Seat No.	

Set P

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: October/November - 2025
Group and Ring Theory (2317101)

Day & Date: Wednesday, 29-10-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) S_n is not solvable for n ______.
   a) $n = 3$ b) $n \geq 3$
   c) $n \leq 5$ d) $n \geq 5$

2) Consider the following two statements:
   P: Subgroup of a solvable group is solvable
   Q: Subgroup of a nilpotent group is nilpotent.
   Then,
   a) P is true and Q is false
   b) P is false and Q is true
   c) both P and Q are true
   d) both P and Q are false

3) Every p-group is _______.
   a) nilpotent
   b) solvable
   c) both nilpotent and solvable
   d) neither nilpotent nor solvable

4) If G is a group and X is a G-set, then ______.
   a) G_x is a subgroup of G b) X_g is a subgroup of G
   c) G_x is a subgroup of X d) X_g is a subgroup of X

5) Every group of order $p^r, r > 1$ is ______.
   a) simple b) abelian
   c) cyclic d) not simple

6) Which of the following is correct?
   a) $2\mathbb{Z}$ is prime ideal of $\mathbb{Z}$ b) $2\mathbb{Z}$ is maximal ideal of $\mathbb{Z}$
   c) both (a) and (b) d) neither (a) nor (b)

7) Which of the following is not a principal ideal of $\mathbb{Z}[x]$?
   a) $< 2 >$ b) $< 3 >$
   c) $< x >$ d) $< 4, x^2 >$

8) If *P* is a prime ideal of a ring *R*, then $ab \in P$ implies.

a) $a \in P \text{ and } b \in P$ b) $a \in P \text{ or } b \in P$

c) $a \notin P \text{ and } b \notin P$ d) $a \notin P \text{ or } b \notin P$

B) State whether true or false. **04**

a) $\mathbb{Z}_6$ is an integral domain.

b) D_4 is an abelian group.

c) Every nilpotent group is solvable.

d) $f(x) = x^2 + 1$ is reducible over $\mathbb{C}$.

Q.2 Answer the following. (Any Six) **12**

a) Define: Normal series

b) Define: Subnormal series

c) Define: Sylow-p-subgroup

d) Define: Conjugate of an element.

e) Define: Module

f) Define: Associate elements and unit in a ring

g) Using Eisenstein's criteria, show that $f(x) = x^2 - 2 \in Z[x]$ is irreducible over $\mathbb{Q}$.

h) State the division algorithm theorem for division of polynomial in a polynomial ring.

Q.3 Answer the following. (Any Three) **12**

a) If *F* is a field and $f(x) \in F[x]$. If deg $f(x) = 2 \text{ or } 3$ then prove that $f(x)$is reducible over *F* if and only if $f(x)$ has a zero in *F.*

b) Prove that homomorphic image of a solvable group is solvable.

c) If *X* is a non-empty set and G is group such that X is a G −set. Define a relation $'\sim'$ on X by $x \sim y \Leftrightarrow x = yg$ for some $g \in G, \forall x, y \in X$.

d) If *p* is an irreducible in a PID and p divides the product $a_1 a_2 \dots a_n$, for $a_i \in D$, then prove that $p | a_i$, for at least one i.

Q.4 Answer the following. (Any Two) **12**

a) State and prove Sylow's second theorem.

b) If G is finite group and X is a finite G-set, then prove that $|xG| = [G : G_x], \forall x \in X$.

c) Prove that every Euclidean domain is a principal ideal domain.

Q.5 Answer the following. (Any Two) **12**

a) Let *D* be a PID. Then prove that every element that is neither 0 nor unit is a product of irreducible.

b) If $\mathbb{F}$ is a field, then prove that $\mathbb{F}[x]$ is a UFD.

c) Prove that no group of order 96 is simple.

Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: October/November - 2025
Real Analysis (2317102)

Day & Date: Thursday, 31-10-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) Consider the following statements:
   I) Function having only one point discontinuity is integrable.
   II) Function having finite no. of points of discontinuity is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

2) For any partition P, the norm of partition is defined as $\mu(P) =$ _____.
   a) $\max P$ $\min P$
   c) $\min \Delta x_i$ d) $\max \Delta x_i$

3) By first mean value theorem, if a function f is continuous on $[a, b]$ then there exist a number ξ in $[a, b]$ such that $\int_a^b f(x)dx =$ _____.
   a) $f(\xi)(a-b)$ b) $f(\xi)(b-a)$
   c) $f(\xi)(a+b)$ d) $f'(\xi)(a-b)$

4) The directional derivative of $f(x, y) = xy$ at point (1,1) in the direction (1,0) is _____.
   a) 1 b) (1,1)
   c) y d) x

5) If S is convex set then _____ for all $x, y \in S$
   a) $L(x, y) \subseteq S$ b) $L(x, y) \supseteq S$
   c) $L(x, y) = S$ d) None of these

6) Riemann sum for a function f on $[a, b]$ is defined as $S(P, f) =$ ____.
   a) $\sum_{i=1}^{n} M_i \triangle x_i$ b) $\sum_{i=1}^{n} m_i \triangle x_i$
   c) $\sum_{i=1}^{n} f(t_i) \triangle x_i$ d) $\sum_{i=1}^{n} (M_i - m_i) \triangle x_i$

7) A necessary and sufficient condition for integrability of a bounded function is _____.
   a) $lim_{\mu(P)\to\infty}\big(U(P,f) - L(P,f)\big) = 0$
   b) $lim_{\mu(P)\to\infty}\big(U(P,f) + L(P,f)\big) = 0$
   c) $lim_{\mu(P)\to 0}\big(U(P,f) + L(P,f)\big) = 0$
   d) $lim_{\mu(P)\to 0}\big(U(P,f) - L(P,f)\big) = 0$

8) If P_1 and P_2 are two partitions of $[a, b]$ then their common refinement is given by $P^* =$ _____.
   a) $P_1 \cap P_2$ b) $P_1 + P_2$
   c) $P_1 - P_2$ d) $P_1 \cup P_2$

B) Fill in the blanks. **04**

1) For $\int_1^2 f(x)dx$, the value of Δx_i (length of n equal sub intervals) is _____.
2) The condition of _____ is necessary for a function to assume its mean value ξ in given interval by first mean value theorem.
3) The upper integral of a function f on $[a, b]$ is defined as _____.
4) The partial derivatives of a function describe the rate of change of a function in the direction of _____.

Q.2 Answer the following. (Any Six) **12**

a) Define
   i) Upper Sum
   ii) Lower Sum
b) Check whether the function $f(x) = x^2 + 4x + 3$ have local extrema or not.
c) Write short note on Primitive of function.
d) State Second Fundamental theorem of Integral Calculus.
e) Find the directional derivative of $f(x, y) = x^3 + xy$ at point (1, 3) in the direction (1, -1).
f) Write Mean Value theorem for the functions f from $R^n \to R$.
g) If $\int_{-1}^2 x^2 dx = 3$ then find its mean value.
h) Define: Directional derivative.

Q.3 Answer the following. (Any Three) **12**

a) If $f(x, y) = (xy, x^2 + y, x + y^2)$ then find $Df(x, y)$
b) If P_1, P_2 are any two partitions then with usual notations prove that $L(P, f, \alpha) \leq U(P, f, \alpha)$
c) If f is bounded and integrable on $[a, b]$ and $K > 0$ is a number such that $|f(x)| \leq K$ for all $x \in [a, b]$ then prove that
$|\int_a^b f(x)dx| \leq K|b - a|$
d) Solve $\int_0^3 (2x + 5)dx$

Q.4 Answer the following. (Any Two) **12**

a) Prove that: A necessary and sufficient condition for the integrability of a bounded function f is that for every $\in > 0$ there corresponds $\delta > 0$ such that for every partition P of $[a,b]$ with norm $\mu(P) < \delta, U(P,f) - L(P,f) < \in$

b) If P^* is a refinement of a partition P then for a bounded function f prove that

i) $L(P^*,f,) \geq L(P,f,)$

ii) $U(P^*,f,) \leq U(P,f,)$

c) Prove that: The oscillation of a bounded function f on an interval $[a,b]$ is the supremum of the set $\{|f(x_1) - f(x_2)|/x_1, x_2 \in [a,b]\}$ of numbers.

Q.5 Answer the following. (Any Two) **12**

a) If f and all its partial derivatives of order less than m are differentiable at each point of an open set S in R^n and a, b are two points of S such that $L(a,b) \subseteq S$ then prove that there is a point z on the line segment $L(a,b)$ such that

$$f(b) - f(a) = \sum_{k=1}^{m-1} \frac{1}{k!} f^{(k)}(a; b-a) + \frac{1}{m!} f^{m}(z; b-a)$$

b) If f have a continuous n^{th} (for some integer $n \geq 1$) derivative in the open interval (a,b) and for some interior point c in (a,b) we have, $f'(c) = f''(c) = \cdots = f^{n-1}(c) = 0$ but $f^n(c) \neq 0$ then prove that for n even, f has local minimum at c if $f^n(c) > 0$ and f has local maximum at c if $f^n(c) < 0$. Also prove that if n is add, there is neither a local maximum nor a local minimum at c.

c) If a function f is bounded and integrable on $[a,b]$ then prove that the function F defined as, $F(x) = \int_a^x f(t)dt; a \leq x \leq b$ is continuous on $[a,b]$. Furthermore if f is continuous at a point c of $[a,b]$ then prove that F is derivable at c and $F'(c) = f(c)$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: October/November - 2025
Number Theory (2317107)

Day & Date: Monday, 03-11-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) The largest integer value of $[\pi]$ is ______.
a) 1 b) 3
c) 2 d) 4

2) Which of the following is true?
a) $\varphi(n)$ is always an even number
b) $\varphi(n)$ is always an odd number
c) $\varphi(n)$ is even for infinitely many values of n
d) $\varphi(n)$ is even for only finitely many values on n

3) If 'a' has order $k\ (mod\ n)$ then a^h has order $k\ (mod\ n)$ iff ______.
a) $\gcd(k,h) = 2$ b) $\gcd(a,h) = 1$
c) $\gcd(a,k) = 2$ d) $\gcd(k,h) = 1$

4) The system of linear congruences $ax + by \equiv r(mod\ n)$ and $cx + dy \equiv s(mod\ n)$ has a unique solution $(mod\ n)$, whenever _______.
a) $\gcd(a,b) = 1$ b) $\gcd(c,d) = 1$
c) $\gcd(ad - bc, n) = 1$ d) $\gcd(ad + bc, n) = 1$

5) Using Euclidean algorithm find the integers x and y such that $\gcd(1769, 2378) = 1769x + 2378y$.
a) $x = 39, y = -29$ b) $x = 29, y = 39$
c) $x = -39, y = 29$ d) $x = -29, y = 39$

6) The primitive roots of 82 are ________.
a) 75 b) 15
c) 69 d) all of these

7) If p is a prime and $d|p-1$ then the congruence $x^d - 1 \equiv 0(mod\ p)$ has _______ solutions.
a) exactly p b) exactly d
c) more than d d) pd

8) The value of $\sum_{n=1}^{2} \mu(n!) =$______.

a) 0 b) 1
c) -1 d) 2

B) Fill in the blanks. **04**

1) If $a > 1$ and m, n are positive integers then $\gcd(a^m - 1, a^n - 1) =$ ______.
2) If the orders of a_1 and a_2 modulo n be k_1 and k_2 respectively and $\gcd(k_1, k_2) = 1$. Then the order of $a_1 a_2 (mod\ n)$ is ______.
3) The remainder when the sum $S = 1! + 2! + 3! + \cdots + 999! + 1000!$ is divisible by 8.
4) $\gcd(ka, kb) = k.\gcd(a, b)$ if _______.

Q.2 Answer the following. (Any Six) **12**

a) Define the following terms:
i) Square free integers
ii) Linear Congruence

b) If $a = bq + r$ then show that $\gcd(a, b) = \gcd(b, r)$.

c) Find $\tau(n)$ and $\sigma(n)$ for $n = 756$.

d) If $ac \equiv bc\ (mod\ n)$ then show that $a \equiv b \left(mod \frac{n}{d}\right)$, where $d = \gcd(c, n)$.

e) Find the last two digits of the number 9^{9^9}.

f) Find the highest power of 13 contained in $4000!$.

g) Find the primitive roots of 10.

h) Find $\gcd$ of 12 and 20 and express the $\gcd$ as linear combination of 12 and 20.

Q.3 Answer the following. (Any Three) **12**

a) Prove that τ and σ are the multiplicative functions.

b) If a is an odd integer, then show that $\frac{a^4+4a^2+11}{16}$ is an integer.

c) Show that the sum of positive integers less than n and relatively prime to n is equal to $\frac{1}{2} n\varphi(n)$.

d) Construct the index table for 17 with primitive root 5.

Q.4 Answer the following. (Any Two) **12**

a) Show that if one of the two integers $2a + 3b$ or $9a + 5b$ is divisible by 17 then so can the other.

b) State and prove Fermat's theorem.

c) Solve the congruence $x^3 \equiv 5(mod\ 13)$.

Q.5 Answer the following. (Any Two) **12**

a) Write a note on Fermat factorization method and factorize 340663.

b) If p is a prime and $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0, a_n \not\equiv 0(mod\ p)$ is a polynomial of degree $n \geq 1$ with integral coefficients then show that $f(x) \equiv 0(mod\ p)$ has at least n incongruent solutions $mod\ p$.

c) Solve the system of linear congruence's;
$x \equiv 3(mod\ 5), x \equiv 5(mod\ 7), x \equiv 10(mod\ 11)$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: October/November – 2025
Research Methodology in Mathematics (2317103)

Day & Date: Thursday, 06-11-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. (MCQ) **08**

1) Data can be collected by _____ method.
a) observation b) mailing of questionnaire
c) personal interview d) all of the above

2) The longform of SCI is _____.
a) Science citation index b) Scopus citation index
c) Science citation India d) Scopus citation India

3) One of the objective of research is to test the hypothesis of a casual relationship between variables, such studies are known as _____ research studies.
a) formulative b) descriptive
c) hypothesis testing d) diagnostic

4) The research concerned with subjective assessment of attitudes, opinions, and behavior is called _____.
a) Qualitative research
b) Quantitative research
c) Experimental research
d) Inferential research

5) In Research, a complete enumeration of all items in the "population" is known as a _____.
a) Census Universe b) Census Population
c) census inquiry d) none of these

6) If the population from which sample is to be drawn does not constitute a homogeneous group then _____ sampling techniques is applied so as to obtain a representative sample.
a) Cluster b) stratified
c) Quota d) Area

7) The maximum value of h such that the given author/journal has publisher at least h papers that have each been cited at least h times is known as _____.

a) i-10 index b) citation
c) h-index d) impact factor

8) The UGC CARE Group I includes journals found qualified through _____.

a) scopus b) web of science
c) UGC CARE protocols d) SCI

B) State true or false. **04**

1) Research methods constitute a part of the research methodology.
2) A citation is a reference to a source.
3) UGC CARE list is divided into six groups.
4) The purpose of abstract is to summarize the contents of research article.

Q.2 Answer the following. (Any Six) **12**

a) Define: h-index, i10 index.
b) Define Research: Give two definitions.
c) Write note on Impact factor of research journal.
d) Write note on Abstract of research paper.
e) Explain the need of UGC CARE list.
f) What is Deliberate sampling?
g) Write note on Key words.
h) Write long form of AMS and UGC CARE.

Q.3 Answer the following. (Any Three) **12**

a) Write note on “Preparation of the report or the thesis.”
b) Write note on the Motivation in research.
c) Write different types of methods of collection of data.
d) Explain the terms: Lemma, theorem, corollary and preposition.

Q.4 Answer the following. (Any Two) **12**

a) Give details about “Words versus symbols”.
b) Write short note on collecting the data.
c) Explain Applied Vs. Fundamental research.

Q.5 Answer the following. (Any Two) **12**

a) Write an expository note on Keywords and Subject classification.
b) Write the problems encountered by researchers in India.
c) Write detail information about different types of sampling.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: October/November - 2025
Field Extension Theory (2317201)

Day & Date: Tuesday, 28-10-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) The splitting field of $x^2 - 1$ over Q is ______.
a) $Q(i)$ b) R
c) Q d) C

2) The number of automorphisms on a field of real numbers is / are ______.
a) 1 b) 0
c) 2 d) finite

3) The number π is algebraic over ______.
a) R b) Q
c) $Q(i)$ d) $Q(\sqrt{2})$

4) $O(G(C, R)) =$ ______.
a) 1 b) 0
c) 3 d) 2

5) For every prime p and every positive integer m there exist a finite field with ______ elements.
a) m^p b) p^m
c) $m.p$ d) None of these

6) For a field of characteristic zero ____.
I. Every finite extension is simple extension.
II. Every finite extension is separable extension.
a) only I is true b) only II is true
c) Both are true d) Both are false

7) An ideal $N = < p(x) >$ of $F[x]$ is a maximal ideal if $p(x)$ is ______ polynomial.
a) minimal b) monic
c) reducible d) irreducible

8) The subfield of K generated by $F(F \subseteq K), a\ b\ \in K$ is given by _____.

a) $F(a, b)$ b) $F(b, a)$
c) $F(b)(a)$ d) All of these

B) State whether the following statements are True or False. **04**

1) Every rational number is left fixed by any automorphism on any extension field K.
2) Set of all constructible number of R may or may not form subfield.
3) Every finite extension is normal extension.
4) $\sqrt{2}$ is algebraic of degree 1 over R.

Q.2 Answer the following. (Any Six) **12**

a) Define.
  i) Fixed field
  ii) Galois group
b) Find splitting field of $x^2 - 2$ over Q.
c) Construct a field with 4 elements.
d) Prove or disprove: Doubling the cube is impossible.
e) Find the fixed field of $G(Q(i), Q)$.
f) If a and b are constructible numbers then prove that $a + b$ and $a - b$ are also constructible.
g) Check whether $3 + \sqrt{2}$ is algebraic over Q or not.
h) Define.
  i) Simple extension
  ii) Finite extension

Q.3 Answer the following. (Any Three) **12**

a) Prove that: The Galois group of a polynomial over a field F of characteristic zero is isomorphic to a group of permutation of its roots.
b) Find all possible automorphisms on a field of rational numbers.
c) Find the fixed field of
  i) $G(Q(2^{1/3}),Q)$
  ii) $G(C, Q)$
d) Prove that: Every finite extension is an algebraic extension.

Q.4 Answer the following. (Any Two) **12**

a) Prove that: A field of characteristic zero is perfect.
b) If K is a normal extension of a field of characteristic 0 and T be a subfield of K containing F then prove that T is a normal extension of F iff $\sigma(T) \subseteq T$ for all $\sigma \in G(K, F)$.
c) Prove that: The polynomial $f(x) \in F[x]$ has a multiple root iff $f(x)$ and $f'(x)$ have nontrivial common factor.

Q.5 Answer the following. (Any Two) **12**

a) If L is a finite extension of K and if K is finite extension of F then prove that L is finite extension of F and $[L:F] = [L:K][K:F]$

b) If $a \in K$ be algebraic over F then prove that any two minimal monic polynomial for a over F are equal.

c) Find Galois group of $x^3 - 1$.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: October/November - 2025
General Topology (2317202)

Day & Date: Thursday, 30-10-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) The number of open sets in an in-discrete T-space is ______.
   a) 0 b) 1
   c) 2 d) 3

2) If $< X, \mathfrak{J} >, < X^*, \mathfrak{J}^* >$ are two T-spaces and if $f: X \to X^*$ is a function, then f is continuous at $x \in X$ if ______
   a) $< X^*, \mathfrak{J}^* >$ is an indiscrete T-space
   b) $< X^*, \mathfrak{J}^* >$ is co-finite T-space
   c) $< X, \mathfrak{J} >$ is co-countable T-space
   d) All of the above

3) In any discrete T-space $< X, \mathfrak{J} >$ with $A \subseteq X, \bar{A} =$ ______.
   a) $\emptyset$ b) A
   c) X d) $i(A)$

4) In any discrete T-space $< X, \mathfrak{J} >, b(A) =$ ______.
   a) $i(E) \cap i(X - E)$ b) $i(E) \cap e(X - E)$
   c) $X - [i(E) \cap i(X - E)]$ d) $X - [i(E) \cap b(E)]$

5) Every T_2 space is _______.
   a) T_1 b) T_0
   c) both T_1 and T_0 d) neither T_1 and T_0

6) $< \mathbb{R}, \mathfrak{J}_u >$ is ______.
   a) compact but not countably compact
   b) countably compact but not compact
   c) both compact and countably compact
   d) neither compact nor countably compact

7) Being a Lindelof space is not _______.
   a) a topological property
   b) hereditary property
   c) closed hereditary property
   d) None of these

8) A regular space is normal if it is _______.
   a) connected b) T_1
   c) T_2 d) compact

B) State whether True or False. **04**

1) Every T_3 space is T_1.
2) Every singleton set in T_1 is open.
3) Closure of a set *A* is the largest closed set containing *A.*
4) Every normal space is T_1.

Q.2 Answer the following. (Any Six) **12**

a) Define: Normal space
b) Define: Regular space
c) Define: Continuity of a function at a point
d) Define: Limit point of a set
e) If $X = \{a, b, c, d, e\}, \mathfrak{I} = \{\emptyset, \{a\}, \{a, b\}, \{b\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}, X\}$, then find $i(\{a, b, c\})$.
f) Define: Dense in itself set
g) Define: Homeomorphism
h) If $< X, \mathfrak{I} >, < X, \mathfrak{I}^* >$ are any two T-spaces and $i: X \to X$ is an identity map, then prove that i is continuous on X iff $\mathfrak{I}^* \leq \mathfrak{I}$.

Q.3 Answer the following. (Any Three) **12**

a) If $< X, \mathfrak{I} >, < X^*, \mathfrak{I}^* >$ are any two T-spaces then prove that a function $f: X \to X^*$ is continuous iff $f[c(E)] \supseteq c^*[f(E)], E \subseteq X$.
b) Prove that being a T_1 space is a topological property.
c) If C is a connected subset of a T-space $< X, \mathfrak{I} >$ and if $X = A|B$, then prove that either $C \subseteq A$ or $C \subseteq B$.
d) If X is any non-empty set and $p \in X$ is a fixed element. Define $\mathfrak{I} = \{X\} \cup \{A \subset X | p \notin A\}$. Then prove that $\mathfrak{I}$ is a topology on X.

Q.4 Answer the following. (Any Two) **12**

a) Prove that a topological space $< X, \mathfrak{I} >$ is compact iff every family of closed sets having the finite intersection property has a non-empty intersection.
b) If any T-space $< X, \mathfrak{I} >$, prove that $\bar{A} = A \cup d(A)$.
c) Prove that a topological space $< X, \mathfrak{I} >$ is normal iff for any closed set F and an open set G containing F, there exists an open set H such that $F \subseteq H \subseteq \overline{H} \subseteq G$.

Q.5 Answer the following. (Any Two) **12**

a) If $X = \{a, b, c, d\}$ and $\mathfrak{I} = \{\emptyset, \{a, c\}, \{a, d\}, \{a, c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X$. Then find the derived set of $A = \{b, c, d\}$.
b) If any T-space $< X, \mathfrak{I} >$, prove that $i(E) = E'^{-'}$.
c) A T-space $< X, \mathfrak{I} >$ is a T_1 space iff $\{x\}$ is a closed set in X for each $x \in X$.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: October/November – 2025
Complex Analysis (2317207)

Day & Date: Saturday, 01-11-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) If $\gamma: [0, 1] \to \mathbb{C}$ is a closed rectifiable curveand $a \notin \{\gamma\}$, then $\frac{1}{2\pi i}\int_{\gamma} \frac{1}{z-a} dz$ is ______.
   a) Purely Imaginary b) Purely Real
   c) Whole number d) Integer

2) Residue of the function $f(z) = e^{\frac{1}{z}}$ at $z = 0$ is ______.
   a) $\frac{1}{2!}$ b) $\frac{-1}{2!}$
   c) 1 d) -1

3) Every non-constant polynomial is an ______.
   a) Analytic function b) Entire function
   c) Non-bounded function d) All of these

4) The function $f(z) = z^m$ at $z = \infty$ has _____.
   a) non-isolated essential singularity
   b) pole of order m
   c) pole of order $m + 1$
   d) removable singularity

5) Laurent series expansion of the function $\frac{1}{z^3-3z+2}$ for $|z| > 2$ is _____.
   a) $\sum_{n=0}^{\infty} \frac{2^n - 1}{z^{n+1}}$ b) $\sum_{n=0}^{\infty} \frac{2^n}{z^{n+1}}$
   c) $\sum_{n=0}^{\infty} \frac{2^n + 1}{z^{n+1}}$ d) $\sum_{n=0}^{\infty} \frac{2^n}{z^n}$

6) Which of the following mapping does not change the shape of the figure but it changes size of the figure?
   a) Rotation b) Translation
   c) Magnification d) Bilinear Transformation

7) If z is any complex number then $|z+5|^2+|z-5|^2=75$ represents ______.
   a) a circle b) an ellipse
   c) a triangle d) straight line

8) The radius of convergence of the power series $\sum_{n=0}^{\infty}(n+2i)^n z^n$ is ______.
   a) 0 b) 1
   c) ∞ d) n^2+4

B) Fill in the blanks. **04**

1) The nature of the singularity of function $\frac{1}{\cos z-\sin z}$ at $z=\frac{\pi}{4}$ is _____.
2) The value of $\int \frac{1}{z^2}\,dz,$ where the contour is the unit circle traversed clockwise is ______.
3) If z_1, z_2, z_3, z_4 be the four distinct points in C_∞ then the cross ratio (z_1, z_2, z_3, z_4) is real iff ______.
4) If image of an open set is not open under an analytic function, then the function is ______.

Q.2 Answer the following. (Any Six) **12**

a) State Taylors Theorem.
b) Show that the order of zero of the Polynomial equals the order of its first non-vanishing derivative.
c) Define the following terms:
   i) Singular point of an analytic function
   ii) Zero's of an analytic function
d) Illustrate the construction of cross ratio.
e) Distinguish between pole and essential singularity.
f) Find Res $(f;2)$ for $f(z)=\frac{z^2}{(z-1)^2(z-2)^2}$
g) What is Cauchy estimate theorem?
h) Define critical point with examples.

Q.3 Answer the following. (Any Three) **12**

a) Show that $\int_0^{\pi}\frac{1}{a+\cos\theta}d\theta=\frac{\pi}{\sqrt{a^{2-1}}}(a>1)$
b) Define the Mobius transformation. Also show that the Mobius transformation is the composition of translation, dilation and inversion.
c) If f is analytic in the disk $B(a,R)$ and suppose that γ is a closed rectifiable curve in $B(a,R)$ then prove that $\int_\gamma f=0$.
d) Show that the rational function has no singularities other than poles.

Q.4 Answer the following. (Any Two) **12**

a) If G is a region and $f: G \rightarrow C$ be an analytic function such that there is a point 'a' in G with $|f(z)| \leq |f(a)| \forall z \in G$ then prove that f is a constant.

b) State and prove Cauchy residue theorem.

c) Prove that all the roots of equation $z^7 + 10z^3 + 14 = 0$ lie within annulus $1 < |z| < 2$.

Q.5 Answer the following. (Any Two) **12**

a) If G is an open subset of the complex plane C and $f: G \rightarrow C$ be an analytic function. If γ is a closed rectifiable curve in G such that, $\eta(\gamma; w) = 0$; $\forall\ w \in C - G$. Then for $a \in G - \{\gamma\}$ prove that,

$$f(a).\eta(\gamma; a) = \frac{1}{2\pi i}\int_{\gamma} \frac{f(w)}{w - a} dw$$

b) Evaluate $\int_0^\infty \frac{1}{1+x^2} dx$

c) Show that the set of all bilinear transformation forms a non-abelian group under composition.

Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination: October/November - 2025 Functional Analysis (2317301)

Day & Date: Wednesday, 29-10-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) In a Banach space, every closed subspace is ______.
   a) open b) complete
   c) compact d) bounded

2) By closed graph theorem, if B and B' are Banach spaces and T is a linear transformation of B into B' then T is continuous mapping iff ______.
   a) its graph is closed set b) its graph is open set
   c) its graph is finite set d) its graph is countable set

3) In a Hilbert space, for any $x, y \in H$ the vectors x, y are said to be orthogonal if ______.
   a) $\langle x, y\rangle \neq 0$ b) $\langle x, y\rangle = 0$
   c) $\langle x, y\rangle \leq 0$ d) $\langle x, y\rangle \geq 0$

4) A continuous linear transformation $T: N \to N'$ is said to be open mapping if for every open set G in N, T(G) is ______ in N'.
   a) closed b) bounded
   c) open d) finite

5) By Schwartz' inequality, If x and y are two vectors in an inner product space then
   $|\langle x, y\rangle| \leq$_____.
   a) $\| x \| + \| y \|$ b) $\| x \| . \| y \|$
   c) $\| x \| - \| y \|$ d) $|x| . |y|$

6) If M is closed linear subspace of a Hilbert space, then M is closed iff ______.
   a) $M = M^{\perp}$ b) $M \subseteq M^{\perp\perp}$
   c) $M = M^{\perp\perp}$ d) $M^{\perp\perp} \subseteq M$

7) If X and Y are normed linear spaces, $T: X \rightarrow Y$ is a linear transformation such that T^{-1}exist then T^{-1} is said to be bounded if ______ for a positive constant m.
   a) $m \parallel x \parallel \leq \parallel T(x) \parallel$ b) $m \parallel x \parallel \geq \parallel T(x) \parallel$
   c) $m \parallel x \parallel < \parallel T(x) \parallel$ d) $m \parallel x \parallel > \parallel T(x) \parallel$

8) In a normed linear space $B(N, N'), (T_1 + T_2)(x) =$ ______.
   a) $T_1(x) + T_2(x)$ b) $T_1(x) - T_2(x)$
   c) $T_1(x).T_2(x)$ d) any of these

B) Fill in the blanks. **04**

1) In Hilbert space X, with usual notations, $\langle x, y + z\rangle =$______.
2) In a normed linear space, the triangular inequality property is given as, ______.
3) Every Cauchy sequence in complete normed linear space is ______.
4) If H is a Hilbert space, $x \in H \text{ and } x \perp x$ then x must be _____.

Q.2 Answer the following. (Any Six) **12**

a) Define norm and Banach space.
b) State Riesz theorem.
c) With usual notation prove that: $S(0; r) = r.S(0; 1)$
d) Define Graph of T.
e) Prove that: Every closed subspace of Banach space is complete.
f) If X is a complex IPS then Prove that
$< x, ay + bz >= \bar{a} < x, y > +\bar{b} < x, z >$
g) Prove that $\parallel x + y \parallel^2 + \parallel x - y \parallel^2 = 2. \parallel x \parallel^2 + 2. \parallel y \parallel^2$
h) With usual notation prove that: $H^{\perp} = \{0\}$

Q.3 Answer the following. (Any Three) **12**

a) If X is a normed linear space over the field F and M is closed subspace of X, define $\parallel . \parallel_1 : \frac{X}{M} \rightarrow R$ by $\parallel . \parallel_1 = inf\ \{\parallel x + m \parallel / m \in M\}$ then prove that $\parallel . \parallel_1$ is a norm on $\frac{X}{M}$
b) If $T: X \rightarrow Y$ be any linear transformation then prove that T is bounded if and only if T maps bounded sets in X into bounded sets in Y.
c) Prove that $B(X, Y)$ is subspace of $L(X, Y)$.
d) If S is a non-empty subset of a Hilbert space H, then show that $S^{\perp}$ is a closed linear subspace of H and hence Hilbert space.

Q.4 Answer the following. (Any Two) **12**

a) If M be a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$.
b) If X is a normed linear space and $S = \{x \in X: \parallel X \parallel \leq 1\}$ be subspace of X such that X be Banach space if and only if S is complete.
c) If N normed linear space then prove that each vector x in N induces functional F_x on N^*defined by, $F_x(f) = f(x)$ for every $f \in N^*$ such that $\parallel F_x \parallel = \parallel x \parallel$ and also prove that the mapping $J : N \rightarrow N^{**}$ defined by $J(x) = F_x$ for every $x \in N$ given an isometric isomorphism of N into N^{**}.

Q.5 Answer the following. (Any Two) **12**

a) If M is a closed linear subspace of a Hilbert space $H \; and \; x$ be a vector not in $M \; and \; d = d(x, M)$ then prove that there exists a unique vector $y_0 \; in \; M$ such that $\| \; x - y_0 \; \| = d$.

b) If T be an operator on a Hilbert space H then prove that there exists a unique operator $T^* \text{ on } H$ such that for all $x, y \; \in H, < Tx, y > = < x, T^* y >$

c) State and prove Open Mapping Theorem.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination: October/November – 2025
Linear Algebra (2317302)

Day & Date: Friday, 31-10-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) What is the relationship between a singular matrix and its eigenvalues?
a) All eigenvalues are non-zero
b) At least one eigenvalue is zero
c) All eigenvalues are positive
d) All eigenvalues are negative

2) What is the norm of the vector $u = (4, -3, -2, 1)$ with respect to the Euclidean inner product in R^4?
a) $\sqrt{30}$ b) 30
c) 0 d) $\sqrt{26}$

3) A linear operator T is self-adjoint if T = ________.
a) T^* b) $T^{**} = T^*T$
c) T^{**} d) $TT^{**} = T^{**}T$

4) Which of the following statements is true about the roots of the minimal and characteristic polynomials of a matrix?
a) The roots of the characteristic polynomial are the same as the roots of the minimal polynomial.
b) The minimal polynomial has roots that are not eigenvalues of the matrix.
c) The characteristic polynomial has roots that are not eigenvalues of the matrix.
d) The roots of the minimal polynomial are a proper subset of the eigenvalues.

5) Which of the following functions $T : R^2 \to R^2$ is a linear transformation?
a) $T(x, y) = (x + 1, y)$ b) $T(x, y) = (x + y, 0)$
c) $T(x, y) = (x + 1, y - 1)$ d) $T(x, y) = (x - 1, y)$

6) If dim $V(F) = n$ then $V^*(F) =$ ______ where V^* is dual of V.

a) n b) $2n$

c) $n+1$ d) 0

.

7) If T be an operator on a complex inner product space V. If T is self-adjoint, which of the following must be true?

a) T is unitary b) The eigenvalues of T are real

c) T is a normal operator d) Both (b) and (c)

8) If λ is characteristic value of a linear operator T then the ______ multiplicity of λ is defined to be the multiplicity of x as a root of the characteristic polynomial of T.

a) Minimal b) Geometric

c) Algebraic d) Unique

B) State True or False: **04**

1) If V be the vector space over the field F and subspace $S = \{0\}$ of V then $S^{\perp} = 0$.
2) Every square matrix satisfies its characteristic equation.
3) If V be inner product space, $x \in V$ then the norm of vector x defined as $|| \mathcal{X} || = |x|$.
4) If a matrix has n distinct eigenvalues, then it is diagonalizable.

Q.2 Answer the following. (Any Six) **12**

a) If S_1 & S_2 are subsets of a vector space V such that $S_1 \subseteq S_2$ then prove that $S_2^0 \subseteq S_1^0$.

b) If V be an inner product space over the field F then prove that.

1) $< a\alpha - b\beta, \gamma > = a < \alpha, \gamma > - b < \beta, \gamma >$
2) $< \alpha, a\beta + b\gamma > = \bar{a} < \alpha, \beta > + \bar{b} < \alpha, \gamma >$

c) Define:

1) Linear Operator
2) Linear Functional

d) What is Jordan canonical form?

e) Show that the mapping $T: R^2 \to R^2$ defined by $T(x, y) = (y + 2z, 2x + y)$ is a Linear Transformation.

f) If $B = \{(1,0), (0,1)\}$ is a basis of $R^2(R)$ then find the dual basis of B.

g) If $A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$ then prove that $A^2 - 5A + 4I = 0$.

h) Define the following terms:

1) Annihilating polynomial.
2) Minimal polynomial.

Q.3 Answer the following. (Any Three) **12**

a) Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 3 & 3 & 2 \\ -3 & -1 & 0 \end{bmatrix}$ over the field of complex numbers.

b) Prove that any orthogonal set of non-zero vectors in an inner product space V is linearly independent.

c) If V be an n dimensional vector space over the field F and W be a subspace of V then prove that $W^{00} = W$.

d) Prove that the matrix $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$ is diagonalizable.

Q.4 Answer the following. (Any Two) **12**

a) If V be an inner product space and T be a self-adjoint operator on V then show that each characteristic value is real and characteristic vector associated with distinct characteristic values are orthogonal.

b) If V be a finite dimensional vector space over the field F and $B = \{\alpha_1, \alpha_2, ---, \alpha_n\}$ be a basis for V then prove that there is uniquely determined basis $B^* = \{f_1, f_2, ---, f_n\}$ such that $f_i(\alpha_j) = \delta_{ij}$.

c) Find the Jordan canonical form of the matrix $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$.

Q.5 Answer the following. (Any Two) **12**

a) If T is a linear operator and V be a finite dimensional inner product space then prove that there exists a unique linear operator T* on V such that

$$< T(\alpha), \beta >=< \alpha, T^*(\beta) > \forall \alpha, \beta \in V.$$

b) State and prove Cayley Hamilton Theorem.

c) If $\beta_1 = (3,0,4), \beta_2 = (-1,0,7) \text{and } \beta_3 = (2,9,11)$ then find the orthogonal and orthonormal basis for R^3 with the standard inner product by using Gram Schmidt orthogonalization process.

Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination: October/November - 2025
Advanced Discrete Mathematics (2317306)

Day & Date: Monday, 03-11-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) Every chain is ______.
a) Modular b) Not Modular
c) May be modular d) None of these

2) The graph $K_{m.n}$ has ______ vertices.
a) $m+n$ b) $m-n$
c) $m \times n$ d) $2(m+n)$

3) 12_{C_r} is greatest when r is equal to _______.
a) 7 b) 6
c) 12 d) 0

4) The number of simple graphs on four vertices, three of which have degree 3 and the remaining vertex has degree one is ______.
a) 4 b) 5
c) 2 d) such graph not exist

5) Let G be a connected graph with vertex set V. For each $v \in V$, the eccentricity of v i.e. $e(v)$ is given by ______.
a) $\max \{d(u,v)/u \in V\}$
b) $\min \{d(u,v)/u \in V\}$
c) $\max \{d(u,v)/u \in V, u \neq v\}$
d) $\min \{d(u,v)/u \in V, u \neq v\}$

6) Let $S = \{ a, b, c \}$ then the least and greatest element in the POSET $(P(S), \subseteq)$ are ______.
a) φ and S b) φ and $P(S)$
c) φ and $\{a, b\}$ d) S and $P(s)$

7) The generating function of the sequence $0, 1, 2\ 3, \ldots$ is ______.
a) $\sum_{r=0}^{\infty} x^r$ b) $\sum_{r=0}^{\infty} (r+1).x^r$
c) $\sum_{r=0}^{\infty} r.x^{-r}$ d) $\sum_{r=0}^{\infty} r.x^r$

8) A lattice $(L,\ \vee,\ \wedge)$ is distributive lattice if and only _____.
a) if it does not contain the five-element diamond
b) if it contains the five element diamond
c) if it contains the five element pentagonal
d) both b and c

B) Fill in the blanks. **04**
1) The number of distinct simple graphs with up to three vertices are ______.
2) If no two distinct elements of a POSET are comparable then it is called ______.
3) If any five integers from 1 to 8 are chosen, then at least two of them will have a sum ______.
4) If the task A can be performed in exactly 15 ways and a task B can be performed in exactly10 ways, then the number of ways of performing task A or task B is ______.

Q.2 Answer the following. (Any Six) **12**
a) Prove that in a distributive lattice, if an element has a complement, then it is unique.
b) State and prove Handshaking lemma.
c) Show that D_{21} is a finite Boolean algebra under partial order of Divisibility.
d) Define tree and spanning tree.
e) Show that the number of combinations of n different things taken any number at a time $2^n - 1$.
f) Prove that any tree with at least two vertices is a bipartite graph.
g) Define the Complete and Bipartite graph.
h) Show that in a group of 13 children, there must be at least two who were born in same month.

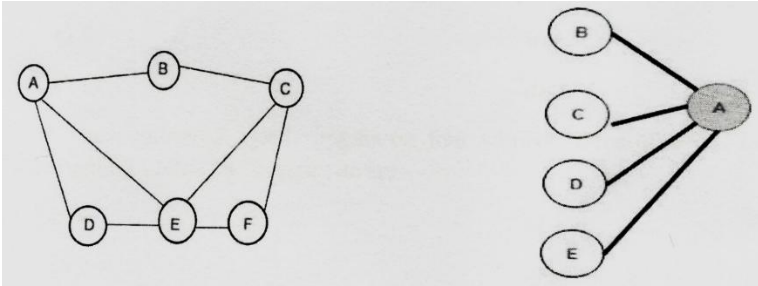
Q.3 Answer the following. (Any Three) **12**
a) In how many ways can 7 boys and 5 girls be seated in a row so that no two girls may seat together.
b) Define isomorphism of graph with two examples.
c) Show that every chain is a distributive lattice.
d) If G be a connected graph with n vertices and $n-1$ edges then prove that G is a Tree.

Q.4 Answer the following. (Any Two) **12**
a) If $(A, \lesssim_1)$ and $(B, \lesssim_2)$ are Posets then show that $(A \times B, \lesssim)$ is a Poset with partial order defined by,
$(a, b) \lesssim (a', b')$ if $a \lesssim_1 a'$ in $A, b \ \lesssim_2 b'$ in B.
b) Solve the recurrence relations.
i) $y_{n+2} - y_{n+1} - 2y_n = n^2$
ii) $y_n - 4y_{n-1} + y_{n-2} = n + 4^n$
c) If G be a graph with n vertices and q edges, $w(G)$ denotes the number of connected components in G then prove that G has atleast $n - w(G)$ edges.

Q.5 Answer the following. (Any Two) **12**

a) Write a short note on Hasse diagram of the Poset. Draw the Hasse diagram of the Poset $(P(S), \subseteq)$ where $P(S)$ is the power set on $S = \{a, b, c, d\}$.

b) Find the distance and diameter of the following graphs.



c) Among the integers 1 to 1000. Find how many of them are not divisible by 3, nor by 5, nor by 7.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination: October/November – 2025
Partial Differential Equations (2317401)

Day & Date: Tuesday, 28-10-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) Every integral generated by one parameter family of characteristics is an ______.
- a) envelope
- b) circle
- c) cone
- d) integral surface

2) The condition that the surfaces $f(x, y, z) = c$ forms a family of equipotential surfaces is that _______.
- a) $\frac{\nabla^2 f}{|\nabla f|^2} = 0$
- b) $\frac{\nabla f}{|\nabla^2 f|^2} = 0$
- c) $\frac{\nabla^2 f}{|\nabla f|^2}$ is function of f only
- d) $\frac{\nabla^2 f}{|\nabla f|^2}$ is not function of f

3) The equations $f(x, y, p, q) = 0$ and $g(x, y, p, q) = 0$ are compatible if ______.
- a) $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0$
- b) $\frac{\partial(f,g)}{\partial(x,p)} - \frac{\partial(f,g)}{\partial(y,q)} = 0$
- c) $\frac{\partial(f,g)}{\partial(y,p)} + \frac{\partial(f,g)}{\partial(x,q)} = 0$
- d) $\frac{\partial(f,g)}{\partial(y,p)} - \frac{\partial(f,g)}{\partial(x,q)} = 0$

4) The integral surface passing through the curve $x_0 = 0, y_0 = s^2$, $z_0 = -s$ of the partial differential equation $(x^2 + y^2)p + 2xyq = (x + y)z$ is _____.
- a) $z^2 = y(x + y)^2$
- b) $z^2(y - x) = y(x + y)^2$
- c) $z(y^2 - x^2) = y(x + y)$
- d) $z^2(y^2 - x^2) = y(x + y)^2$

5) The complete integral of $z^3 = pqxy$ is _______.

a) $x^a y^b = exp\left(2\sqrt{\frac{ab}{z}}\right)$ b) $xy = exp\left(\sqrt{\frac{ab}{z}}\right)$

c) $x^a y^b = exp\left(\sqrt{\frac{ab}{z}}\right)$ d) $2x^a y^b = exp\left(\sqrt{\frac{ab}{2z}}\right)$

6) In the parametric equation of curve $x = f_1(t), y = f_2(t), z = f_3(t)$ the condition for the parameter t to be an arc length of curve is ______.

a) ${f_1'}^2 + {f_2'}^2 + {f_3'}^2 = 1$ b) ${f_1'}^2 + {f_2'}^2 + {f_3'}^2 = 0$

c) ${f_1'}^2 + {f_2'}^2 = 0$ d) $f_1 + f_2 + f_3 = 1$

7) The general integral of the partial differential equation $x(x+y)p = y(x+y)q - (x-y)(2x+2y+z)$ is ______.

a) $F\big(xy, (x+y)(x+y-z)\big) = 0$

b) $F\big(xy, (x+y)(x+y+z)\big) = 0$

c) $F\big(xy, (x-y)(x+y-z)\big) = 0$

d) $F\big(xy, (x+y-z)\big) = 0$

8) The Pfaffian differential equation in more than two variables ______.

a) is integrable b) is not integrable

c) has integrating factor d) may not be integrable

B) Fill in the blanks. **04**

1) The condition $X^{-}.curlX^{-} = 0$ is equivalent to $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) - Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0.$

2) The characteristic curves of $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$ are $y - x = c, 4y - x = d$.

3) The Lagrange's auxiliary equation for the partial differential $Pp + Qq = R$ is $\frac{dx}{p} = \frac{dy}{Q} = \frac{dz}{R}$

4) A function $f(x, y)$ is said to be a homogeneous function of x and y of degree n if $f(\lambda x, \lambda y) = \lambda^n f(x, y)$

Q.2 Answer the following. (Any Six) **12**

a) Discuss the method of finding the integral surface of non-linear partial differential equations.

b) Define semi linear and quasi linear partial differential equations.

c) Find the partial differential equation which represents the set of all right circular cones with z-axis as the axis of symmetry.

d) Show that the partial differential equation $u_{xx} + x^2 u_{yy} = 0$ has elliptic canonical form.

e) Write a note on Dirichlet problem.

f) Show that there always exists an integrating factor for a Pfaffian differential equation in two variables.

g) Define second order partial differential equation.
h) What is family of equipotential surfaces.

Q.3 Answer the following. (Any Three) **12**

a) Find a partial differential equation by eliminating arbitrary constant from $z = x + ax^2y^2 + b$

b) Prove that a necessary and sufficient condition that there exist a relation between two functions $u = u(x, y)$ and $v = v(x, y)$ a relation $F(u, v) = 0$ or $u = H(v)$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.

c) Show that the surfaces $x^2 + y^2 + z^2 = r^2, r > 0$ forms a family of equipotential surfaces and find the general form of corresponding potential function.

d) Prove that the solution of Dirichlet problem if it exists is unique.

Q.4 Answer the following. (Any Two) **12**

a) Find the complete integral of $(p^2 + q^2)y - qz = 0$ by Charpit's method.

b) Obtain D-Alembert's solution of the one-dimensional wave equation which describes the vibration of infinite length string.

c) Solve $z + 2u_z - \left(u_x + u_y\right)^2 = 0$ by Jacobi's method.

Q.5 Answer the following. (Any Two) **12**

a) Prove that the singular integral is also the solution of the first order partial differential equation.

b) Prove that a necessary and sufficient condition that the Pfaffian differential equation $\bar{X}\,\overline{dr} = 0$ be integrable is that $\bar{X}.curl\,\bar{X} = 0$.

c) Find the integral surface of $x^2p + y^2q + z^2 = 0$ which passes through the curve $xy = x + y, z = 1$.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination: October/November - 2025 Integral Equations (2317402)

Day & Date: Thursday, 30-10-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) An integral equation $g(x)u(x) = f(x) + \int_a^b K(x,t)u(t)dt$ is said to be of the first kind if ______.
   a) $g(x) = 0$ b) $g(x) = 1$
   c) $f(x) = 0$ d) $f(x) = 1$

2) Which of the following is not a degenerate kernel?
   a) $K(x,t) = x + t$ b) $K(x,t) = x - t$
   c) $K(x,t) = \cos(x + t)$ d) $K(x + t) = e^{x/t}$

3) Which of the following is an example of symmetric kernel?
   a) $K(x,t) = e^{ixt}$ b) $K(x,t) = e^{\frac{x}{t}}$
   c) $K(x,t) = (x - t)$ d) $K(x,t) = i(x - t)$

4) $\int_a^x y(t)dt^3 =$ _______.
   a) $(x - t)^2 y(t)$ b) $\int_a^x \frac{(x-t)^2}{2} y(t)dt$
   c) $\int_a^x \frac{(x-t)^3}{3!} y(t)dt$ d) $\int_a^x \frac{(x-t)^3}{3} y(t)dt$

5) An initial value problem gets converted into ______.
   a) Volterra integral equation
   b) Fredholm integral equation
   c) Singular integral equation
   d) None of the above

6) Which of the following is always a solution for a homogeneous Fredholm integral equation?
   a) $y(x) = 0$ b) $y(x) = 1$
   c) $y(x) = x$ d) $y(x) = -1$

7) Fredholm integral equation cannot be solved by _______.
   a) Separable kernel method
   b) Successive approximation method
   c) Laplace transform
   d) All of the above

8) $G(x,t)$ satisfies the differential equation of the given BVP as a function of a _______.
   a) x b) t
   c) both x and t d) neither x nor t

B) State whether True or False: **04**

1) Every homogeneous Fredholm integral equation has eigenvalues and eigen functions.
2) $y(x) = 1$ is a solution of $y(x) = 2 - \int_0^1 y(t)dt$.
3) Every boundary value problem possesses Green's function.
4) If the kernel $K(x,t)$ is symmetric then its iterated kernels are symmetric.

Q.2 Answer the following. (Any Six) **12**

a) Define symmetric kernel.
b) Define: nth Iterated kernel
c) Define: Separable kernel
d) Give Leibnitz formula for differentiation under integral sign.
e) Show that $y(x) = x$ is a solution of $y(x) = x - \int_{-1}^{1} y(t)dt$.
f) Convert the following into an integral equation:
$y'' + y = x, y(0) = 0, y'(0) = 0$.
g) Define eigenvalue and eigen function for the Fredholm integral equation.
h) Define: Resolvent kernel.

Q.3 Answer the following. (Any Three) **12**

a) Write a note on first, second, third kind and homogeneous Volterra integral equation.
b) Define Green's function.
c) Convert the following into an integral equation without substitution method:
$$y'' + y = 0, y(0) = y'(0) = 0.$$
d) Solve: $\int_0^\infty F(x) cospx\, dx = \begin{cases} 1-p, & 0 \le p \le 1 \\ 0, & p > 1 \end{cases}$

Q.4 Answer the following. (Any Two) **12**

a) Solve by using resolvent kernel:
$$y(x) = 1 + x^2 + \int_0^x \frac{(1+x^2)}{(1+t^2)} y(t)dt$$
b) Find the eigenvalues and eigen functions: $y(x) = \lambda \int_0^1 (2xt - 4x^2) y(t)\, dt$.
c) Prove that the eigen functions of a symmetric kernel, corresponding to different eigenvalues are orthogonal.

Q.5 Answer the following. (Any Two) **12**

a) Solve: $Y'(t) = t + \int_0^t Y(t-x)\cos x\,dx, Y(0) = 4.$

b) Solve: $y(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)y(t)dt$

c) Using the method of successive approximations, solve:

$$y(x) = 1 + \int_0^x (x-t)y(t)dt, y_0(x) = 1$$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination: October/November – 2025
Measure and Integration (2317405)

Day & Date: Saturday, 01-11-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) A subset E of X is said to be μ^* measurable, if for any set A _______.
   a) $\mu^*(A) \le \mu^*(A \cup E) + \mu^*(A \cup E^c)$
   b) $\mu^*(A) \ge \mu^*(A \cup E) + \mu^*(A \cup E^c)$
   c) $\mu^*(A) = \mu^*(A \cup E) + \mu^*(A \cup E^c)$
   d) both b and c

2) A real valued function ∅ defined on X is called as simple function if ∅ assumes ______ of values.
   a) infinite number b) finite number
   c) unique d) any number

3) With usual notations which of the following relation is true?
   a) $\mu_*(E) \le \mu^*(E), \forall E \in \mathrm{A}$
   b) $\mu_*(E) \ge \mu^*(E), \forall E \in \mathrm{A}$
   c) $\mu_*(E) = \mu^*(E), \forall E \in \mathrm{A}$
   d) $\mu_*(E) < \mu^*(E), \forall E \in \mathrm{A}$

4) Consider the following statements:
   I) Every algebra is semi-algebra.
   II) Every algebra is sigma-algebra
   a) only I is true b) only II is true
   c) Both are true d) Both are false

5) If $\mathcal{A}$ is an algebra then the collection of sets that are countable union of sets in $\mathcal{A}$ is called ______ set.
   a) $\mathcal{A}_\delta$ b) $\mathcal{A}_\sigma$
   c) $\mathcal{A}_{\delta\sigma}$ d) $\mathcal{A}_{\sigma\delta}$

6) If $(X, \mathcal{B}, \mu)$ be a measure space, $E \subseteq X$ then E is called finite measure if _______.
   a) $\mu(X) < \infty$ b) $\mu(\mathcal{B}) < \infty$
   c) $\mu(E) < \infty$ d) All of the above

7) If f_n is a sequence of non negative measurable function which converges almost everywhere to f and $f_n \leq f$ $\forall n$ then Monotone convergence theorem says that $\int f$ ______.
   a) $\leq lim \int f_n$
   b) $\geq lim \int f_n$
   c) $= lim \int f_n$
   d) All of these

8) If μ and v are two measures on $(X, \mathcal{B})$ then the measure v is said to be absolutely continuous with respect to μ if $\mu(E) = 0$ for any $E \in \mathcal{B}$ implies _____.
   a) $v(E) \neq 0$
   b) $v(E) > 0$
   c) $v(E) < 0$
   d) $v(E) = 0$

B) State True or False: **04**

1) The collection $\mathcal{R}$ of measurable rectangles is a $\sigma -$ algebra.
2) Every null set has a measure zero.
3) Hahn decomposition of a set is unique for a given signed measure.
4) If an extended real valued function f defined on X is measurable then $f^2 + 3$ is also measurable function.

Q.2 Answer the following. (Any Six) **12**

a) Define Measure and Measure space.
b) Define integration of simple function w.r.t. measure μ.
c) Define Locally measurable set and saturated measure.
d) If c is a constant and f is a measurable function defined on X then prove that $f + c$ is a measurable function.
e) State the Generalized Lebesgue convergence theorem.
f) Prove that: Every measurable subset of positive set is itself positive.
g) Define Radon Nikodym derivative.
h) If E is μ^* is measurable set then prove that E^c is also measurable set.

Q.3 Answer the following. (Any Three) **12**

a) If v is a signed measure and μ is a measure such that $v \perp \mu$ and $v << \mu$ then prove that $v = 0$.
b) Prove that: The set of locally measurable sets form σ-algebra.
c) State and Prove Lebesgue convergence theorem.
d) Prove that: The collection $\mathcal{R}$ of measurable rectangles forms semi algebra.

Q.4 Answer the following. (Any Two) **12**

a) Show that the triplet $(\mathcal{R}, \mathcal{M}, \mu)$ is a measure space where $\mathcal{M}$ is set of Lebesgue measurable sets and μ is set function defined by $\mu(E) = |E|$ is E is finite, $\mu(E) = \infty$ is E is infinite.
b) If μ_1 and μ_2 are measures on a measurable space $(X, \mathcal{B})$ such that atlcast one of them is finite and $v(E) = \mu_1(E) - \mu_2(E)$ for all $E \in \mathcal{B}$ then prove that v is a signed measure.

c) Show that the condition of σ-finiteness is essential in Radon Nikodym theorem.

Q.5 Answer the following. (Any Two) **12**

a) If $\mathcal{R}$ is a measurable rectangle and $x \in X$ is any element then for $E \in R_{\sigma\delta}$ prove that E_x is measurable subset of Y.

b) Define product measure and prove that if E is measurable subset $X \times Y$ then

i) $(E^c)_x = E_x^c$

ii) $(\cup_{i=1}^{\infty} E_i)_x = \cup_{i=1}^{\infty} (E_i)_x$

c) Prove that: The union of countable collection of positive sets is a positive set w.r.t signed measure.

Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS)
Examination: October/November - 2025
Partial Differential Equations (MSC15402)

Day & Date: Thursday, 30-10-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **10**

1) Suppose that $u(x, y)$ is harmonic in a bounded domain D and is continuous on $\overline{D} = D \cup B$, where B is boundary of D. Then $u(x, y)$ attains its minimum ______.
   a) on B
   b) inside D but not on B
   c) outside D but not on B
   d) inside D as well as on B

2) The complete integral of $px + qy - q^2 = 0$ is ______.
   a) $z = ax + by + a + b$
   b) $2z = (ax + y)^2 + b$
   c) $z = (ax - y)^2 + b$
   d) $z = (ax + y)^2 + b$

3) The Charpit's equation for partial differential equation $xpq + yq^2 - 1 = 0$ are ______.
   a) $\frac{dx}{q} = \frac{dy}{xp} = \frac{dz}{2(xpq + yq^2)} = \frac{-dp}{q} = \frac{dq}{p}$
   b) $\frac{dx}{x} = \frac{dy}{x + y} = \frac{dz}{2(xpq + q^2)} = \frac{dp}{p} = \frac{dq}{q}$
   c) $\frac{dx}{xq} = \frac{dy}{xp + 2yq} = \frac{dz}{2(xpq + yq^2)} = \frac{dp}{-pq} = \frac{dq}{-q^2}$
   d) $\frac{dx}{q} = \frac{dy}{p} = \frac{dz}{(xpq + yq^2)} = \frac{dp}{q} = \frac{dq}{p}$

4) The characteristic curves for the equation $xz_y - yz_x = z$ are ______.
   a) Straight line passing through origin
   b) Circle with Centre at origin
   c) Parabola with vertex at origin
   d) Rectangular hyperbola

5) The condition $X^{-}.curlX^{-} = 0$ is equation to ______.
   a) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) - Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
   b) $P\left(\frac{\partial Q}{\partial x} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial z}\right) = 0$
   c) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
   d) $P\left(\frac{\partial Q}{\partial z} + \frac{\partial R}{\partial y}\right) - Q\left(\frac{\partial P}{\partial x} + \frac{\partial R}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$

6) Which of the following is not true?
   a) Two dimensional Laplace equation is elliptic
   b) One dimensional wave equation is hyperbolic
   c) One dimensional heat equation is hyperbolic
   d) Canonical form of one dimensional wave equation is of the form $\frac{\partial^2 z}{\partial u\ \partial v} = 0$

7) Every integral generated by one parameter family of characteristics is an _______.
   a) envelope
   b) circle
   c) cone
   d) integral surface

8) Every one parameter family of surface $f(x, y, z) = c$ is a family of equipotential surfaces ______.
   a) True
   b) False
   c) need not be true
   d) None of these

9) The complete integral of the partial differential equation $p = (z + qy)^2$ is ______.
   a) $z = ax + 2\sqrt{ay} + b$
   b) $2z = ax + 2\sqrt{ay} + b$
   c) $2z = ax + 2ay + b$
   d) $z = ax - 2\sqrt{ay} + b$

10) The condition that the surfaces $f(x, y, z) = c$ forms a family of equipotential surfaces is that ______.
   a) $\frac{\nabla^2 f}{|\nabla f|^2} = 0$
   b) $\frac{\nabla f}{|\nabla^2 f|^2} = 0$
   c) $\frac{\nabla^2 f}{|\nabla f|^2}$ is function of f only
   d) $\frac{\nabla^2 f}{|\nabla f|^2}$ is not function of f

B) Write True/False. **06**

1) The partial differential equation obtained by eliminating arbitrary constant from the relation
$z^2(1+a^3) = 8(x+ay+b)^3$ is $p^3 + q^3 = 27z$.
2) A partial differential equation $pq = z$ is linear.
3) The differential equation $yzdx + xzdy + xydz = 0$ is not integrable.
4) The solution of Dirichlet problem if it exists is unique.
5) Charpit's method is used to solve a non-linear partial differential equation.
6) The first boundary value problem is called as The Neumann problem.

Q.2 Answer the following. **16**

a) Define:
i) Pfaffian Differential Equation
ii) Second Order Partial Differential Equation

b) Find a partial differential equation by eliminating arbitrary function from the relation $z = x + y + F(xy)$.

c) Show that the solution of Neumann problem is either unique or it differs from one another by constant.

d) Find the partial differential equation satisfied by all the surfaces of the form $F(u,v) = 0$ where $u = u(x,y,z)$, $v = v(x,y,z)$ and F is arbitrary function of u and v.

Q.3 Answer the following. **16**

a) Prove that a necessary and sufficient condition for the integrability of

$dz = \varphi(x,y,z)dx + \Psi(x,y,z)dy$ is $[f,g] = \frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$

b) Show that the equations $f = p^2 + q^2 - 1 = 0$ & $g = (p^2+q^2)x - pz = 0$ are compatible and find the one parameter family of common solution.

Q.4 Answer the following. **16**

a) Find the complete integral of $z^2(1+p^2+q^2) = 1$ by Charpit's method.

b) Reduce the equation $x^2u_{xx} - y^2u_{yy} = 0$ to a canonical form.

Q.5 Answer the following. **16**

a) Find the complete integral of $(p^2+q^2)x = pz$ and hence find the integral surface through the curve $x = 0, z^2 = 4y$.

b) Describe Jacobi's method of solving a first order partial differential equation.

Q.6 Answer the following. **16**

a) Define Pfaffian differential equation and check the integrability of $(y^2+yz)dx + (xz+z^2)dy + (y^2-xy)dz = 0$, Further find the solution.

b) Find the integral surface of the given p.d.e.
$(2xy-1)p + (z-2x^2)q = 2(x-yz)$ which passes through the curve $x_0(s) = 1, y_0(s) = 0, z_0(s) = s$.

Q.7 Answer the following. **16**

a) Find the condition that a one parameter family of surfaces forms a family of equipotential surfaces.

b) Find the general integral of $(x^2 + y^2)p + 2xyq = (x + y)z$.

SLR-HO-25

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: October/November - 2025
Integral Equations (MSC15403)

Day & Date: Saturday, 01-11-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) An integral equation of the form $y(x) = f(x) + \lambda \int_y^x k(x,t)y(t)dt$ is _____.
a) Fredholm equation of the first kind
b) Fredholm equation of the second kind
c) Volterra equation of the first kind
d) Volterra equation of the second kind

2) If a function $f(x)$ is a solution to the integral equation $f(x) = \int_a^x f(t)dt$ then the value of $f(a)$ is _____.
a) 1 b) 0
c) e^x d) e^a

3) The kernel $k(x,t)$ of a Fredholm integral equation is called symmetric if _____.
a) $k(x,t) = k(t,x)$ b) $k(x,t) = -k(t,x)$
c) $k(x,t) = k(-t,-x)$ d) $k(x,t) = k(x+t)$

4) If $K(x,t) = t - x$ be the kernel of a Volterra integral equation and $\lambda = 1$. Then which the following is the resolvent kernel?
a) $\cos(t-x)$ b) e^{t-x}
c) $\sin(t-x)$ d) $\cosh(t-x)$

5) The eigen value of the homogeneous integral equation $y(x) = \lambda \int_0^x e^x e^t \, y(t)dt$ is _____.
a) $\frac{2}{e-1}$ b) $\frac{5}{e+1}$
c) $\frac{2}{e^2-1}$ d) $\frac{2}{e^2+1}$

6) A function $\varphi(x)$ is said to be normalised if $\| \varphi(x) \| =$ _____.
a) 1 b) 0
c) -1 d) 5

7) Which of the following statements is true for a Volterra integral equation?
a) The upper limit of integration is a constant
b) The upper limit of integration is the variable of the function
c) The lower limit of integration is the variable of the function
d) The integral is always non-zero

8) If y be the solution to the Volterra integral equation $y(x) = e^x + \int_0^x \frac{1+x^2}{1+t^2} y(t)dt$. Then which of the following statements are true?
a) $y(1) = \left(1 + \frac{\pi}{4}\right)e$ b) $y(1) = \left(1 - \frac{\pi}{4}\right)e$
c) $y(1) = \left(1 + \frac{\pi}{2}\right)e$ d) $y(1) = \left(1 - \frac{\pi}{2}\right)e$

9) Which of the following kernel is a not a symmetric kernel?
a) $k(x,t) = \sin(x+t)$ b) $k(x,t) = x + t$
c) $k(x,t) = \sin(x-t)$ d) $k(x,t) = x^2t^2$

10) If $y(x)$ be the solution of the Fredholm integral equation $y(x) = x + \int_0^1 xy(t)dt$ then $y(2) =$ ______.
a) 4 b) 0
c) 2 d) 1

B) Write True/False. **06**

1) The function $y(x) = e^x(2x - \frac{2}{3})$ is the solution of the Fredholm integral equation $y(x) + 2\int_0^1 e^{x-t} y(t)dt = 2xe^x$
2) The continuous functions $f(x)$ and $g(x)$ on $[a,b]$ are said to orthogonal on $[a,b]$ if $\int_a^b f(x)g(x)dx = 1$
3) The integral equation $y(x) - \lambda\int_0^1 (3x-2)t\, y(t)dt = 0$ has no characteristic numbers.
4) A symmetric kernel is always zero.
5) The sequence of eigen functions of symmetric kernel can be made orthonormal.
6) There is no universal method for solving all integral equations. The solution method depends on the specific form of the equation.

Q.2 Answer the following. **16**

a) Show that $y(x) = 1$ is solution of the Fredholm Integral Equation.

$$y(x) + \int_0^1 x(e^{xt} - 1)y(t)dt = e^x - x$$

b) Prove that: If the kernel is symmetric then all its iterated kernel is also symmetric.

c) Solve the Integral Equation $y(x) = 1 + \int_0^x y(t)dt$ where, $y_0(x) = 0$ by using successive approximation method.

d) Write a note on Iterated Kernel and Resolvent Kernel.

Q.3 Answer the following. **16**

a) Convert the differential equation $y'' - 5y' + 6y = 0$ into the integral equation with initial conditions $y(0) = 0$ and $y'(0) = -1$.

b) Using the resolvent kernel find the solution of the integral equation

$$y(x) = (1 + x^2) + \int_a^x \frac{1 + x^2}{1 + t^2} y(t)dt$$

Q.4 Answer the following. **16**

a) Prove that the formula for converting multiple integral into single ordinary integral is

$$\int_a^x y(t)dt^n = \int_a^x \frac{(x-t)^{n-1}}{(n-1)!} y(t)dt$$

b) Find the value of λ for which the integral equation $y(x) = \lambda \int_0^1 (6x - t)\, y(t)dt$ has a non-trivial solution.

Q.5 Answer the following. **16**

a) Explain the procedure of conversion of Initial Valued Problem into Volterra Integral Equation.

b) Find the resolvent kernel of the kernel $k(x, t) = e^x - t$.

Q.6 Answer the following. **16**

a) Solve the symmetric integral equation $y(x) = (x + 1)^2 + \lambda \int_{-1}^1 (xt + x^2t^2) y(t)dt$ by using Hilbert Gram-Schmidt Theorem.

b) Prove that the eigen functions of symmetric kernel corresponding to different eigen values are orthogonal.

Q.7 Answer the following. **16**

a) Explain the method of Greens function for solving the ordinary differential equation

b) Solve the integral equation $\int_0^\infty F(x) \cos px \, dx = \begin{cases} 1 - p & ; 0 \le p \le 1 \\ 0 & ; p > 1 \end{cases}$ by Fourier transform Method.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: October/November - 2025
Operations Research (MSC15404)

Day & Date: Tuesday, 04-11-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q.No.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **10**

1) Games which involve more than two players are called _____.
   a) conflicting games b) negotiable games
   c) n-person game d) All of the above

2) The set of all feasible solution of a linear programming problem is _____ set.
   a) Convex b) Concave
   c) Strictly convex d) Strictly concave

3) Consider the following statements _____.
   I) The closed ball in R^3 is a convex set.
   II) A hyperplane in R^n is a convex set.
   a) only I is true b) only II is true
   c) both are true d) both are false

4) A saddle point in game exists when _____.
   a) maximin value = maximax value
   b) minimax value = minimum value
   c) minimax value = maximin value
   d) All of the above

5) The best use of linear programming problem is to find an optimal use of _____.
   a) Money b) Machine
   c) Manpower d) All of the above

6) The dual of the primal problem is obtained by, _____.
   a) transposing the co-efficient matrix and reverting the inequalities
   b) interchanging the role of constant terms and the co-efficient of the objective function
   c) minimizing the objective function instead of maximizing it
   d) All of the above

7) The dual simplex method works towards _____ while simplex method works towards _____
   a) optimality, feasibility
   b) feasibility, optimality
   c) boundedness, basic solution
   d) finiteness, basic solution

8) Simplex method is developed by American mathematician _____.
   a) Frank Wolf
   b) Martin Beale
   c) Ralph E. Gomory
   d) George Dantzig

9) In a mixed strategy game _____.
   a) No saddle point exist
   b) Each player selects the same strategy without considering
   c) Each player always selects same strategy
   d) All of the above

10) In dual simplex method, _____ variables are not required.
   a) Slack
   b) Surplus
   c) Original
   d) Artificial

B) Fill in the blanks. **06**

1) A game is said to be fair if both upper and lower values of the game are same and are _____.
2) If a primal LPP has a finite solution then the dual LPP should have _____ solution.
3) To convert ≥ inequality constraints into equality constraints, we must add a _____.
4) A quadratic form $Q(X)$ is positive definite iff $Q(X)$ is _____ for all $x \neq 0$
5) If pth variable of the primal is unrestricted in sign then the pth constraint of dual is _____.
6) Gomory's cutting plane method will take the help of _____ method to solve the given integer programming problem.

Q.2 Answer the following. **16**

a) Prove that: The dual of the dual of a given primal is primal.
b) Write general form of Quadratic programming problem.
c) Define :
   i) Extreme point of convex set
   ii) Convex hull
d) Write the rules for determining a saddle point in Game theory.

Q.3 Answer the following. **16**

a) Solve the following problem by Simplex method.
$Max\ Z = 3x_1 + 2x_2$ subject to the constraints $x_1 + x_2 \leq 4, x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

b) Prove that: The set of all convex combinations of a finite number of points $x_1, x_2 \dots \dots x_n$ is a convex set.

Q.4 Answer the following. **16**

a) If k^{th} constraint of the primal is an equality then prove that the dual vaiable w_k is unrestricted in sign.

b) Solve the following problem by Dual Simplex method.
$Min\ Z = 2x_1 + x_2$ subject to the constraints
$3x_1 + x_2 \geq 3, 4x_1 + 3x_2 \geq 6, x_1 + 2x_2 \ \geq 3$ and $x_1, x_2 \geq 0$

Q.5 Answer the following.

a) Find the saddle point and solve the game: **10**

Player B

Player A

	B_1	B_2	B_3	B_4
A_1	1	7	3	4
A_2	5	6	4	5
A_3	7	2	0	3

b) Write an algorithm of Big-M method for solving linear programming problem. **06**

Q.6 Answer the following. **16**

a) Find the optimum integer solution to the following IPP by Gomory's cutting plane method.
$Max\ Z = x_1 \ + \ 2x_2$ subject to the constraints
$2x_2 \leq 7, x_1 + x_2 \leq 7, 2x_1 \leq 11$ and $x_1, x_2 \geq 0$ and are integers.

b) Prove that: The intersection of two convex sets is a convex set.

Q.7 Answer the following. **16**

a) If X is any feasible solution to the primal problem and W is any feasible solution to the dual problem then prove that $CX \leq b^T W$.

b) Prove that: The collection of all feasible solutions to linear programming problem constitutes a convex set whose extreme point corresponds to the basic feasible solution.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: October/November - 2025
Numerical Analysis (MSC15408)

Day & Date: Friday, 07-11-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Questions no. 1 & 2 are compulsory.
2) Attempt any Three Question from Q No.3 to Q No.7
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **10**

1) The approximate value of $y(0.1)$ from $\frac{dy}{dx} = x^2y - 1, y(0) = 1$ is ______.
a) 0.900 b) 0.222
c) 1.001 d) 0.994

2) What is the value of the Trace of the matrix $\begin{bmatrix} 3 & 0 & 2 \\ 8 & 5 & 5 \\ 1 & 2 & 2 \end{bmatrix}$?
a) -5 b) 0
c) 24 d) 10

3) If $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}$, the eigenvalue corresponding to the eigenvector is ______.
a) 1 b) 4
c) -4.5 d) 6

4) Gauss-Seidel iterative method is used to solve ______.
a) differential equation
b) system of linear equations
c) system of non-linear equations
d) partial differential equation

5) If E_R is a relative error then the percentage error is given by _____.
a) $E_p = E_R \times 100$ b) $E_p = -E_R \times 100$
c) $E_p = E_R \times 10$ d) $E_p = \frac{E_R}{100}$

6) The digits that are used to express a number is called ______.
a) significant digit b) significant figure
c) both a and b d) error

7) How many real roots does the equation $\sin x - x = 0$ have?

a) 2 b) 3
c) 1 d) infinite

8) The eigenvalues of 4×4 matrix $[A]$ are given as 2,-3,13, and 7 then the |det(A)| is _______.

a) 546 b) 25
c) 19 d) 37

9) The method of false position is also known as ______.

a) Secant Method b) Newton-Raphson Method
c) LU-decomposition d) Regula Falsi Method

10) The equation $f(x)$ is given as $x^3 + 4x + 1 = 0$. Considering the initial approximation at $x = 1$. Then the value of x_1 is given as _______.

a) 0.6712 b) 0.1856
c) 0.1429 d) 1.8523

B) Write True / False. **06**

1) The n^{th} degree polynomial has n real or complex roots.
2) The Bisection method is guaranteed to converge if $|f'(x)| > 1$.
3) The Secant method is similar to the Newton Raphson method, but it uses an approximation of the derivative.
4) The order of convergence of the Bisection method is 2.
5) The Newton Raphson method fails if $f'(x)$ is zero.
6) The root/roots of the equation $e^x - 4x = 0$ lying between 0 and 1.

Q.2 Answer the following. **16**

a) Define eigen values and eigen vectors.
b) Construct a formula for Newton-Raphson method.
c) Evaluate the sum $S = \sqrt{43} + \sqrt{47} + \sqrt{5}$ correct to three significant figures and find absolute and relative error.
d) Write a note on Euler's Modified method.

Q.3 Answer the following. **16**

a) Reduce the matrix $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 7 & 9 \\ 5 & 8 & -2 \end{bmatrix}$ to the tridiagonal form.
b) Explain the construction of Gauss elimination method.

Q.4 Answer the following. **16**

a) Find a real root of the equation $x^3 - x - 1 = 0$ by bisection method, correct upto three decimal places.
b) Describe rate of convergence of secant method.

Q.5 Answer the following. **16**

a) Find the largest eigen value of $\begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix}$ by using Rayleigh's power method.

b) Find fourth approximation of the solution of initial value problem $\frac{dy}{dx} = x + y, y(0) = 1$ by Picard's method and estimate $y(0.8)$.

Q.6 Answer the following. **16**

a) Solve the following system of equations.
$5x + y - z = 9, x + 4y + 2z = 16, x - 2y + 5z = 18$
by using Gauss-Seidel method.

b) Find a real root of the equation $x^4 - x - 10 = 0$ by using secant method.

Q.7 Answer the following. **16**

a) Find a real root of the equation $f(x) = \cos x - xe^{-x} = 0$ by method of False position, correct upto three decimal places.

b) Find an approximate value of $y(1.2)$ and $y(1.4)$ for the initial value problem
$\frac{dy}{dx} = \left(\frac{2x-1}{x^2}\right) y + 1, y(1) = 2$ Using Runge-Kutta method.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: October/November - 2025
Probability Theory (MSC15410)

Day & Date: Friday, 07-11-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Questions no. 1 & 2 are compulsory.
2) Attempt any Three Question from Q No.3 to Q No.7
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **10**

1) If $\{A_n\}$ is decreasing sequence of sets, then it converges to ______.
a) $\lim\inf A_n$ b) $\lim \text{suf} A_n$
c) both (a) and (b) d) None of the above

2) If events A and B are independent events, then which of the following is correct?
a) $P(A \cap B) = P(A) + P(B)$
b) $P(A \cup B) = P(A) + P(B) - P(A) * P(B)$
c) $P(A \cup B) = P(A) * P(B)$
d) $P(A \cup B) = P(A) - P(B)$

3) If X_n is a degenerate random variable for all n and X is identical random variable to X_n, then $\{X_n\}$ converges to X in ______.
a) r^{th} mean and in probability
b) probability and in distribution
c) r^{th} mean, in probability and in distribution
d) r^{th} mean, almost sure, in probability and in distribution

4) The sequence of sets $\{A_n\}$, where $A_n = \left(0, 2 + \frac{1}{n}\right)$ converges to _______.
a) (0,2) b) (0,2]
c) [0,3) d) [0,2]

5) Indicator function is a ______.
a) Simple function b) Elementary function
c) Arbitrary function d) All of these

6) If a r.v. X is symmetric about zero, then the characteristic function $\varphi_x(t)$ of X is ______.
a) Real b) doesn't exist
c) Complex d) None of these

7) If F_1 and F_2 are two fields defined on subsets of Ω, then which of the following is/are always a field?
a) $F_1 \cup F_2$ b) $F_1 \cap F_2$
c) both (a) and (b) d) neither (a) nor (b)

8) If events A, B and C are mutually independent, then which of the following is not correct?
a) A and B are pairwise independent
b) A and C are pairwise independent
c) B^c and C are independent
d) All are correct

9) A well-defined collection of sets is called as ______.
a) Subset b) Superset
c) Class d) None of these

10) Expectation of a simple non-negative random variable satisfies ____.
a) Linearity property
b) Scale preserving property
c) Non-negativity property
d) All of these

B) Fill in the blanks. **06**
1) Convergence in probability implies ______ convergence.
2) If P is a probability measure defined on (Ω, A), then $P(\Omega) =$ ______.
3) If $A \subset B$, then $P(A) \ldots.\ P(B)$.
4) The σ-field generated by the intervals of the type $(-\infty, x), x \in R$ is called ______.
5) The convergence in ______ is also called as a weak convergence.
6) If Ω contains 2 elements, then the largest field of subsets of Ω contains ______sets.

Q.2 Answer the following. **16**
a) Prove that inverse mapping preserves all set relations.
b) Prove or disprove: Arbitrary union of fields is a field.
c) Define mixture of two probability measures. Show that mixture is also a probability measure.
d) Write a note on Lebesgue-Stieltjes measure.

Q.3 Answer the following. **16**
a) State and prove monotone convergence theorem.
b) Prove that an arbitrary random variable can be expressed as a limit of sequence of simple random variables.

Q.4 Answer the following. **16**
a) Prove that inverse image of σ-field is also a σ-field.
b) Prove that probability measure is a continuous measure.

Q.5 Answer the following. **16**

a) State and prove Fatou's lemma.

b) Prove or disprove:

i) Convergence in distribution implies convergence in probability

ii) Convergence in probability implies convergence in distribution

Q.6 Answer the following. **16**

a) Prove that expectation of a random variable X exists, if and only if $E|X|$ exists.

b) Define the characteristic function of a random variable. Also state its inversion theorem and uniqueness property.

Q.7 Answer the following. **16**

a) Discuss, in details, σ-field induced by r.v. X.

b) Define field and σ-field. Show that there exist classes which are field but not σ-field .

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: October/November - 2025
Algebra - II (MSC15201)

Day & Date: Tuesday, 28-10-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) The splitting field of $x^2 - 1$ over Q is ______.
a) $Q(i)$ b) R
c) Q d) C

2) The number of automorphisms on a field of real numbers is / are ______.
a) 1 b) 0
c) 2 d) finite

3) The number π is algebraic over ______.
a) R b) Q
c) $Q(i)$ d) $Q(\sqrt{2})$

4) $O(G(C, R)) =$ ______.
a) 1 b) 0
c) 3 d) 2

5) For every prime p and every positive integer m there exist a finite field with ______ elements.
a) m^p b) p^m
c) $m.p$ d) None of these

6) For a field of characteristic zero ____.
I. Every finite extension is simple extension.
II. Every finite extension is separable extension.
a) only I is true b) only II is true
c) Both are true d) Both are false

7) An ideal $N = < p(x) >$ of $F[x]$ is a maximal ideal if $p(x)$ is ______ polynomial.
a) minimal b) monic
c) reducible d) irreducible

8) The subfield of K generated by $F(F \subseteq K), a\ b\ \in K$ is given by _____.

a) $F(a,b)$
b) $F(b,a)$
c) $F(b)(a)$
d) All of these

B) State whether the following statements are True or False. **04**

1) Every rational number is left fixed by any automorphism on any extension field K.
2) Set of all constructible number of R may or may not form subfield.
3) Every finite extension is normal extension.
4) $\sqrt{2}$ is algebraic of degree 1 over R.

Q.2 Answer the following. (Any Six) **12**

a) Define.
   i) Fixed field
   ii) Galois group

b) Find splitting field of $x^2 - 2$ over Q.

c) Construct a field with 4 elements.

d) Prove or disprove: Doubling the cube is impossible.

e) Find the fixed field of $G(Q(i), Q)$.

f) If a and b are constructible numbers then prove that $a + b$ and $a - b$ are also constructible.

g) Check whether $3 + \sqrt{2}$ is algebraic over Q or not.

h) Define.
   i) Simple extension
   ii) Finite extension

Q.3 Answer the following. (Any Three) **12**

a) Prove that: The Galois group of a polynomial over a field F of characteristic zero is isomorphic to a group of permutation of its roots.

b) Find all possible automorphisms on a field of rational numbers.

c) Find the fixed field of
   i) $G(Q(2^{1/3}),Q)$
   ii) $G(C,Q)$

d) Prove that: Every finite extension is an algebraic extension.

Q.4 Answer the following. (Any Two) **12**

a) Prove that: A field of characteristic zero is perfect.

b) If K is a normal extension of a field of characteristic 0 and T be a subfield of K containing F then prove that T is a normal extension of F iff $\sigma(T) \subseteq T$ for all $\sigma \in G(K,F)$.

c) Prove that: The polynomial $f(x) \in F[x]$ has a multiple root iff $f(x)$ and $f'(x)$ have nontrivial common factor.

Q.5 Answer the following. (Any Two) **12**

a) If L is a finite extension of K and if K is finite extension of F then prove that L is finite extension of F and $[L:F] = [L:K][K:F]$

b) If $a \in K$ be algebraic over F then prove that any two minimal monic polynomial for a over F are equal.

c) Find Galois group of $x^3 - 1$.