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
MATHEMATICS (Paper – XIX)
Operations Research (Elective – II)

Day and Date : Thursday, 23-4-2015 Total Marks : 70
Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. **1** and Q. No. **2** are **compulsory**.*
*3) Attempt **any three** from Q. No. **3** to Q. No. **7**.*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) The given payoff matrix of a game is transposed. Which of the following is not true ?
a) value of the game changes
b) saddle point of a game if exist, changes
c) player B has as many strategies as A had, and A has as many strategies as B
d) optimum strategies of both players does not change

2) Quadratic programming is concerned with the NLPP of optimizing the quadratic objective function subject to ___________
a) linear inequality constraints
b) non-linear inequality constraints
c) non-linear equality constraints
d) no constraint

3) Which of the following methods of solving a Quadratic programming problem is based on modified simplex method ?
a) Wolfe's method
b) Beale's method
c) Frank-Wolfe method
d) Fletcher's method

4) Given an LPP to Maximize $Z = -5x_2$ subject to $x_1 + x_2 \leq 1$, $0.5x_1 + 5x_2 \geq 0$ and $x_1 \geq 0$, $x_2 \geq 0$. Then we have ______________

a) no feasible solution

b) unbounded solution

c) unique optimum solution

d) multiple optimum solution

5) Consider the LPP

Minimize $Z = 3x_1 + 5x_2$

Subject to the constraints,

$x_1 + 2x_2 \leq 4$, $2x_1 + x_2 \geq 6$ and $x_1, x_2 \geq 0$

The problem represents a :

a) Zero-one IPP b) Pure IPP

c) Mixed IPP d) Non-IPP

B) Fill in the blanks : **5**

1) The basic solution to the system is called ____________ if one or more of the basic variables vanish.

2) If all variables of an IPP are either 1 or 0, then problem is called __________

3) If either the primal or the dual problem has an has unbounded objective function value then the other has _____________

4) Dual simplex method is applicable to those LPPs that start with infeasible but otherwise _________________

5) A game in said to be determinable if ____________

C) State whether the following statements are **True** or **False** : **4**

1) A slack variable cannot be present in the optimum basis of an LPP.

2) If an LPP has unbounded solution, the objective function will always be unbounded.

3) The dual LPP must always be of minimization type.

4) For a bounded primal problem, the dual would be infeasible.

2. a) Answer the following : **6**
   1) Explain the use of artificial variables in linear programming .
   2) Define :
      i) Convex polyhedron
      ii) Convex function.

   b) Write short notes on the following : **8**
      i) Two phase method of solving LPP.
      ii) Primal-dual relationship.

3. a) State and prove basic duality theorem. **8**

   b) Use dual simplex method to solve the following LPP : **6**

   Minimize $Z = 10x_1 + 6x_2 + 2x_3$

   Subject to the constraints

   $-x_1 + x_2 + x_3 \geq 1, \; 3x_1 + x_2 - x_3 \geq 2$

   and $x_1, x_2, x_3 \geq 0$.

4. a) Describe Gomory's method of solving an all integer LPP. **7**

   b) Use Branch and Bound method to solve the following IPP : **7**

   Maximize $Z = 7x_1 + 9x_2$

   subject to the constraints,

   $-x_1 + 3x_2 \leq 6, \; 7x_1 + x_2 \leq 35, \; x_2 \leq 7$

   and $x_1, x_2 \geq 0$ and are integers.

5. a) Derive the K-T conditions for an optimal solution to a QPP. **6**

   b) Solve the following LPP using Beale's method : **8**

   Maximize $Z = 2x_1 + 3x_2 - 2x_2^2$

   Subject to the constraints,

   $x_1 + 4x_2 \leq 4, \; x_1 + x_2 \leq 2$

   and $x_1, x_2 \geq 0$.

6. a) Explain Maximin and Minimax principle used in game theory. **6**

b) Solve the following game by LP technique. **8**

$$\begin{bmatrix} 9 & 1 & 4 \\ 0 & 6 & 3 \\ 5 & 2 & 8 \end{bmatrix}$$

7. a) Explain the theory of dominance in the solution of rectangular game. Illustrate with the following example. **8**

Player B

$$\text{Player A} \quad \begin{array}{c} \\ \mathbf{I} \\ \mathbf{II} \\ \mathbf{III} \\ \mathbf{IV} \end{array} \begin{array}{c} \begin{array}{cccc} \mathbf{I} & \mathbf{II} & \mathbf{III} & \mathbf{IV} \end{array} \\ \begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix} \end{array}$$

b) Define the following : **6**

i) Saddle point

ii) Basic feasible solution

iii) Two person zero sum game.

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SLR-AP – 426

Seat No.	

M.Sc. (Part – II) (Semester – IV) Examination, 2015
MATHEMATICS (Paper – XX)
Probability Theory (Elective – III)

Day and Date : Saturday, 25-4-2015 Total Marks : 70

Time : 3.00 p.m. to 6.00 p.m.

Instructions : *1) Attempt **five** questions.*
*2) Q. No. (**1**) and Q. No. (**2**) are **compulsory**.*
*3) Attempt **any three** from Q. No. (**3**) to Q. No. (**7**)*
*4) Figures to the **right** indicate **full** marks.*

1. A) Select correct alternative : **5**

1) Let $X_n \xrightarrow{P} X$ then __________

a) $X_n \xrightarrow{r} X$ b) $X_n \xrightarrow{a.s.} X$

c) $X_n \xrightarrow{L} X$ d) None of these

2) The range of an Indicator function I(A) is __________

a) {0, 1} b) $\{0, \infty\}$ c) {–1, 1} d) $\{-\infty, \infty\}$

3) If $A_n = \left(1 + \frac{1}{n}, 2 + \frac{1}{n}\right)$, $n \geq 1$ then $\lim\limits_{n \to \infty} A_n$ is equal to __________

a) (1, 2) b) [1, 2] c) [1, 2) d) (1, 2]

4) WLLN states that sample mean converges in __________ to population mean.

a) almost surely b) probability

c) r^{th} mean d) distribution

5) Every field contains

a) Ω b) ϕ

c) both (a) and (b) d) neither (a) nor (b)

B) Fill in the blanks : **5**

1) Characteristic function $\phi_x(0) =$ ______________

2) A ______________ linear combination of indicator functions is called simple function.

3) Minimal field containing A is _____________

4) Let $\{A_n\}$ be a sequence of events with $\sum_{n=1}^{\infty} P(A_n) < \infty$. Then $P(\overline{\lim} A_n) =$ _________

5) A sequence of sets $\{A_n\}$ is said to be monotonic increasing if __________ .

C) State whether the following statements are **True** or **False** : **4**

1) Any polynomial of random variables is also a random variable.

2) Any elementary function can be expressed as simple function.

3) Every σ-field is a field.

4) Characteristic function uniquely determines distribution function.

2. a) i) Define field and σ-field. Give an example of a field which is not a σ-field ? **6**

ii) Define lim inf and lim sup of a sequence of sets.

b) Write short notes on the following. **8**

i) Mixture of probability measures.

ii) Kolmogrov three series criteria for almost sure convergence.

3. a) Prove that a non-empty set which is closed under complementation and countable union is a σ-field.

b) With usual notations prove that

$\overline{\lim} (A_n \cup B_n) = \overline{\lim} A_n \cup \overline{\lim} B_n$ **(7+7)**

4. a) State and prove monotone convergence theorem.

b) Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and let C be a constant. Show that

i) $X_n + C \xrightarrow{L} X + C$

ii) $CX_n \xrightarrow{L} CX, C \neq 0$. **(6+8)**

5. a) Define expectation of simple random variable and arbitrary random variable. If $X \geq 0$ a.s. then show that $E(X) \geq 0$.

   b) For two random variables X and Y show that E (X + Y) = E (X) + E (Y). **(8+6)**

6. a) Define probability measure and conditional probability measure. Show that conditional probability measure satisfies properties of a probability measure.

   b) Define measurable function. Show that an indicator function defined on (Ω, F) is F-measurable. **(8+6)**

7. a) Define characteristic function of a random variable x. Show that characteristic function is real if X is symmetric about origin.

   b) Obtain characteristic function when the distribution random variable of X is :

      i) Poisson with parameter λ

      ii) Binomial with parameters n and p. **(6+8)**

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