Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: March/April - 2025
Group and Ring Theory (2317101)

Day & Date: Thursday, 15-May-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All Questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) ______ is non commutative ring.
a) Z b) Q
c) $M_2(R)$ d) None of these

2) What is the identity element in the group $G = \{2,4,6,8\}$ under multiplication modulo 10?
a) 2 b) 4
c) 6 d) 8

3) The polynomial $x^2 + 1$ is irreducible over _____.
a) C b) $Q(i)$
c) $Q(-i)$ d) R

4) Consider the following statements
I) Every finite group G has at least one principal series.
II) Any two principal series of a group G are isomorphic.
a) Both statements are true b) Both statements are false
c) Only I is true d) Only II is true

5) Which of the following is false?
a) $Z(G)$ is always a normal subgroup of G.
b) The intersection of any two normal subgroup of a group G is normal subgroup.
c) If N and M are normal subgroups of G then NM is also a normal subgroup of G
d) None of these

6) Which of the following is not PID?
a) Z b) $Z[x]$
c) Q d) Z_7

7) If F is a field then number of ideal / ideals is/are _____.
a) 0 b) 2
c) 1 d) 3

8) Cyclic group of order 5 has only _____ generators.
a) 6 b) 4
c) 5 d) 1

B) State whether the following statements are true or false. **04**
1) If G is abelian $\Leftrightarrow Z(G) = G$.
2) S_3 is nilpotent.
3) Every group of prime order is simple.
4) If X is a G-set then G_x is subgroup of G.

Q.2 Answer the following. (Any Six) **12**
a) Define the following terms:
1) Fixed sets 2) Orbit in X
b) Show that every abelian group is solvable.
c) Explain refinement of subnormal series.
d) Explain concept of primitive polynomial.
e) Define principal ideal with one example.
f) Define the following terms:
1) Equivalence class 2) p - group
g) Explain two examples of group action on a set.
h) If R is a ring and $f(x)$, $g(x)$ be non zero polynomials in $R[x]$ where degree $f(x) = n$ and deg $g(x) = m$ then prove that $\deg(f(x) + g(x)) \leq \max(m, n)$

Q.3 Answer the following (Any three). **12**
a) Define the following terms with one example each:
1) unique factorization domain
2) Euclidean domain
b) Prove that: Any group of order p^n is nilpotent.
c) If $f(x) = x^3 + x^2 - 2x - 1 \in Z[x]$ then check the irreducibility over Q.
d) If G' be the commutator subgroup of group G then prove that G is abelian $\text{iff}\ G' = \{e\}$, e being identity element of G.

Q.4 Answer the following (Any two). **12**
a) State and prove second sylow theorem.
b) Show that: No group of order 30 is simple.
c) State and prove Eisenstein Criteria for irreducibility of polynomials over set of rational numbers.

Q.5 Answer the following (Any two). **12**
a) Prove that: Any two composition series of group G are isomorphic.
b) If X be any G-set then prove that $|xG| = (G: G_x)$ for any $x \in X$ where $(G: G_x)$ is index of G_x in G
c) If F is a field then prove that an element $a \in F$ is zero of $f(x) \in F[x]$ $\text{iff}(x - a)$ is a factor of $f(x)$ in $F[x]$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: March/April - 2025
Real Analysis (2317102)

Day & Date: Saturday, 17-May-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) The lower integral of a function f on [a,b] is
a) infimum of set of upper sums
b) infimum of set of lower sums
c) supremum of set of upper sums
d) supremum of set of lower sums

2) Find the mean value of $\int_0^1 x dx$ in $[0,1]$
a) 0 b) 1
c) $\frac{1}{2}$ d) $-\frac{1}{2}$

3) The partial derivatives of a function describes the rate of change of a function in the direction of ____.
a) X-axis b) Y-axis
c) Z-axis d) co-ordinate axis

4) If f is non-negative function on $[a,b]$ such that $\int_0^1 f(x)dx = 0$ then for all $x \in [a,b]$
a) $f(x) = 0$ b) $f(x) \geq 0$
c) $f(x) \leq 0$ d) $f(x)$ do not exist

5) Consider the following statements:
a) W(P,f) is always non-negative.
b) W(P,f)= U(P,f)- L(P,f)
c) only I is true b) only II is true
d) both are true d) both are false

6) For any two partitions P_1, P_2 following relation holds
a) $L(P_1, f) \leq U(P_2, f)$ b) $L(P_2, f) \leq U(P_1, f)$
c) both a) and b) d) none of these

7) If P* is refinement of P then ____.
a) $P \subseteq P^*$ b) $P^* \subseteq P$
c) $P \subset P^*$ d) $P^* \subset P$

8) If $f(x) = x$ on $[0,1]$ and divide the interval into two equal sub intervals then $L(P, f) =$ ____.

a) 0.25 b) 2.5
c) 0 d) 0.75

B) Fill in the blanks: **04**

1) A function $f = (f_1, f_2, \dots . f_n)$ has continuous partial derivative on an open set S in R^n and the Jacobian determinant is non zero at some point a in S then there is an n-ball B(a) on which *f* is ____.
2) The directional derivative of $f(x, y) - x^2 y$ at point $(1,2)$ in the direction (1,1) is ____.
3) If P_1 and P_2 are two partitions of $[a, b]$ then their common refinement is given by $P^* =$
4) A bounded function f is integrable on $[a, b]$ if the set of points of discontinuity has ____ limit points.

Q.2 Write short answers. (Any Six) **12**

a) Define: Upper Integral and Lower Integral.
b) Examine whether the function $f(x) = x2 + 4x + 3$ on $[-10,10]$ have local extrema or not.
c) Find the directional derivative of $f(x, y) - x^2 + y^2$ at point $(1,2)$ in the direction $(2,3)$.
d) State first fundamental theorem of calculus.
e) Write second definition of integrability (Using Riemann sum).
f) Write Short note on Primitive of function.
g) Define Norm and Refinement of Partition
h) Write note on Directional derivative.

Q.3 Write short notes. **12**

a) Solve. $\int_0^3 (4x + 5)dx$
b) Prove that : The oscillation of a bounded function *f* on an interval $[a, b]$ is the supremum of the set $\{|f(x_1) - f(x_2)|/x_1, x_2 \in [a, b]\}$ of numbers.
c) If f is differentiable function at c with total derivative T_c then prove that the directional derivative $f'(c; u)$ exists for every u in R^n and also prove that $T_c(u) = f'(c; u)$
d) Check whether directional derivative of following function exists at 0 in the direction of $u - (u_1, u_2)$

$$f(x) = \begin{cases} \dfrac{x.y}{x+y} & \text{if } (x, y) \neq (0,0) \\ 0, & \text{if } (x, y) = (0,0) \end{cases}$$

Q.4 Answer the following: **12**

a) If a function f is bounded and integrable on $[a,b]$ then prove that the function *F* defined as, $F(x) = \int_a^x f(t)\, dt;\ a \le x \le b$ is continuous on $[a,b]$. Furthermore if *f* is continuous at a point c of $[a,b]$ then prove that *F* is derivable at c and $F'(c) = F(c)$

b) If f_1 and f_2 are two bounded and integrable functions on $[a,b]$ then prove that $f_1 + f_2$ is also integrable on $[a,b]$ and also prove that

$$\int_a^b (f_1 + f_2)\, dx = f1dx + f2dx$$

c) Prove that : Every continuous function is integrable.

Q.5 Answer the following: **12**

a) If S is an open subset of R^n and $f: S \to R^m$ is differentiable at each point of S, x and y are two points in S such that $L(x,y) \subseteq S$ then prove that for every vector a in R^m there is a point z on $L(x,y)$ such that , $a.\{f(y) - f(x)\} = a.\{f'(z)(y - x)\}$

b) If $B = B(a;r)$ is an n-ball in $R^n, \partial B$ denotes its boundary, $\partial B = \{x/||x - a|| = r\}$ denote its closure, $f = \{f_1, f_2, \dots f_n$ be continuous on $\bar{B}$ and assume that all partial derivative $D_j f_i(x)$ exist if $x \in B.$ Assume further that $f(x) \neq f(a)$ if $x \in \partial B$. and that the Jacobian $J_f(x) \neq 0$ for each $x \in B$ then prove that $f(B)$ the image of B under f contains an n-ball with center at $f(a)$.

c) If f have a continuous n^{th} (for some integer$n \ge 1$) derivative in the open interval (a,b) and for some interior point c in (a,b) we have, $f'(c) = f''(c) = - - -= f^{n-1}(c) = 0$ but $f^n(c) \neq)0$ then prove that for n even, f has local minimum at $f^n(c) > 0$ and f has local maximum at c if $f^n(c) < 0$. Also prove that if n is add, there is neither a local maximum nor a local minimum at c.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: March/April - 2025
Number Theory (2317107)

Day & Date: Monday, 19-May-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) Which of the following is not a square free integer?
a) 105 b) 30
c) 40 d) 10

2) Consider the statements _____.
i) If $a^k \equiv b^k (mod\ m)$ then $a \equiv b\ (mod\ m)$ for all $k \geq 1$
ii) If $a \equiv b(mod\ m)$ then $c \equiv b\ (mod\ m)$ then $a + c \equiv b + d(mod\ m)$
a) Only I is true b) Only II is true
c) both I and II are true d) both I and II are false

3) The linear congruence $ax \equiv b\ (mod\ n)$ has a solution iff _____.
a) $\gcd(a,b)\ |n$ b) $\gcd(b,n)\ |a$
c) $\gcd(a,n)\ |b$ d) $\gcd(a,n) = a$

4) Let 'p' be a prime number and 'a' be an integer such that $p|a$ then which of the followings are true?
a) $a^p \equiv a(mod\ p)$ b) $a^p \equiv 0(mod\ p)$
c) $a \equiv 0(mod\ p)$ d) all of these

5) Which of the following is a perfect square?
a) $(10000)!$ b) $(95)!$
c) $(40)!$ d) none of these

6) The congruence $x^2 \equiv -1(mod\ p)$, p is a prime, has a solution if and only if _____.
a) $p \equiv -1(mod\ 4)$ b) $p \equiv 0(mod\ 4)$
c) $p \equiv 1(mod\ 4)$ d) $p \equiv 1(mod\ p^2)$

7) The number of integers less than 1896 and relatively prime to 1896 are _____.
a) 620 b) 624
c) 108 d) 312

8) For any positive integer $n, \varphi(n) =$ _____.

a) $n\sum_{d|n} \frac{\mu(d)}{d}$ b) $n\sum_{d|n} \mu(d)$

c) $\sum_{d|n} \frac{\mu(d)}{d}$ d) $d\sum_{d|n} \frac{\mu(d)}{d}$

B) Fill in the blanks. **04**

1) If a is an primitive root modulo n and b, k are any integers, then $ind.b^k \equiv$ _____.
2) If n is a positive integer such that $n \geq 5 \sum_{k=1}^{n} \mu(k!) =$ _____.
3) Solution of $47x \equiv 11(mod\ 249)$ is _____.
4) The greatest integer value of $x = -5.9$ is _____.

Q.2 Answer the following. (Any Six) **12**

a) Find the highest power of 11 contained in 6000!.

b) If $k > 0$ be any integer then show that $gcd(ka, kb) = k\ \gcd(a, b)$.

c) Show that every integer is of the form $3q\ or\ 3q \pm 1$.

d) Find the order of 5 modulo 29.

e) Define the following terms:

i) Primitive roots

ii) Order of an integer modulo n

f) If n is an odd integer then show that $\varphi(2n) = \varphi(n)$.

g) What is Multiplicative function.

h) Factorize 2279.

Q.3 Answer the following. (Any three). **12**

a) Solve $17x \equiv 9(mod\ 276)$.

b) If f is multiplicative function and $S(n) = \sum_{d|n} f(d)$ then prove that $S(n)$ is also multiplicative function.

c) Prove that the linear Diophantine equation $ax + by = c$ has a solution iff $d|c$ where $d = \gcd(a, b)$.

d) Construct the index table for 17 with primitive root 5.

Q.4 Answer the following. (Any two) **12**

a) State and Prove Wilson's theorem

b) If $\gcd(a, b)$ then the equation $ax + by = c$ has a solution iff $d|c$, further if (x_0, y_0) is a solution of $ax + by = c$ then show that all the other solutions are in the form $x_1 = x_0 - \frac{b}{d}t, y_1 = y_0 + \frac{a}{d}t$ for any integer t.

c) If p is a prime and $f(x) = a_n x^n + a_{n-1}x^{n-1} + - - - + a_1 x + a_0, a_n \not\equiv 0(mod\ p)$ is a polynomial of degree n≥ 1 with integral coefficients then show that $f(x) \equiv 0(mod\ p)$ has at least n incongruent solutions mod p.

Q.5 Answer the following. (Any two) **12**

a) State and prove Chinese Reminder Theorem.

b) Find the primes not exceeding 155 by using the method Sieve of Eratosthenes.

c) If $n = \mathrm{p_1 k_{1_{p_2}} k_2} - - - \mathrm{p_r r}$ is a prime factorization of $n > 1$ then show that $\sum_{d|n} \mu(d)\varphi(d) = (2 - p_1)(2 - p_2) - - - (2 - p_r)$.

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Set P

M.Sc. (Mathematics) (Semester - I) (New) (NEP CBCS) Examination: March/April - 2025
Research Methodology in Mathematics (2317103)

Day & Date: Saturday, 24-May-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) "Gathering knowledge for knowledge's sake" is termed as _____ research.
a) Pure b) Basic
c) Fundamental d) All of the above

2) _____ approach involves the construction of an artificial environment within which relevant information and data can be generated.
a) Simulation b) Inferential
c) Experimental d) Qualitative

3) The maximum value of h such that the given author/journal has published at least h papers that have each been cited at least h times is known as _____.
a) i-10 index b) citation
c) h-index d) impact facto

4) In Research, a complete enumeration of all items in the "population" is known as a _____.
a) Census Universe b) Census Population
c) census inquiry d) none of these

5) What is a primary feature of a "theorem" in mathematics?
a) It is a statement that can be directly observed
b) It requires a rigorous proof for validation
c) It is only conjectural and has no proof
d) It is used for experimental data analysis

6) _____ is the act of publishing borrowed ideas or words as though they are your own.
a) Plagiarism b) reference
c) abstract d) Archive

7) The longform of AMS is _____.
- a) Annual Mathematics Society
- b) American Mathematics Source
- c) Annual Mathematics Source
- d) American Mathematical Society

8) A public notice was issued by the UGC, on the _____ to announce the establishment of a dedicated Consortium for Academic and Research Ethics (CARE) to carry out this mandate.
- a) 28th of November, 2018 b) 28th of November, 2020
- c) 28th of November, 2024 d) 28th of November, 1990

B) State whether following statements are true or false. **04**
1) UGC-CARE is a quality mandate for Indian academia.
2) A corollary is a direct or easy consequence of lemma, theorem or proposition.
3) Deliberate sampling is known as probability sampling.
4) Good research is not replicable.

Q.2 Answer the following. (Any Six) **12**
- **a)** Define Research. (give two definitions)
- **b)** Write Motivation in Research.
- **c)** Write longform of AMS and SCI.
- **d)** Explain the terms: Lemma, theorem, corollary and preposition.
- **e)** Define Citation and impact factor.
- **f)** What is extensive literature survey?
- **g)** Write importance of conclusion in research article.
- **h)** Define: h-index, i10 index

Q.3 Answer the following. (Any three). **12**
- **a)** Write an expository note on Research Approaches.
- **b)** Give the difference between Research methods and Research Methodology.
- **c)** Write note on: The Role of examples in research article.
- **d)** State the qualities of good research.

Q.4 Answer the following. (Any two) **12**
- **a)** Write detail information about different types of sampling.
- **b)** Write the problems encountered by researchers in India.
- **c)** Write an expository note on UGC CARE list journal including objective, need and scope of UGC CARE.

Q.5 Answer the following. (Any two) **12**
- **a)** Explain six different types of Research.
- **b)** Write the tex file format of Research article.
- **c)** Write short note on collecting the data.

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Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: March/April - 2025
Field Extension Theory (2317201)

Day & Date: Wednesday, 14-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) Every automorphism of every field E leaves _____ elements of E fixed.
a) at least one b) at least two
c) two d) one

2) Two finite field of same order are _____.
a) homomorphic b) isomorphic
c) not isomorphic d) none of these

3) Characteristic of an integral domain is _____.
a) either zero or prime number
b) always prime number
c) always zero
d) never zero

4) A field K is regarded as a vector space over _____ of K.
a) any subset b) any subfield
c) any subring d) any subgroup

5) A field C of complex numbers is _____ extension of field R of real numbers.
a) finite b) simple
c) algebraic d) All of these

6) $O(G(Q(\sqrt{2}), Q))$ is _____.
a) equal to 1 b) equal to 2
c) less than or equal to 1 d) less than or equal to 2

7) If a & b are algebraic over F of degree m & n respectively then $a+b$ is algebraic of degree _____.
a) $m+n$ b) atmost $m+n$
c) mn d) atmost mn

8) If characteristic of F is zero and $f(x) \in F[x]$ is irreducible then $f(x)$ has _____ roots.
a) multiple b) distinct
c) imaginary d) real

B) Write True/False. **04**

1) Every complex number is algebraic over R.
2) It is not possible to find an extension of finite field.
3) Fixed field of $G(K,F)$ if contained in F.
4) π is algebraic over R.

Q.2 Answer the following. (Any Six) **12**

a) Check whether $\sqrt{5}+2^{1/3}$ is algebraic over Q or not.
b) Find degree and basis of $Q(2^{1/3}, i)$, over Q.
c) Write short note on elementary symmetric functions.
d) Prove or disprove: Doubling the cube is impossible.
e) Construct a field with 9 elements.
f) Prove that R is not normal extension of Q.
g) Define:
i) Separable element ii) Perfect fields
h) Define: Algebraic element and its degree

Q.3 Answer the following. (Any Three) **12**

a) With usual notations Prove or disprove that: $Q\left(\sqrt{3}+\sqrt{7}\right)=Q(\sqrt{3},\sqrt{7})$
b) If $a\in K$ be algebraic over F and $p(x)$ be minimal polynomial for a over F then prove that $p(x)$ is irreducible over F.
c) Check whether $\sqrt{5-\sqrt{11}}$ is algebraic over Q or not.
d) If $f(x)\in f[x]$ be of degree $n\geq 1$ then prove that there is a finite extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

Q.4 Answer the following. (Any Two) **12**

a) Prove that: Any finite extension of a field of characteristic zero is a simple extension.
b) If K is a field and $\sigma_{1,}\sigma_2 \dots \sigma_n$ are n distinct automorphisms of K then prove that it is impossible to find the elements $a_{1,}a_2 \dots.. a_n$ not all zero in K such that $a_1\sigma_1(u)+a_2\sigma_2(u)+\cdots+a_n\sigma_n(u)=0$ for all $u\in K$.
c) If L is a finite extension of K and K is finite extension of F then prove that L is finite extension of F.

Q.5 Answer the following. (Any Two) **12**

a) Prove that: The polynomial $f(x)\in F[x]$ has a multiple root iff $f(x)$ and $f'(x)$ have a nontrivial common factor.
b) If K be an extension of F and $a\in K$ be algebraic over F then prove that $F(a)$ is isomorphic to $\frac{F[x]}{V}$ where V is an ideal of $F[x]$ generated by the minimal polynomial for a over F.
c) Find Galois group of x^2-7 over Q.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: March/April - 2025
General Topology (2317202)

Day & Date: Friday, 16-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) In a T-space $< X, \Im >$, a subset E of X is closed if______.
   a) $i(E) \subset E$ b) $E \subset c(E)$
   c) $c(E) = E$ d) $E' = c(E)$

2) In a T-space $< X, \Im >, i(E)$ is _______.
   a) E'^{-} b) $E'^{-\prime}$
   c) $E^{-\prime}$ d) E^{-}

3) In any T-space $< X, \Im >, b(E) =$ _______.
   a) $X - i(E)$ b) $X - i(X - E)$
   c) $X - e(E)$ d) $X - [i(E) \cup e(E)]$

4) In a discrete T-space $< X, \Im >$,, every subset of X is _______.
   a) an open set b) a closed set
   c) both open and closed d) neither open nor closed

5) A co-countable topology on a finite set X reduces to _______.
   a) discrete topology b) indiscrete topology
   c) co-countable topology d) usual topology

6) In any T-space $< X, \Im >$, consider the following two statements.
   $\mathrm{P}: i(A \subset A)$ $\mathrm{Q}: A \subset i(A)$. Then,
   a) Only statement P is true b) Only statement Q is true
   c) both P and Q are true d) both P and Q are false

7) A discrete T-space $< X, \Im >$ is connected if $|X| =$ _______.
   a) 1 b) 2
   c) 3 d) ∞

8) Every homeomorphism between any two T-spaces is _______.
   a) a bijective map b) an open map
   c) a closed map d) all of the above

B) State whether true or false. **04**

1) In any co-finite T-space $< X, \mathfrak{I} >$, a subset A of X is open if $X - A$ is finite.
2) In any T-space, finite intersection of closed sets is closed.
3) Every continuous map between two T-spaces is a closed map.
4) Every compact space is locally compact.

Q.2 Answer the following. (Any Six) **12**

a) Define derived set.

b) Define Hausdorff space.

c) Prove that closed subset of a locally compact space is locally compact.

d) Prove that in any T-space $< X, \mathfrak{I} >$, prove that $x \in d(A) \Rightarrow x \in d(A - \{x\})$, where $A \subset X$.

e) If $< X, \mathfrak{I} >, < X, \mathfrak{I}^* >$ are two T-spaces and if $i: X \to X$ is an identity function. Then prove that i is an open mapping iff $\mathfrak{I}^* \geq \mathfrak{I}$.

f) Define compact space and connected space.

g) If $< X, \mathfrak{I} >$ is a discrete T-space and $< X^*, \mathfrak{I}^* >$ is any T-space, then prove that any function $f: X \to X^*$ is continuous function.

h) Define second axiom space.

Q.3 Answer the following. (Any Three) **12**

a) If $< X, \mathfrak{I} >$ and $< X^*, \mathfrak{I}^* >$ are two T-spaces and $f: X \to X^*$ is a function, then prove that f is an open map iff $f[i(E)] \subseteq i^*[f(E)]$, for any $E \subseteq X$.

b) In any T-space $< X, \mathfrak{I} >$, prove that $d(A \cup B) = d(A) \cup d(B)$ for any two sets $A, B \subset X$.

c) Let $p \in X$ be any fixed element. Define $\mathfrak{I} = \{\emptyset\} \cup \{A \subset X | p \in A\}$ is a topology on X.

d) Show that continuous image of a Lindelof space is a Lindelof space.

Q.4 Answer the following. (Any Two) **12**

a) In any separable space, prove that any countable family of mutually disjoint open sets is countable.

b) If $< X, \mathfrak{I} >$ and $< \mathbb{R}, \mathfrak{I}_u >$ are T-spaces and $E \subseteq X$. Then the characteristic function $\chi_E: X \to \mathbb{R}$ is continuous on X iff E is both open and closed.

c) If $< X, \mathfrak{I} >$ and $< X^*, \mathfrak{I}^* >$ are two T-spaces and $f: X \to X^*$ is a function, then prove that f is continuous iff inverse image of every closed set in X^* is a closed set in X.

Q.5 Answer the following. (Any Two) **12**

a) If $X = \{a, b, c, d, e\}, \mathfrak{I} = \{\emptyset, \{a, b\}, \{c, d\}, \{e\}, \{a, b, e\}, \{c, d, e\}, \{a, b, c, d\}, X\}$, then find $d(\{a, b, d\})$.

b) Prove that being a T_2 space is a topological property.

c) Prove that being a countably compact space is a topological property.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: March/April - 2025
Complex Analysis (2317207)

Day & Date: Tuesday, 20-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) If f has a simple pole at $z = a$, then $f(z) = \frac{g(z)}{z-a}$ gives Res $(f; a) =$ _____.
a) $g'(a)$
b) $\frac{g'(a)}{z-a}$
c) $\frac{g(a)}{2a}$
d) $g(a)$

2) If $f: C \to C$ defined by $f(z) = z^2 + 1$ is an analytic function then the set of zeros of the function f is _____.
a) C
b) $\{0, i, -i\}$
b) $\{i, -i\}$
d) $\{0,1,-1\}$

3) Every mobius transformation have _____ fixed point.
a) At least one
b) at most one
c) at most two
d) at least two

4) Critical points of $w = \frac{az+\beta}{yz+\delta}, \alpha\delta - \beta\gamma \neq 0$ are _____.
a) $-\frac{\delta}{\gamma}$
b) $-\frac{\delta}{\gamma}$ and 0
c) $-\frac{\delta}{\gamma}$ and ∞
d) ∞ and 0

5) The pole of function $f(z) = \frac{\cos z}{\sin z}$ are at _____.
a) $\frac{(2n+1)\pi}{2}, n$ is any integer
b) $\frac{2n\pi}{3}, n$ is any integer
c) $n\pi, n$ is any integer
d) $z = 0$

6) The bilinear transformation which maps the points $z = 1, z = 0$, $z = -1$ of z-plane into $w = i, w = 0, w = -i$ of w-plane respectively is _____.
a) $w = z$
b) $w = iz$
c) $w = i(z + 1)$
d) $w = z + 2$

7) Which of the following functions does represent the series $\sum_{n=0}^{\infty} \frac{z^n}{n!} for\ |z| < \infty$?

a) $\sin z$ b) $\cos z$

c) e^z d) $\frac{e^z}{n!}$

8) A polygon with three sides is called _____.

a) Circle b) Simple curve

c) Triangular path d) Open set

B) Fill in the blanks. **04**

1) Singularities of rational functions are _____.

2) A polynomial with no zeros in C is a _____ polynomial.

3) The fixed points of the mapping $f(z) = \frac{2iz+5}{z-2i}$ are _____.

4) If $s(z) = \frac{z+2}{z+3}$ then $S^{-1}(z)$ is _____.

Q.2 Answer the following. (Any Six) **12**

a) Define residue at infinity.

b) Find the fixed points of $f(z) = \frac{3z+2}{2-4z}$

c) Find Laurent series expansion of $\frac{1}{z^2-3z+2}$ for $|z| > 2$

d) Define the following terms:

i) Removable singularity

ii) Residue of an analytic function

e) Show that a Mobius map is uniquely determined by its action on any three distinct points in C_∞.

f) State Morera's Theorem.

g) Show that the function $e^{\frac{-1}{z^2}}$ has no singularities.

h) Find all the zeros of $f(z) = \sin z$ and $g(z) = \cos z$

Q.3 Answer the following. (Any three). **12**

a) Write a note on Translation and Rotation mapping.

b) Calculate residue of $\frac{z^2}{(z-1)(z-2)^2}$

c) If $\gamma: [0,1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that, $\frac{1}{2\pi!}\int_\gamma \frac{dz}{z-a}$ is an integer.

d) State and prove Liouville's theorem.

Q.4 Answer the following. (Any two) **12**

a) If f has an isolated singularity at $z = a$ and $\lim_{z \to a}(z - a)f(z) = 0$ then prove that the point $z = a$ is removable singularity of f.

b) If f is analytic in the disk $B(a, R)$ and suppose that γ is a closed rectifiable curve in $B(a, R)$ then prove that $\int_\gamma f = 0$.

c) Evaluate $\int_0^{2\pi} \frac{1}{1+a\sin\theta} d\theta (-1 < a < 1)$

Q.5 Answer the following. (Any two) **12**

a) State and prove Argument Principle.

b) If z_1, z_2, z_3, z_4 are the four distinct points in C_∞ then prove that the cross ratio (z_1, z_2, z_3, z_4) is real iff all four points lie on a circle or straight line.

c) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination: March/April - 2025
Functional Analysis (2317301)

Day & Date: Thursday, 15-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All Questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Multiple choice questions. **08**

1) Consider the statements.
   I) Every finite dimensional normed linear space is a Banach space.
   II) Every Banach space is finite dimensional linear space.
   a) only I is true b) only II is true
   c) both are true d) both are false

2) In a normed linear space, a non-zero vector x can be converted to unit vector by _____.
   a) $\frac{x}{\|x\|}$ b) $\frac{x}{|x|}$
   c) $\|x\|$ d) None of these

3) If N and N' are normed linear spaces and $T: N \to N'$ then graph of T is gives as $T_G =$ ______.
   a) $\{(x, T(x))/x \in N'\}$ b) $\{(x, T(x))/x \in N\}$
   c) $\{(x, T(x))/x \in T\}$ d) $\emptyset$

4) In a quotient space $\frac{N}{M}$, the addition is defined as $(x + M) + (y + M) =$ _____.
   a) $x + y + 2M$ b) M
   c) $x + y + M$ d) None of these

5) A non-empty subset of a Hilbert space H is said to be an orthonormal set if it contains ______.
   a) orthogonal unit vectors
   b) mutually orthogonal vectors
   c) mutually orthogonal unit vectors
   d) None of these

6) A linear transformation E on a linear space L into itself is called idempotent ______.
   a) $E^{\perp} = E$ b) $E = E'$
   c) $E^2 = E$ d) None of these

7) The set of bounded linear transformation $B(X,Y)$ is complete if ____.
   a) X is open b) Y is closed
   c) X is complete d) Y is complete

8) If H is a Hilbert space then the conjugate space H^* is also a Hilbert space with the inner product defined by, $\langle F_x, F_y \rangle =$ _____.
   a) $\langle x, y \rangle$ b) $\langle x, 0 \rangle$
   c) $\langle y, x \rangle$ d) $\langle 0, y \rangle$

B) Fill in the blanks. **04**

1) An operator N is said to be normal operator if it commutes with Its ______.
2) On finite dimensional spaces, all norms are ______.
3) A normed linear space X is said to be complete if every cauchy sequence is _____ in X.
4) An operator T on Hilbert space is said to be self adjoint if _____.

Q.2 Answer the following. (Any Six) **12**

a) Define Inner Product and Inner Product Space.

b) Prove that: $\|x+y\|^2 - \|x-y\|^2 = 4.$ Real part of $\langle x, y \rangle$.

c) Prove that: Every closed subspace of Banach space is complete.

d) Define Banach space with one example.

e) With usual notation prove that: $d(\alpha x, \alpha y) = \alpha d(x,y)$ for all $x, y \in V$, $\alpha \in F$.

f) Prove that: Every convergent sequence in a normed linear space is Cauchy.

g) State and prove Pythagoras theorem.

h) Define Orthogonal vectors and orthogonal complement of set.

Q.3 Answer the following. (Any three) **12**

a) If X is a normed linear space over the field F and M is closed subspace of X, define $\| . \|_1 : \frac{X}{M} \to R$ by $\| . \|_1 = in\, f\{\|x+m\| / m \in M\}$ then prove that $\| . \|_1$ is a norm on $\frac{X}{M}$.

b) Prove that $B(X,Y)$ is subspace of $L(X,Y)$.

c) State and Prove Riesz Lemma.

d) If $S(x,r)$ be an open sphere in B with centre at x and radius r, S_r is the open with centre at origin and radius r then prove the following results,

1) $S(x,r) = x + S(0,r)$ or $x + S_r$
2) $S_r = r.S_1$ or $S(0,r) = rS(0,r)$

Q.4 Answer the following. (Any two). **12**

a) If X is an inner product space, then prove that $||x|| = < x, x >^{\frac{1}{2}}$ is a norm on $X.$

b) If x and y are two vectors in a Hilbert space then prove that $4 < x, y >= \|x + y\|^2 - \|x - y\|^2 + i\|x + iy\|^2 - i\|x - iy\|^2$.

c) If M be a linear subspace of a Hilbert space H then M is closed if and only if $M = M^{\perp\perp}$.

Q.5 Answer the following. (Any two) **12**

a) If M be a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$.

b) If a Banach space B is direct sum of linear space M and N , $z = x + y$ be unique representation of vector $z \in B$ where $x \in M,\ y \in N,$ define the function $\| . \|_1$ on B by $\|z\|_1 = \|x\| + \|y\|$ then show that $\| . \|_1$is a norm on B and $< B, \| . \|_1 >$ is a Banach space if M and N are closed subspaces of B.

c) If y be a fixed vector in a Hilbert space H and f_y be a function defined as, $f_y(x) =< x, y >$ for every $x \in H$ then prove that f_y is functional on H and $\|y\| = \|f_y\|$.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination: March/April - 2025
Linear Algebra (2317302)

Day & Date: Saturday, 17-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. (MCQ) **08**

1) If $f: \mathbb{R}^2 \to \mathbb{R}$ is a linear functional defined by $f(x, y) = x + 4y, \forall (x, y) \in \mathbb{R}^2$. Then $rank(f) =$
   a) 0 b) 1
   c) 2 d) 3

2) If V is a finite-dimensional vector space and $T : V \to V$ is a linear transformation, f, m denote the characteristic and minimal polynomial, then which of the following is true?
   a) m divides f b) f divides m
   c) $f = m$ d) None of these

3) The characteristic values of the matrix $\begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix}$ is
   a) $1,0$ b) $1, -1$
   c) $2, -2$ d) $2, 1, 0$

4) If V is a vector space and $T : V \to V$ is a linear transformation, then which of the following subspaces of V are invariant under T?
   a) $\{0\}$ b) V
   c) $N(T)$ d) All of the above

5) If E is a projection defined on a vector space V, then $I - E$
   a) is also a projection map b) is a zero map
   c) is an identity map d) is not a linear transformation

6) If the vector $\alpha \in V$ is a cyclic vector for a transformation T defined on a vector space V, then
   a) $Z(\alpha; T) = \{0\}$ b) $Z(\alpha; T) = \alpha$
   c) $Z(\alpha; T) = V$ d) None of the above

7) If V is a vector space, then $\{0\}^{\perp} =$ ______.
   a) $\{0\}$ b) V
   c) singleton set d) proper non-trivial subspace

8) If V is an inner product space, then for any $\alpha, \beta \in V, < \alpha \backslash c\beta >=$

a) $< \beta \backslash \alpha >$ b) 0

c) $\bar{c} < \beta \backslash \alpha >$ d) $\bar{c} < \alpha \backslash \beta >$

B) Write true/ False. **04**

1) Every linear functional defined on a vector space is a linear transformation.
2) If V is a vector space of dimension n, then any subspace of V of dimension $n-2$ is known as a hyperspace.
3) A form f is said to be Hermitian if $f(\alpha, \beta) = \overline{f(\beta, \alpha)}$.
4) Nilpotent matrix is never diagonalizable over the field $\mathbb{F}$.

Q.2 Answer the following. (Any Six) **12**

a) Define linear functional.

b) If V is a vector space and W is its subspace of V, then prove that $W^{\perp}$ is a subspace of V.

c) Define a characteristic value and characteristic vector of a linear transformation.

d) Find the characteristic values of the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

e) If V is a vector space and $T, U : V \to V$ are linear transformations such that $TU = UT$, then prove that $N(U)$ is invariant under T.

f) Define Unitary operator.

g) Define diagonalizable operator.

h) Define similar matrices.

Q.3 Answer the following. (Any Three) **12**

a) If V is an inner product space then prove that

$< \alpha | \beta >= \frac{1}{4} \| \alpha + \beta \|^2 + \frac{1}{4} \| \alpha - \beta \|^2$

b) If S is a non-empty subset of a finite-dimensional vector space V, then $(S^{\circ})^{\circ}$ is the subspace spanned by S.

c) Prove that any orthonormal set of non-zero vectors is linearly independent.

d) If T is a linear transformation from an inner product space V to V, then prove the following that T preserves the norm implies T preserves the inner product.

Q.4 Answer the following. (Any Two) **12**

a) Consider the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ Prove that A is diagonalizable over $\mathbb{R}$.

b) If V is a finite-dimensional vector space over the field $\mathbb{F}$ and if W is a subspace of V, then prove that $dimW + dimW^{\circ} = dimV$.

c) If V, W are vector spaces and $T: V \to W$ is a linear transformation, then prove that $T\alpha = c\alpha \Longrightarrow f(T)\alpha = f(c\alpha)$.

Q.5 Answer the following. (Any Two) **12**

a) Let V be a finite dimensional inner product space. If T, U are linear operators on V and c is a scalar, then prove that

a) $(T + U)^* = T^* + U^*$

b) $(cT)^* = \bar{c}T^*$

c) $(TU)^* = U^*T^*$

b) For any linear operator T on a finite-dimensional inner product space V, prove that there exists a unique linear operator T^* on V such that

$< T\alpha|\beta >=< \alpha|T^*\beta >$for all $\alpha, \beta \in V$.

c) Orthonormalize the set $\{(1, 0, 1), (0, 1, 1), (1, 3, 3)\}$ in $\mathbb{R}^3$ equipped with standard inner product.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (New) (NEP CBCS) Examination: March/April - 2025
Advanced Discrete Mathematics (2317306)

Day & Date: Monday, 19-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) The least and the greatest elements of a lattice are called _____.
a) bounds b) universal bounds
c) units d) All of these

2) If A and B are finite sets then $|A - B| =$ _____.
a) $|A| - |A \cap B|$ b) $|A| + |B| + |A \cap B|$
b) $|B| - |A \cap B|$ d) $|A| + |B|$

3) The complete bipartite graph $K_{m,n}$ is isomorphic to _____.
a) K_{m+n} b) K_{mn}
c) K_{m-n} d) $K_{n,m}$

4) A graph G is called 1-connected if _____.
a) G is connected
b) G is complete
c) G has spanning tree
d) G is connected with at least two vertices

5) A chain that is not a subset of larger chain is called _____
a) antichain b) length of a chain
c) maximal chain d) isthmus

6) Consider the statement _____.
I) There is no horizontal line in a Hasse diagram of a POSET.
II) $(N, >)$is not a POSET.
a) Only I is true b) Only II is true
c) Both I and II are true d) Both I and II are false

7) The recurrence $T(n) = 2T(n-1) + n$ for $n \geq 2$ and $T(1) = 1$ evaluates to _____.
a) $2^{n+1} - n - 2$ b) $2^n - n$
c) $2^n + n$ d) $2^{n+1} - 2n - 2$

8) The adjacency matrix is _____ about its diagonal.
a) unitary b) orthogonal
c) skew-symmetric d) symmetric

B) Fill in the blanks. **04**

1) The edges of a graph G which are not in spanning tree are called as _____.
2) The edges e and f which connects the same end points are called _____.
3) The order and degree of the recurrence relation $a_n = 2a_{n-1}$ is _____.
4) An expression for geometric series $(1 + x)^n$

Q.2 Answer the following. (Any Six) **12**

a) Define Spanning subgraph and complement of graph.
b) Show that the pentagonal lattice is not modular.
c) If G be a connected graph then show that G is tree iff every edge of G is bridge.
d) Find the complement of each element of D_{42}.
e) Show that the number of permutations of n different things taken r at a time, when things may be repeated any number of times is n^r.
f) Draw a tree with 10 Vertices.
g) What is recurrence relation.
h) What is the smallest integer n such that the complete graph K_n has at least 500 edges.

Q.3 Answer the following. (Any three). **12**

a) Given any two vertices u and v of a graph G, then prove that every $u - v$ walk contain a $u - v$ path.
b) Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?
c) Show that an acyclic graph with n vertices is tree iff it contains precisely $(n - 1)$ edges.
d) If $(L,\vee,\wedge)$ is a distributive lattice then show that if an element has a complement, then this complement is unique

Q.4 Answer the following. (Any two) **12**

a) Find the closed form of generating function of
i) $1, (1 + 2), (1 + 2 + 3), (1 + 2 + 3 + 4) - - -$
ii) $1^2, (1^2 + 2^2), (1^2 + 2^2 + 3^2) - - -$
b) Show that a graph G is connected if and only if given any pair u and v of vertices there is path from u to v.
c) Show that a graph G is connected if and only if it has a spanning tree.

Q.5 Answer the following. (Any two) **12**

a) State and prove Bridge Theorem.

b) Define non-homogeneous recurrence relation and solve

$y_n - 7y_{n-1} + 12y_{n-2} = n4^n$

c) Define finite Boolean algebra and show that D_{42} is a finite Boolean algebra under partial order of Divisibility.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (Old) (CBCS) Examination: March/April - 2025
Functional Analysis (MSC15301)

Day & Date: Thursday, 15-May-2025 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question 1 & 2 is compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If N and N' are normed linear spaces and $T : N \to N'$ then graph of T is gives as T_G = _____.
a) $\{(x, T(x))/x \in N\}$ b) $\{(x, T(x))/x \in N'\}$
c) $\{(x, T(x))/x \in T\}$ d) $\emptyset$

2) A projection E on a linear space L determines two linear subspaces M and N such that L = _____.
a) $M + N$ b) $M \cup N$
c) $M \oplus N$ d) $M \cap N$

3) A subset S of a normed linear space $\langle X, \|\cdot\|\rangle$ is bounded if there exist a positive constant K such that _____ for all $x \in S$
a) $\|x\| \leq K$ b) $\|x\| \geq K$
c) $\|x\| < K$ d) $\|x\| > K$

4) In a normed linear space, the triangular inequality property is given as,
a) $\|x + y\| \leq \|x\| + \|y\|$ b) $\|x + y\| \geq \|x\| + \|y\|$
c) $\|x + y\| = \|x\| + \|y\|$ d) $\|x - y\| \leq \|x\| - \|y\|$

5) By Schwartz' inequality, If x and y are two vectors in an inner product space then
$|\langle x, y\rangle|$ = _____.
a) $\|x\| + \|y\|$ b) $\|x\| \cdot \|y\|$
c) $\|x\| - \|y\|$ d) $|x| \cdot |y|$

6) In a Hilbert space, for any $x, y \in H$ the vectors x, y are said to be orthogonal if _____.
a) $\langle x, y\rangle \neq 0$ b) $\langle x, y\rangle = 0$
c) $\langle x, y\rangle \leq 0$ d) $\langle x, y\rangle \geq 0$

7) A continuous linear transformation $T : N \to N'$ is said to be open mapping if for every open set G in $N, T(G)$ is _____ in N'.

a) closed b) bounded
c) open d) finite

8) Consider the following statements:

I) Every Cauchy sequence in normed linear space is convergent.
II) Every convergent sequence in normed linear space is Cauchy.

a) only I is true b) only II is true
c) both are true d) both are false

9) A linear transformation E on a linear space L into itself is called idempotent _____.

a) $E^{\perp} = E$ b) $E = E'$
c) $E^2 = E$ d) none of these

10) A normed linear space X is said to be complete if every cauchy sequence is _____ in X.

a) divergent b) finite
c) bounded d) convergent

B) Fill in the blanks. **06**

1) In the set of all bounded linear transformations $B(X, Y)$ the scalar multiplication is defined as $(\alpha.T)(x)$ =_____.
2) In Hilbert space X, with usual notations, $\langle x, y + z\rangle$ = _____.
3) If $T : X \to Y$ is a linear transformation and T is bounded then T maps bounded sets in X into _____ sets in Y.
4) A non-empty subset of a Hilbert space H is said to be an orthonormal set if it contains
5) On finite dimensional spaces, all norms are _____.
6) The set of bounded linear transformation $B(X, Y)$ is complete if _____.

Q.2 Answer the following question **16**

a) Show that $|\|x\| - \|y\|| \le \|x - y\|, \forall x, y \in V$
b) State and prove Pythagorean theorem.
c) Prove that : Every complete subspace of normed linear space is closed.
d) Define orthogonal vectors and orthogonal complement.

Q.3 Answer the following question **16**

a) If X is a complex IPS then Prove that:

i) $< ax - by, z >= a < x, z > -b < y, z >$
ii) $< x, ay + bz >= \bar{a} < x, y > +\bar{b} < x, z >$
iii) $< x, ay - bz >= \bar{a} < x, y > -\bar{b} < x, z >$
iv) $< x, 0 >= 0$ and $< 0, x >= 0, \forall x, \in X$

b) If H is a Hilbert space then prove that H^* is also Hilbert space with the inner product defined by $< f_x, f_y >=< y, x >$

Q.4 Answer the following question **16**

a) If x and y are two vectors in a Hilbert space then prove that
$4 < x, y >= \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$

b) If M be a linear subspace of a Hilbert space H then prove that M is closed if and only if $M = M^{\perp\perp}$

Q.5 Answer the following question **16**

a) If $T : X \to Y$ be any linear transformation then prove that T is Continuous on X if and only if T bounded on $X.$

b) Prove that $B(X, Y)$ is normed linear space where,
$$\|T\| = \sup\{\|T(x)\|: x \in X, \|x\| \leq 1\}$$

Q.6 Answer the following question **16**

a) If P is projection on Banach space B and If M and N are its range nd null spaces respectively then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.

b) If X is an inner product space, then prove that $\|x\| =< x, x >^{\frac{1}{2}}$ is a norm on X

Q.7 Answer the following question **16**

a) State and Prove Riesz Lemma.

b) Show that the real linear space and complex linear space are Banach spaces under the norm, $\|x\| = |x|, x \in \mathbb{R}\ or\ \mathbb{C}$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (Old) (CBCS) Examination: March/April - 2025
Advanced Discrete Mathematics (MSC15302)

Day & Date: Saturday, 17-May-2025 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question No.1 and 2 are compulsory.
2) Attempt any 3 questions from Q. No.3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternatives: **10**

1) A graph with n vertices will definitely have a parallel edge self-loop if the total number of edges are _____.
a) more than n b) more than n+1
c) more than (n+1)/2 d) than n(n-1)/2

2) In any lattice L, which of the following is true?
a) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
b) $a \wedge (b \vee c) \leq (a \wedge b) \vee (a \wedge c)$
c) $a \wedge (b \wedge c) \geq (a \wedge b) \vee (a \vee c)$
d) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

3) The relation $\{(1,2), (1,3), (3,1), (1,1), (3.3), (3,2), (1,4), (4,2), (3,4)\}$ is _____.
a) Reflexive b) Symmetric
c) Transitive d) None of these

4) For any connected graph G,
a) $rad\ (G) \leq 2\ rad\ (G)$ b) $rad\ (G) \leq diam\ (G)$
c) $diam\ (G) < 2\ rad\ (G)$ d) All of these

5) How many different words can be formed out of the letters of word VARANASI?
a) 64 b) 120
c) 40320 d) 720

6) A tree contains an ____
a) pedant vertex b) Loop
c) isolated vertex d) Parallel edges

7) A graph which consists of disjoint union of trees is called ____.
a) bipartite graph b) Forest
c) complete bipartite graph d) Spanning tree

8) In how many ways can 5 balls be chosen so that 2 are red and 3 are black?

a) 910 b) 990
c) 970 d) 124

9) Which of the following is not correct regarding lattice?

a) $[\{1,2,3,6,9,18\},/\ ]$ is abounded lattice
b) $[1, \leq]$ is not a bounded lattice, where I am the set of integers
c) $[(0,1), <]$ is bounded lattice
d) $[[0,1\ ], <]$ is bounded lattice

10) The complete graph with four vertices has k edges where k is ____.

a) 3 b) 4
c) 5 d) 6

Q.1 B) Write true/false. **06**

1) equal to (=) relation is a relation which is both partial order as well as an equivalence relation.
2) The length of a walk in a graph is total number of edges in a graph.
3) If G be a tree with 71 vertices then it has precisely 72 edges.
4) A Poset in which every pair of elements has both a least upper bound and a greatest lower bound is termed as Chain.
5) The characteristic equation of $a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$ is $r^3 - 8r^2 + 21r - 18 = 0$
6) The edges of a graph G which are not in spanning tree are called as branches.

Q.2 Answer the following. **16**

a) State and prove Hand shaking lemma.
b) If T is a tree with n vertices then prove that T has precisely $n-1$ edges.
c) If $P(11, n) = P(12, n-1)$ then find n.
d) Prove that a non-empty finite partially ordered set has
  i) at most one greatest element
  ii) at most one least element

Q.3 Answer the following: **16**

a) Write a short note on matrix representation of graph with two examples.
b) If u and v are any two vertices of a graph G then show that every u-v walk contains a u-v path.

Q.4 Answer the following: **16**

a) Write a short note on Hasse diagram of the Poset. Draw the Hasse diagram of the Poset $(P(S), \subseteq)$ where $P(S)$ is the power set on $S = [1,2,3,4\}$
b) Show that a graph G is connected if and only if it has a spanning tree.

Q.5 Answer the following: **16**

a) State and prove Bridge Theorem.

b) If $A \precsim_1)$ and $(B, \precsim_2)$ are Posets then show that $(A \times B, \precsim)$ is a Poset with partial order defined by.
$(a, b) \precsim (a', b')$ if $a \precsim_1 a'$ in A and $b \precsim_2 b'$ in B

Q.6 Answer the following: **16**

a) Define homogeneous and non-homogeneous recurrence relation and Solve $y_n = y_{n-1} + y_{n-2}, n \geq 2$ with the initial condition $f_0 = 0, f_1 = 1$

b) Write a note on combination of things not all different.

Q.7 Answer the following: **16**

a) Show that in a complemented distributive lattice, the followings are equivalent:
- i) $a \precsim b$
- ii) $a \wedge b' = 0$
- iii) $a' \vee b = 1$
- iv) $b' \leq a'$

b) Define the following terms with examples
- i) Complete graph
- ii) Regular graph
- iii) Bipartite graph
- iv) Connected graph

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - III) (Old) (CBCS) Examination: March/April - 2025
Linear Algebra (MSC15303)

Day & Date: Monday, 19-May-2025 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) If $f: \mathbb{R}^2 \to \mathbb{R}$ is a linear functional defined by $f(x, y) = x + y, \forall (x, y) \in \mathbb{R}^2$. Then $N(f) =$ _____.
a) $span\,\{(1,1)\}$ b) $span\,\{(1,-1)\}$
c) $span\,\{(2,1)\}$ d) $span\,\{(1,3)\}$

2) A column rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$ is _____.
a) 0 b) 1
c) 2 d) 3

3) Which of the following functional annihilates $S = \{(1,-2)\} \subset \mathbb{R}^2$.
a) $f(x, y) = 1$ b) $f(x, y) = 2x + y$
c) $f(x, y) = x - y$ d) $f(x, y) = x$

4) The subspaces W_1 and W_2 of a vector space V are such that $V = W_1 \oplus W_2$. Then,
a) $V = W_1 + W_2, W_1 + W_2 = \{0\}$
b) $V = W_1 + W_2, W_1 \cup W_2 = \{0\}$
c) $V = W_1 + W_2, W_1 \cap W_2 = \{0\}$
d) $V = W_1 + W_2, W_1 \cup W_2 = V$

5) If V is a 4-dimensional vector space, then dimension of its double dual space V^{**} is _____.
a) 1 b) 2
c) 3 d) 4

6) A form on complex vector space V is called Hermitian if $\forall_{\alpha}\beta \in V$
a) $f(\alpha,\beta) = \overline{f(\beta,\alpha)}$ b) $f(\alpha,\beta) = f(\beta,\alpha)$
c) $f(\alpha,\beta) = \overline{-f(\beta,\alpha)}$ d) $f(\alpha,\beta) = -f(\beta,\alpha)$

7) A finite dimensional complex inner product space is known as ____.
   a) Euclidean space b) Unitary space
   c) Normal space d) none of these

8) If $A = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, then the smallest integer k such that $A^k = 0$ is ____.
   a) 1 b) 2
   c) 3 d) 4

9) If E is a projection map, then _____.
   a) $E^2 = 0$ b) $E^2 = I$
   c) $E^2 = -E$ d) $E^2 = E$

10) Which of the following set is not an orthogonal subset of $\mathbb{R}^2$.
   a) $\{(1,1),(1,-1)\}$ b) $\{(1,2),(-2,1)\}$
   c) $\{(3,-1),(2,1)\}$ d) $\{(1,3),(-3,1)\}$

B) State whether true or false: **06**

1) Rank of any linear functional can be either 0 or 1.
2) Two subspaces W_1, W_2 of a vector space V are independent if $\alpha_1 + \alpha_2 = 0, \alpha_1 \in W_1, \alpha_2 \in W_2 \Rightarrow \alpha_1 = 0$ or $\alpha_2 = 0$.
3) T-cyclic subspace generated by α is 1-dimensional iff *a* is a characteristic vector of T
4) If E is a projection map and I is an identity operator, then $(I - E)$ is an idempotent operator.
5) Every triangulable operator is diagonalizable.
6) In any inner product space $< V, ||\cdot|| >$, for any vector $\alpha \in V, ||\alpha|| \geq 0$

Q.2 Answer the following. **16**

a) If W_1, W_2 are subspaces of a finite-dimensional vector space, then prove that $W_1, = W_2$ iff $W_1^\circ = W_2^\circ$.

b) If V, W are vector spaces over the field $\mathbb{F}$ and if $T: V \to W$ is a linear transformation, then prove that the null space of T^t is the annihilator of range of T. Further, if V, W are finite-dimensional, then prove that $rank\ (T^t) = rank\ (T)$.

c) Find the characteristic polynomial and characteristic roots for a matrix

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

d) If V is a vector space and $T: V \to V$ is a linear transformation, then prove that $N(T), R(T)$ are invariant under T.

Q.3 Answer the following.

a) If f_1, f_2, f_3 functionals defined on $\mathbb{R}^4$ given by _____. **08**

$f_1(x, y, z, t) = x + 2y + 2z + t$

$f_2(x, y, z, t) = 2y + t$

$f_3(x, y, z, t) = -2x - 4z + 3t$

then find the subspace of $\mathbb{R}^4$ annihilated by f_1, f_2, f_3

b) Let A be any $m \times n$ matrix over the field $F.$ Then, prove that row rank of A is equal to the column rank of $A.$ **08**

Q.4 Answer the following.

a) If T is a linear operator on the finite-dimensional space $V, c_1, c_2, \cdots, c_k$ are the distinct characteristic values of T and if W_i are the spaces of characteristic vectors associated with the characteristic value c_i then if $W = W_1 + \cdots + W_k$ then prove that dim $W = \dim W_1 + \cdots + \dim W_k$. Also, if $\mathfrak{B}_i$ is an ordered basis for W_i, then prove that $\mathfrak{B} = (\mathfrak{B}_{1,} \cdots, \mathfrak{B}_r)$. **08**

b) Let $< V, || \cdot || >$ be a real normed linear space. Then, prove that **08**

$< \alpha|\beta >= \frac{1}{4}\|\alpha + \beta\|^2 - \frac{1}{4}\|\alpha - \beta\|^2$

Q.5 Answer the following.

a) Consider the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ Prove that A is diagnoalizable over $\mathbb{R}$ and find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix. **08**

b) Find the rational canonical form for the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ **08**

Q.6 Answer the following.

a) Define Hermitian form. If V is a complex vector space and f is a form on V such that $f(\alpha, \alpha)$ is real for every $\alpha.$ Then, prove that f is Hermitian. **08**

b) Find the minimal polynomial for the matrix $A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ **08**

Q.7 Answer the following.

a) Define normal operator. Let V be a finite dimesnional inner product space. If T, U are linear operators on V and c is a scalar, then prove that **08**

i) $(T + U)^\star = T^\star + U^\star$

ii) $(cT)^\star = \bar{c}T^\star$

iii) $(TU)^\star = U^\star T^\star$

iv) $(T)^\star = T$

b) Orthonormalize the set $\{(3,0,4), (-1,0,7), (2,9,11)\}$ in $\mathbb{R}^3$ equipped with standard inner product. **08**

Seat No.	

Set P

M.Sc. (Semester - III) (Old) (CBCS) Examination: March/April - 2025
MATHEMATICS
Differential Geometry (MSC15306)

Day & Date: Thursday, 22-May-2025 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. No. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If $V = xU_1 - y^2U_3, f = x^2y + z^3$ then $V[f] =$ _____.
   a) $2x^2y + z^3$ b) $2x^2y - 3y^2z^2$
   c) $4x^2y + y^2z^2$ d) $4x^2y + 6y^4z$

2) If β is a reparametrization of α by h then _____.
   a) $\beta'(s) = h'(s)$ b) $\beta'(s) = h'(s)\,\alpha(h(s))$
   c) $\beta'(s) = h'(s)\,\alpha'(h(s))$ d) $\beta'(s) = h(s)\,\alpha(h(s))$

3) The regularity condition $\frac{d\alpha}{dt} \neq 0, \forall t \in I$ implies that the set $\left\{\frac{d\alpha}{dt}\right\}$ is ______.
   a) Linearly dependent b) Linearly independent
   c) Empty d) Singleton

4) The arc length of the circle $\alpha(t) = (a\cos t, a\sin t, 0); 0 \leq t \leq 2\pi$ is _____.
   a) 2π b) $2\pi a$
   c) 0 d) 1

5) If a unit speed curve has curvature identically zero then it is a ____.
   a) Circle b) Ellipse
   c) Triangle d) Straight line

6) The set of all tangent vectors in E^3 is denoted by ______.
   a) $U_{p\in E^3} T_p(E^3)$ b) $T_p(E^3)$
   c) T_p d) All of these

7) If φ and Ψ are 1-forms then $\varphi \wedge \Psi =$ ______.
   a) $\Psi \wedge \varphi$ b) $-\Psi \wedge \varphi$
   c) $-\Psi \wedge -\varphi$ d) $-\varphi \wedge \varphi$

8) If v and w be tangent vectors at the same point p then $v \times w$ is orthogonal to _____.
   a) v b) w
   c) Both a and b d) None of these

9) The one-one condition _____.
   a) smoothness
   b) linearity
   c) proper patch
   d) avoids cutting of the surface

10) If $v_p = (v_1, v_2, v_3)$ is a tangent vector to E^3 at a point p then $v_p[f] =$ _______.
   a) $\sum_i v_i \frac{\partial f}{\partial x_i}(p)$
   b) $\sum_i v_i(p)$
   c) 0
   d) $\frac{\partial f}{\partial x_i}(p)$

B) Write true/ False. **06**

1) Parametric equations of a curve are unique.
2) A curve can have different speed through reparametrization.
3) For any 2-form $d^2w = 0$
4) A vector of norm one is called unit vector.
5) The curvature and torsion influence the shape of the curve.
6) A regularity condition says that the speed is always non-zero.

Q.2 Answer the following. **16**

a) Show that Rotation is an orthogonal transformation.

b) Find the directional derivative $\overline{v_p}\,[f]$ when $\bar{v} = (2,-1,3), p = (2,0,-1)$ for
   i) $f = y^2 z$
   ii) $f = e^x \cos y$

c) Define:
   i) Tangent vector in E^3
   ii) Natural coordinates

d) Check $M: x^2 + y^2 + 3z^2 = 1$ is surface or not.

Q.3 Answer the following.

a) If v and w be tangent vectors at the same point p. Then prove that $v \times w$ is orthogonal to v and w and has length $\|v \times w\|^2 = (v \cdot v)(w \cdot w) - (v \cdot w)^2$ **08**

b) Prove that: **08**
   i) $\|p + q\| \leq \|p\| + \|q\|; p, q \in E^3$
   ii) $\|ap\| = |a|.\|p\|$

Q.4 Answer the following.

a) Prove that every isometry of E^3 can be uniquely described as orthogonal transformation followed by translation. **08**

b) Show that $M: Z = = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$ is a surface and $X(u,v) = (a\cos u \cos v,\ b \cos u \sin v,\ c \sin u)$ defined on D is a parametrization where $D: \frac{-\pi}{2} < u, v < \frac{\pi}{2}$. **08**

Q.5 Answer the following.

a) Show that a surface obtained by rotating a curve is a surface. **10**

b) For a patch $X: D \to E^3$, if $E = X_u . X_u, F = X_u . X_v, G = X_v . X_v$, then prove that X is regular iff $EG - F^2 \neq 0$. **06**

Q.6 Answer the following.

A) Prove that: **08**

i) If S and T are Translation then show that $ST = TS$ is also translation.

ii) If T is translation by a then T^{-1} is translation by $-a$.

B) Write a note on Reparametrization of a curve $\alpha(t)$. **08**

Q.7 Answer the following.

a) Prove that a mapping $X: D \to \mathbb{R}^3$ is regular iff $X_u \times X_v \neq 0, \forall (u, v) \in D$. **08**

b) Compute the Frenet apparatus for the curve $\alpha(t) = (e^t \cos t, e^t \sin t, e^t)$ **08**

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination: March/April - 2025
Partial Differential Equations (2317401)

Day & Date: Wednesday, 14-May-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) The problem of finding a harmonic function $u(x, y)$ in D such that it coinsides with f on boundary B is called _____.
a) Neumann problem b) Wave equation
c) Dirichlet problem d) Laplace equation

2) A set of those points of a 3-dimensional space which are expressed as function of two parameters is called a _____.
a) Surface b) Plane
c) Direction ratio d) Tangent to curve

3) Canonical form of $Z_{xx} - 6z_{xy} + 9z_{yy} + 2p + 3q - z = 0$ is _____.
a) $\frac{\partial^2 z}{\partial u \partial v} = \frac{z}{9} - \frac{\partial z}{\partial u} - \frac{1}{3}\frac{\partial z}{\partial v}$ b) $\frac{\partial^2 z}{\partial v^2} = \frac{z}{9} - \frac{\partial z}{\partial u} - \frac{1}{3}\frac{\partial z}{\partial v}$
c) $\frac{\partial^2 z}{\partial \alpha^2} + \frac{\partial^2 z}{\partial \beta^2} = \frac{\partial z}{\partial \alpha} + \frac{\partial z}{\partial \beta}$ d) $\frac{\partial^2 z}{\partial \alpha^2} - \frac{\partial^2 z}{\partial \beta^2} = \frac{\partial z}{\partial \alpha} - \frac{\partial z}{\partial \beta}$

4) The partial differential equation which represents the set of all right circular cones with z-axis as the axis of symmetry is _____.
a) $yp - xq = 0$ b) $yp + xq = 0$
c) $xp + yq = 0$ d) $xp - yq = 0$

5) Elimination of a function f from $z = f\left(\frac{y}{x}\right)$ gives a partial differential equation _____.
a) $x\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ b) $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$
c) $\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$ d) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

6) The Lagrange's auxiliary equation for the partial differential $P_p + Q_q = R$ is _____.

a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$
b) $\frac{dx}{P} = \frac{dy}{Q}$
c) $\frac{dx}{P} = \frac{dz}{R}$
d) $\frac{dy}{Q} = \frac{dz}{R}$

7) The equations $f(x,y,p,q) = 0$ and $g(x,y,p,q) = 0$ are compatible if _____.

a) $\frac{\partial(f,g)}{\partial(x,p)} + \frac{\partial(f,g)}{\partial(y,q)} = 0$
b) $\frac{\partial(f,g)}{\partial(x,p)} - \frac{\partial(f,g)}{\partial(y,q)} = 0$
c) $\frac{\partial(f,g)}{\partial(y,p)} + \frac{\partial(f,g)}{\partial(x,q)} = 0$
d) $\frac{\partial(f,g)}{\partial(y,p)} - \frac{\partial(f,g)}{\partial(x,q)} = 0$

8) The parametric equations of a curve and a surface are _____.

a) unique
b) same
c) not unique
d) None of these

B) Fill in the blanks. **04**

1) The integral surface of $z = p^2 - q^2$ which passes through the curve $4z + x^2, y = 0$ is $-2z = (x + \sqrt{2}y)^2$.
2) The characteristic curves for the equation $xz_y - yz_x = z$ are circle with Centre at origin.
3) The differential equation $f_{xx} + 2f_{xy} + 4f_{xy} = 0$ is of hyperbolic form.
4) In polar coordinates (r,θ), Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ changers to $\frac{\partial^2 u}{\partial r^2} - \frac{1}{r}\frac{\partial u}{\partial r} - \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$

Q.2 Answer the following. (Any Six) **12**

a) Find the general solution of $(x+1)p + (y+1)q = z$.
b) Show that the equations $xp = yq, z(xp + yq) = 2xy$ are compatible.
c) Define complete integral and singular integral.
d) Write a note on Neumann problem.
e) Find the partial differential equation which represents the set of all spheres with centres on the z-axis and of radius a.
f) Obtain the partial differential equation of first order by eliminating arbitrary constants from the relation $z = (x+a)(y+b)$.
g) Write a note on Cauchy problem.
h) What is integrability of Pfaffian differential equation.

Q.3 Answer the following. (Any Three) **12**

a) Define Interior and Exterior Dirichlet Problem.
b) Find the general solution of
$x(x+y)p - y(x+y)q + (x-y)(2x+2y+z) = 0$.
c) Find the partial differential equation satisfied by all the surfaces of the form $F(u,v) = 0$ where $u = u(x,y,z), v = v(x,y,z)$ and F is arbitrary function of u and v.

d) Describe Jacobi's method of solving a first order partial differential equation.

Q.4 Answer the following. (Any Two) **12**

a) Show that the Pfaffian differential equation $(6x + yz)dx + (xz - 2y)dy + (xy + 2z)dz = 0$ is integrable and hence find the corresponding integral.

b) Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is a complete integral of $z^2(1 + p^2 + q^2) = 1$ then by taking $b = 2a$ show that the subfamily is $(y - 2x)^2 + 5z^2 = 5$ which is a particular solution. Show further that $z = \pm 1$ are the singular integrals.

c) Find the condition that a one parameter family of surfaces forms a family of equipotential surfaces.

Q.5 Answer the following. (Any Two) **12**

a) Solve $x^2 u_x - {u_y}^2 - a{u_z}^2 = 0$ by Jacobis method.

b) Discuss how a general solution is used to determine the integral surface which passes through a given curve.

c) Reduce the equation $u_{xx} + x^2 u_{yy} = 0$ to a canonical form.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination: March/April - 2025
Integral Equations (2317402)

Day & Date: Friday, 16-May-2025 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) $\int_0^x y(t)dt^4 =$ ________.
   a) $\int_0^x y(t)dt$
   b) $\int_0^x \frac{(x-t)^2}{2} y(t)dt$
   c) $\int_0^x \frac{(x-t)^3}{3} y(t)dt$
   d) $\int_0^x \frac{(t-x)^3}{6} y(t)dt$

2) Eigenvalues of symmetric kernel of a Fredholm integral equation are ________.
   a) always positive
   b) always negative
   c) always real
   d) purely imaginary

3) Solution of $y(x) = 1 + \int_0^x y(t)dt$ is _______.
   a) 1
   b) e^x
   c) x
   d) None of these

4) If $K(x,t)$ is symmetric kernel, then its iterated kernel _______
   a) $K_2(x,t)$ need not be symmetric
   b) $K_{100}(x,t)$ need not be symmetric
   c) $K_n(x,t)$ is symmetric for all $n \in \mathbb{N}$
   d) $K_n(x,t)$ is never symmetric

5) If $R(x,t;\lambda)$ is the resolvent kernel of a Fredholm integral equation $g(x)u(x) = f(x) + \lambda \int_a^b K(x,t)u(t)dt$ then the resolvent kernel is a solution of _______.
   a) $R(x,t;\lambda) = K(x,t) + \lambda \int_t^x K(x,z)R(z,t;\lambda)dt$
   b) $R(x,t;\lambda) = K(x,t) + \lambda \int_a^x K(x,z)R(z,t;\lambda)dt$
   c) $R(x,t;\lambda) = K(x,t) + \lambda \int_a^b K(x,z)R(z,t;\lambda)dt$
   d) All of the above

6) An integral equation $g(x)u(x) = f(x) + \int_a^b K(x,t)u(t)dt$ is said to be of the second kind if
   a) $g(x) = 0$ b) $g(x) = 1$
   c) $f(x) = 0$ d) $f(x) = 1$

7) Which of the following is not a homogeneous Fredholm integral equation?
   a) $y(x) = \int_0^2 y(t)dt$
   b) $x = \int_0^2 \sin t\, y(t)dt$
   c) $y(x) = \int_0^1 \sin(x+t)\, y(t)dt$
   d) All of these

8) Which of the following is not a symmetric kernel?
   a) $K(x,t) = x + t$ b) $K(x,t) = \sin(x-t)$
   c) $K(x,t) = e^{x^2+t^2}$ d) $K(x,t) = \log(x+t)$

B) State whether True or False: 04

1) Every Fredholm integral equation has a trivial solution.
2) A Volterra integral equation involving convolution type kernel is solved by Laplace transform.
3) Volterra integral equation is obtained from boundary value problem.
4) The Green's function exists for every boundary value problem.

Q.2 Answer the following. (Any Six) 12

a) Define resolvent kernel.
b) Define: Convolution type kernel and give one example.
c) Show that $y(x) = x$ is a solution of $y(x) = x - \frac{x^2}{2} + \int_0^x y(t)dt$
d) Find $K_3(x,t)$ for the kernel $K(x,t) = e^{x-t}$ of an Volterra integral equation.
e) Define: Iterated kernel for Volterra integral equation.
f) Show that the kernel $K(x,t) = i(x-t)$ is symmetric.
g) State the Leibnitz formula for the differentiation under integral sign.
h) Convert into an integral equation: $y' - xy = \sin x, y(0) = 5$

Q.3 Answer the following. (Any Three) 12

a) Define Volterra integral equation of first kind, second kind, third kind and homogeneous Volterra integral equation.
b) Define Green's function.
c) Show that $y(x) = 1 - x$ is solution of $\int_e^x e^{x-t} y(t)dt = x$.
d) Convert the following differential equation into integral equation using substitution method.

$$y'' + xy = 1, y(0) = y'(0) = 0.$$

Q.4 Answer the following. (Any Two) **12**

a) Convert into an integral equation: $y'' + \lambda y = x, y(0) = 0, y(1) = 1.$

b) Solve: $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt$

c) Solve by the method of successive approximations:

$$y(x) = 1 + \int_0^x y(t)dt, y_0(x) = 0.$$

Q.5 Attempt the following (Any One). **12**

a) Solve: $y(x) = e^x + \lambda \int_0^1 2e^x e^t y(t)dt$

b) Solve using Laplace transform: $Y(t) = t^2 + \int_0^1 Y(x) \sin(t-x)dx$

c) Solve by using resolvent kernel: $y(x) = e^{x^2} + \int_0^x e^{x^2-t^2} y(t)dt$

Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - IV) (New) (NEP CBCS) Examination: March/April - 2025
Measure and Integration (2317405)

Day & Date: Tuesday, 20-May-2025
Time: 03:00 PM To 05:30 PM

Max. Marks: 60

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **08**

1) Consider the following two statements.
I) Every Semi algebra is algebra
II) Every algebra is σ −algebra
a) both are true
b) only I is true
c) only II is true
d) both are false

2) The collection of sets that are countable union of sets in algebra $\mathcal{A}$ is called as _____ set.
a) A_σ
b) A_δ
b) G_σ
d) G_δ

3) The μ^* outer measure is a measure induced by _____.
a) inner measure
b) signed measure
c) measure on algebra
d) product measure

4) Radon Nikodym theorem holds for _____.
a) locally measurable sets
b) finite measure space
c) σ − finite measure space
d) All of these

5) Every signed measure is a _____ of two measures.
a) sum
b) difference
c) product
d) reciprocal

6) A set with positive measure _____.
a) is a positive set
b) not a positive set
c) need not be a positive set
d) negative set

7) Two measures ν_1 and ν_2 on a measurable space are said to be mutually singular if there are disjoint measurable sets A and B such that $X = A \cap B$ and _____.
a) $\nu_1(A) = \nu_2(B) = 0$
b) $\nu_1(B) = \nu_2(A) = 0$
c) $\nu_1(E) = 0$ implies $\nu_2(E) = 0$
d) Both a and b

8) A measure space $(X, \mathcal{B}, \mu)$ said to be ______ if $\mathcal{B}$ contains all subsets of set of measure zero.

a) saturated b) finite
c) finite d) σ −finite

B) Fill in the blanks. **04**

1) The Jordan Decomposition of signed measure is unique.
2) The collection of all locally measurable sets is σ −algebra.
3) Every Semi-algebra is an algebra.
4) The collection $\mathcal{R}$ of measurable rectangles is an algebra.

Q.2 Answer the following. (Any Six) **12**

a) Define Locally measurable set and saturated measure.
b) If c is a constant and f is a measurable function defined on X then prove that $f + c$ is a measurable function.
c) State the Generalized Lebesgue convergence theorem.
d) Prove that: Every measurable subset of positive set is itself positive.
e) Define x-cross section and y-cross section of set E.
f) Define Semi algebra and Product measure.
g) Define Finite measure and σ −finite measure.
h) If f and g are non-negative extended real valued measurable functions on $(X, \mathcal{B}, \mu)$ and $E \in \mathcal{B}$ then prove that $f \leq g$ a.e on $E \Longrightarrow \int_E f d\mu \leq \int_E g d\mu$

Q.3 Answer the following. (Any three). **12**

a) If $(X, \mathcal{B}, \mu)$ is a measure space and $\mathcal{C}$ be the σ −algebra of locally measurable sets, for any $E \in \mathcal{C}$ define $\bar{\mu}(E) = \mu(E)$ if $E \in \mathcal{B}$ and $\bar{\mu}(E) = \infty$ if $E \notin \mathcal{B}$ then prove that $(X, \mathcal{C}, \bar{\mu})$ is a measure space.
b) If $E_i \in \mathcal{B}, \mu(E_1) < \infty$ and $E_i \supseteq E_{i+1,} \forall i$ then prove that $\mu(\cap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} \mu(E_n)$
c) If $E \subseteq F$ then with usual notations prove that $\mu_*(E) \leq \mu_*(F)$.
d) Prove that: Every σ −finite measure is saturated.

Q.4 Answer the following. (Any two) **12**

a) If $\mathcal{R}$ is a measurable rectangle and $x \in X$ is any element then for $E \in R_{\sigma\delta}$ prove that E_x is measurable subset of Y.
b) State and Prove Monotone convergence theorem
c) Show that: Any two Hahn Decomposition of X differ by null set.

Q.5 Answer the following. (Any two) **12**

a) If μ_1 and μ_2 are measures on a measurable space $(X, \mathcal{B})$ such that atleast one of them is finite and $\nu(E) = \mu_1(E) - \mu_2(E)$ for all $E \in \mathcal{B}$ then prove that ν is a signed measure.

b) Define product measure and prove that if E is measurable subset $X \times Y$ then

i) $(E^c)_x = E_x^c$

ii) $(\cup_{i=1}^{\infty} E_i)_x = \cup_{i=1}^{\infty} (E_i)_x$

c) If $A \in \mathcal{A}$ (Algebra) and $\{A_i\}$ is a sequence of sets in $\mathcal{A}$ such that $A \subseteq \cup_{i=1}^{\infty} A_i$ then prove that $\mu(A) \leq \sum_{i=1}^{\infty} \mu(A_i)$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: March/April - 2025
Measure & Integration (MSC15401)

Day & Date: Wednesday, 14-May-2025 Max. Marks: 80
Time: 03:00 AM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Every signed measure has a _____ Jordan decomposition.
a) more than one b) infinite
c) unique d) finite

2) A subset E of X is said to be μ^* measurable, if for any set A ____.
a) $\mu^*(A) \leq \mu^*(A \cup E) + \mu^*(A \cup E^c)$
b) $\mu^*(A) \geq \mu^*(A \cup E) + \mu^*(A \cup E^c)$
c) $\mu^*(A) = \mu^*(A \cup E) + \mu^*(A \cup E^c)$
d) both b and c

3) Radon Nikodym theorem holds for _____.
a) locally measurable sets b) finite measure space
c) σ –finite measure space d) All of these

4) Every signed measure is a _____ of two measures.
a) sum b) difference
c) product d) reciprocal

5) A set with positive measure _____.
a) is a positive set b) not a positive set
c) need not be a positive set d) negative set

6) Two measures v_1 and v_2 on a measurable space are said to be mutually singular if there are disjoint measurable sets A and B such that $X = A \cap B$ and _____.
a) $v_1(A) = v_2(B) = 0$
b) $v_1(B) = v_2(A) = 0$
c) $v_1(E) = 0$ implies $v_2(E) = 0$
d) both a and b

7) Consider the following two statements.
I) Every Semi algebra is algebra.
II) Every algebra is σ –algebra.
a) both are true b) only I is true
c) only II is true d) both are false

8) The collection of sets that are countable union of sets in algebra $\mathcal{A}$ is called as _____ set.

a) A_σ b) A_δ
c) G_σ d) G_δ

9) The μ^* outer measure is a measure induced by _____.

a) inner measure b) signed measure
c) measure on algebra d) product measure

10) A measure space $(X, \mathcal{B}, \mu)$ said to be _____ if $\mathcal{B}$ contains all subsets of set of measure zero.

a) saturated b) finite
c) complete d) σ –finite

B) State true or false. **06**

1) Lebesgue outer measure is also μ^* outer measure.
2) A measure on an algebra $\mathcal{A}$ is a measure iff $\mathcal{A}$ is a σ –algebra.
3) The collection $\mathcal{R}$ of measurable rectangles is a σ –algebra.
4) Hahn decomposition of a set is unique.
5) The signed measure of any set is always non-negative.
6) Every null set has a measure zero.

Q.2 Answer the following. **16**

a) Define measure space and give one example.
b) Show that: Every σ – finite measure is saturated.
c) Prove that: Every measurable subset of positive set is positive set.
d) If is v signed measure and μ is a measure such that $\nu \perp \mu$ and $\nu \ll \mu$ then prove that $\nu = 0$.

Q.3 Answer the following.

a) If $E \subseteq F$ then with usual notations prove that $\mu_*(E) \le \mu_*(F)$. **08**
b) If $\mathcal{R}$ is a measurable rectangle and $x \in X$ is any element then for $E \in R_{\sigma\delta}$ prove that E_x is measurable subset of Y. **08**

Q.4 Answer the following.

a) If u_1 and u_2 are measures on a measurable space $(X, \mathcal{B})$ such that atleast one of them is finite and $\nu(E) = \mu_1(E) - \mu_2(E)$ for all $E \in \mathcal{B}$ then prove that ν is a signed measure. **08**
b) If ν is a signed measure on measurable space $(X, \mathcal{B})$ then there is a positive set A and negative set B such that $X = A \cup B, A \cap B = \emptyset$ **08**

Q.5 Answer the following.

a) Show that the triplet $(R, \mathcal{M}, \mu)$ is a measure space where $\mathcal{M}$ is set of Lebesgue measurable sets and μ is set function defined by $\mu(E) = |E|$ is E is finite, $\mu(E) = \infty$ is E is infinite. **08**
b) State and Prove Monotone convergence theorem. **08**

Q.6 Answer the following.

- **a)** Prove that: The set of locally measurable sets form σ −algebra. **08**
- **b)** If c is a constant and f, g are measurable function defined on X prove that $f + c, cf, f + g, f - g$ are measurable functions. **08**

Q.7 Answer the following.

- **a)** If $(X, \mathcal{B}, \mu)$ is a measure space and $\mathcal{C}$ be the σ −algebra of locally measurable sets, for any $E \in \mathcal{C}$ define $\bar{\mu}(E) = \mu(E)$ if $E \in \mathcal{B}$ and $\bar{\mu}(E) = \infty$ if $E \notin \mathcal{B}$ if then prove that $(X, \mathcal{C}, \bar{\mu})$ is a measure space. **08**
- **b)** State and Prove Lebesgue convergence theorem. **08**

SLR-ZO-29

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: March/April - 2025
Partial Differential Equations (MSC15402)

Day & Date: Friday, 16-May-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) In the parametric equation of curve $x = f_1(t), y = f_2(t), z = f_3(t)$ the condition for the parameter t to be an arc length of curve is ________.
a) $f_1'^2 + f_2'^2 + f_2'^3 = 1$
b) $f_1'^2 + f_2'^2 + f_2'^3 = 0$
c) $f_1'^2 + f_2'^2 = 0$
d) $f_1 + f_2 + f_3 = 1$

2) The general integral of the partial differential equation $x(x+y)p = y(x+y)q - (x-y)(2x+2y+z)$ is _______.
a) $F(xy, (x+y)(x+y-z) = 0$
b) $F(xy, (x+y)(x+y+z)) = 0$
c) $F(xy, (x-y)(x+y-z)) = 0$
d) $F(xy, (x+y-z)) = 0$

3) The Pfaffian differential equation in more than two variables ______.
a) is integrable
b) is not integrable
c) has integrating factor
d) may not be integrable

4) The Lagrange's auxiliary equation for the partial differential $Pp + Qq = R$ is _______.
a) $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$
b) $\frac{dx}{P} = \frac{dy}{Q}$
c) $\frac{dx}{P} = \frac{dz}{R}$
d) $\frac{dy}{Q} = \frac{dz}{R}$

5) The integral surface of $z = p^2 - q^2$ which passes through the curve $4z + x^2, y = 0$ is _______.
a) $z = (x + 2y)^2$
b) $2z = (x + 2y)^2$
c) $-2z = \left(x + \sqrt{2}y\right)^2$
d) $z = \left(x + \sqrt{2}y\right)^2$

6) The parametric equations of a curve and a surface are _______.
a) unique
b) same
c) not unique
d) none of these

7) The heat equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$ reduces to Laplace equation when the temperature u ________.

a) becomes zero
b) increases with time t
c) decreases with time t
d) does not change with time t

8) The complete integral of $z^3 = pqxy$ is _______.

a) $x^a y^b = exp\left(2\sqrt{\frac{ab}{z}}\right)$
b) $xy = exp\left(\sqrt{\frac{ab}{z}}\right)$
c) $x^a y^b = exp\left(\sqrt{\frac{ab}{z}}\right)$
d) $2x^a y^b = exp\left(\sqrt{\frac{ab}{2z}}\right)$

9) The direction ratios of the normal to the surface $z = f(x, y)$ are _______.

a) $(p, q, 1)$
b) $(p, q, -1)$
c) (p, q)
d) $(p, q, 0)$

10) Canonical form of $z_{xx} - 6z_{xy} + 9z_{yy} + 2p + 3q - z = 0$ is _______.

a) $\frac{\partial^2 z}{\partial u\,\partial v} = \frac{z}{9} - \frac{\partial z}{\partial u} - \frac{1}{3}\frac{\partial z}{\partial v}$
b) $\frac{\partial^2 z}{\partial v^2} = \frac{z}{9} - \frac{\partial z}{\partial u} - \frac{1}{3}\frac{\partial z}{\partial v}$
c) $\frac{\partial^2 z}{\partial \alpha^2} + \frac{\partial^2 z}{\partial \beta^2} = \frac{\partial z}{\partial \alpha} + \frac{\partial z}{\partial \beta}$
d) $\frac{\partial^2 z}{\partial \alpha^2} - \frac{\partial^2 z}{\partial \beta^2} = \frac{\partial z}{\partial \alpha} - \frac{\partial z}{\partial \beta}$

B) Write True or False. **04**

1) If $u(x, y)$ is harmonic in a bounded domain D and is continuous on $\overline{D} = D \cup B$, Where B is boundary of D. Then $u(x, y)$ attains its minimum on B.
2) The characteristic curves for the equation $xz_y - yz_x = z$ are circle with Centre at origin.
3) The condition $X^{-} curl X^{-} = 0$ is equivalent to $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) - Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$
4) Every one parameter family of surface $f(x, y, z) = c$ is a family of equipotential surfaces.
5) The partial differential equation by eliminating arbitrary constants 'a' and 'b' from equation $2z = (ax + y)^2 + b\ is\ px + qy = q^2$
6) Lagrange's method is used to solve non-linear partial differential equations.

Q.2 Answer the following. **16**

a) Show that the necessary condition for the existence of solution of the Neumann problem is that the integral of function f over the boundary B should vanish.

b) Define:
1) Singular Integral
2) Complete Integral

c) Show that there always exists an integrating factor for a Pfaffian differential equation in two variables.

d) Find a partial differential equation by eliminating arbitrary constant from
$z = x + ax^2y^2 + b$

Q.3 Answer the following. **16**

a) Find a complete integral of $f = xpq + yq^2 - 1 = 0$

b) Solve: $(6x + yz)dx + (xz - 2y)dy + (xy + 2z)dz = 0$

Q.4 Answer the following. **16**

a) Find the general solution of $2x(y + z^2)p + y(2y + z^2)q = z^3$

b) Reduce the equation $u_{xx} + x^2u_{yy} = 0$ to a canonical form.

Q.5 Answer the following. **16**

a) Prove that a necessary and sufficient condition that the Pfaffian differential equation $\bar{X}\,\overline{dr} = 0$ be integrable is that $\bar{X} curl\ \bar{X} = 0$.

b) Solve $xu_x + yu_y = {u_z}^2$ by Jacobi's method.

Q.6 Answer the following. **16**

a) Find the general solution of $x(y - z)p + y(z - x)q = z(x - y)$

b) Obtain D-Alembert's solution of the one dimensional wave equation which describes the vibration of a semi-infinite string.

Q.7 Answer the following. **16**

a) Show that the surfaces $x^2 + y^2 + z^2 = r^2, r > 0$ forms a family of equipotential surfaces and find the general form of corresponding potential function.

b) Show that $z = ax + by + a^2 + b^2$ is a complete integral of $z = px + qy + p^2 + q^2$ then by taking $b = a$ find the envelope of the subfamily which is a particular solution. Further find the singular integrals.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: March/April - 2025
Integral Equations (MSC15403)

Day & Date: Tuesday, 20-May-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Select the correct alternative: **10**

1) Which of the following is a not separable kernel?
a) $K(x,t) = \sin h(x-t)$ b) $K(x,t) = \cos h(x-t)$
c) $K(x,t) = \sin(x+t)$ d) $K(x,t) = e^{\frac{x}{t}}$

2) An integral equation $g(x)y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt$ is said to be of the second kind if _____.
a) $g(x) = 0$ b) $g(x) = 1$
c) $f(x) = 0$ d) $f(x) = 1$

3) Solution of $y(x) = \int_0^1 y(t)dt$ is _____.
a) $y(x) = 1$ b) $y(x) = 0$
c) $y(x) = -1$ d) all of the above

4) A Volterra integral equation $y(x) = e^x + \lambda \int_0^x ty(t)dt$ is _____.
a) homogeneous second kind
b) non-homogeneous second kind
c) homogeneous first kind
d) non-homogeneous first kind

5) Which of the following is a convolution type kernel?
a) $K(x,t) = (t-x)^3$ b) $K(x,t) = e^{(t+x)}$
c) $K(x,t) = \sin(tx)$ d) All of the above

6) Integral equation corresponding to the $IVP\ y'(x) - y(x) = 0, y(0) = 1$ is ______.

a) $y(x) = 1 - \int_0^x y(t)dt$

b) $y(x) = -1 + \int_0^x y(t)dt$

c) $y(x) = 1 + \int_0^x y(t)dt$

d) $y(x) = -1 - \int_0^x y(t)dt$

7) Which of the following kernel is not symmetric?

a) $K(x,t) = xt$
b) $K(x,t) = x + t$
c) $K(x,t) = (x + t)^2$
d) $K(x,t) = x - t$

8) A BVP gets converted into _____.

a) Volterra integral equation
b) Fredholm integral equation
c) Singular integral equation
d) None of these

9) An nth iterated kernel of a Volterra integral equation

$$y(x) = f(x) + \lambda \int_a^x K(x,t)y(t)dt$$

a) $K_n(x,t) = \int_a^b K(x,z)K_{n-1}(z,t)dz$

b) $K_n(x,t) = \int_a^b K(x,z)K_{n-2}(z,t)dz$

c) $K_n(x,t) = \int_a^x K(x,z)K_{n-1}(z,t)dz$

d) $K_n(x,t) = \int_t^x K(x,z)K_{n-1}(z,t)dz$

10) Eigen values of symmetric kernel of a Fredholm integral equation are _____.

a) always imaginary
b) always positive
c) always negative
d) always real

B) State whether true or false. **06**

1) If a only solution for a BVP is a trivial solution, then Green's function exist for the BVP.
2) Integral equations with convolution type kernel are solved by using Laplace transform.
3) Every homogeneous Fredholm integral equation has at least one solution.
4) If $K(x,t) = 1; a = 0, b = 1$ is a kernel of a Fredholm integral equation, then the second iterated kernel $K_n(x,t) = 2, \forall n \in \mathbb{N}$
5) A solution of a homogeneous Fredholm integral equation is said to be an eigen function if it is non-zero.
6) $\int_0^x y(t)dt^4 = \int_0^x \frac{(x-t)^3}{24} dt.$

Q.2 Answer the following. **16**

a) Define Symmetric kernel, convolution type kernel, Resolvent kernel, separable kernel.

b) Show that $y(x) = 1 - x$ is solution of $\int_0^x e^{x-t} y(t)dt = x.$

c) Show: $y(x) = \lambda \int_0^1 \sin(\pi x)\cos(\pi t) y(t)dt$

d) Solve: $y(x) = \cos x + \lambda \int_0^\pi \sin x y(t)dt$

Q.3 Answer the following. **16**

a) Convert the following IVP into integral equation:
$y''(x) - 3y'(x) + 2y(x) = 4\sin x, y(0) = 1, y'(0) = -2$

b) Solve by using resolvent kernel method:
$y(x) = e^x - \frac{1}{2}e + \frac{1}{2} + \frac{1}{2}\int_0^1 y(t)dt.$

Q.4 Answer the following. **16**

a) Solve by using resolvent kernel method:
$y(x) = e^x \sin x + \int_0^x \frac{2+\cos x}{2+\cos t} y(t)dt$

b) Find the resolvent kernel of a Fredholm integral equation whose kernel is $K(x,t) = e^x \cos t; a = 0, b = \pi.$

Q.5 Answer the following. **16**

a) Find the green's function for the BVP $y'' + \mu^2 y = 0; y(0) = y(1) = 0.$

b) Solve using Laplace transform: $Y(t) = t^2 + \int_0^t Y(x)\sin(t-x)dx\,.$

Q.6 Answer the following. **16**

a) Convert the following into an integral equation:

$y'' + \lambda y = 0; y(0) = y(l) = 0$

b) Solve: $y(x) = 1 + \int_0^1 (1 + e^{x+t}) y(t)dt$

Q.7 Answer the following. **16**

a) Find the eigenvalues and eigen functions of an integral equation

$y(x) = \lambda \int_0^{\pi} K(x,t) y(t)dt$

Where $K(x,t) = \begin{cases} x(t-1), & 0 \le x \le t \\ t(x-1), & t \le x \le 1 \end{cases}$

b) Solve by using the method of successive approximations:

$y(x) = 1 + x - \int_0^x y(t)dt, y_0(x) = 1$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: March/April - 2025
Operations Research (MSC15404)

Day & Date: Thursday, 22-May-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Consider the following statements:
I) The closed ball in R^3 is a convex set.
II) A hyperplane in R^n is a convex set.
a) only I is true b) only II is true
c) both are true d) both are false

2) A saddle point in game exists when-
a) maximin value=maximax value
b) minimax value= minimum value
c) minimax value=maximin value
d) all of the above

3) The best use of linear programming problem is to find an optimal use of _____
a) Money b) Machine
c) manpower d) all of the above

4) The dual of the primal problem is obtained by _____
a) transposing the co-efficient matrix and reverting the inequalities
b) interchanging the role of constant terms and the co-efficient of the objective function
c) minimizing the objective function instead of maximizing it
d) all of the above

5) The dual simplex method works towards ______ while simplex method works towards ______
a) optimality, feasibility
b) feasibility, optimality
c) boundedness, basic solution
d) finiteness, basic solution

6) Simplex method is developed by American mathematician ______
- a) Frank Wolf
- b) Martin Beale
- c) Ralph E. Gomory
- d) George Dantzig

7) The dual of the primal maximization LPP having m constraints and n non-negative variables should ______
- a) Have n constraints and m non-negative variables
- b) Be a minimization LPP
- c) Both (a) and (b)
- d) None of the above

8) Game theory models are classified by the ______
- a) Number of players b) Sum of all payoffs
- c) Number of strategies d) All of the above

9) The graphical method of linear programming problem uses _____
- a) objective function equation b) constraint equations
- c) linear equations d) all of the above

10) In dual simplex method, ______ variables are not required.
- a) Slack b) Surplus
- c) Original d) Artificial

B) Fill in the blanks. **06**

1) If a primal LPP has a finite solution then the dual LPP should have _____ solution.
2) To convert $\geq$ inequality constraints into equality constraints, we must add a ______
3) A quadratic form $Q(X)$ is positive definite iff $Q(X)$ is —- for all $x \neq 0$
4) The set of all feasible solution of a linear programming problem is ______ set.
5) If pth variable of the primal is unrestricted in sign then the pth constraint of dual is _____
6) Gomory's cutting plane method will take the help of _____ method to solve the given integer programming problem.

Q.2 Answer the following. **16**

a) Prove that: The dual of the dual of a given primal is primal.

b) Write general form of Quadratic programming problem.

c) Define:
1) Extreme point of convex set
2) Convex hull

d) Write the rules for determining a saddle point in Game theory.

Q.3 Answer the following.

a) If X is any feasible solution to the primal problem and W is any feasible solution to the dual problem then prove that $CX < b^T W$. **08**

b) Prove that: The collection of all feasible solutions to linear programming problem constitutes a convex set whose extreme point corresponds to the basic feasible solution. **08**

Q.4 Answer the following.

a) Explain the construction of Kuhn Tucker conditions for Quadratic programming problem. **08**

b) Write an algorithm of Big-M method for solving linear programming problem. **08**

Q.5 Answer the following.

a) Find the optimum integer solution to the following IPP by Gomory's cutting plane method. **08**

Max $Z = x_1 + 2x_2$ subject to the constraints

$2x_2 \leq 7,\ x_1 + x_2 \leq 7,\ 2x_1 \leq 11$ and $x_1, x_2 \geq 0$ are integers.

b) Find the saddle point and solve the game. **08**

	Player B B_1	B_2	B_3	B_4
Player A A_1	1	7	3	4
A_2	5	6	4	5
A_3	7	2	0	3

Q.6 Answer the following.

a) Solve the following problem by Simplex method. **08**

Max $Z = 3x_1 + 2x_2$ subject to the constraints $x_1 + x_2 \leq 4, x_1 - x_2 \leq 2$ and $x_1, x_2 \geq 0$

b) Prove that: The set of all convex combinations of a finite number of points $x_1, x_2, \ldots . x_n$ is a convex set. **08**

Q.7 Answer the following.

a) If k^{th} constraint of the primal is an equality then prove that the dual vaiable w_k is unrestricted in sign. **08**

b) Solve the following problem by Dual Simplex method. **08**

Min $Z = 2x_1 + x_2$ subject to the constraints

$3x_1 + x_2 \geq 3, 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \geq 3$ and $x_1, x_2 \geq 0$

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: March/April - 2025
Numerical Analysis (MSC15408)

Day & Date: Tuesday, 27-May-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) The root of the equation $f(x) = 0$ lies in interval (a, b) if _____.
a) $f(a)f(b) = 0$ b) $f(a)f(b) > 0$
c) $f(a)f(b) < 0$ d) $f(a)f(b) = 1$

2) If $A = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$ then the eigen values of _____.
a) $2,4$ b) $3,1$
c) $2,3$ d) $1,5$

3) The Newton-Raphson algorithm for the function $f(x) = x - x^2$ will be _____.
a) $x_{n+1} = \dfrac{x_n^2}{1 - 2x_n}$ b) $x_{n+1} = \dfrac{-x_n^2}{1 - 2x_n}$
c) $x_{n+1} = 2x_n - x_n^2$ d) $x_{n+1} = 2x_n + x_n^2$

4) How many real roots does the equation $x^2 + 2 = 0$ have?
a) 2 b) 3
c) 1 d) 0

5) If a function is real and continuous in the region from a to b and $f(a), f(b)$ have opposite signs then there is _____ root between a and b.
a) no real b) real
c) rational d) irrational

6) The approximate value of $y(0.1)$ from $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$ is _____.
a) 0.900 b) 0.222
c) 1.001 d) 0.994

7) Gauss-Seidel iterative method is used to solve _____.
- a) differential equation
- b) system of linear equations
- c) system of non-linear equations
- d) partial differential equation

8) If $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -5 \\ 0 & 0 & -6 \end{bmatrix}$ then the largest eigen value of A is _____.
- a) 1 b) 5
- c) 4 d) 6

9) In Newton-Raphson method if the curve $f(x)$ is constant then _____.
- a) $f(x) = 0$ b) $f'(x) = c$
- c) $f''(x) = 0$ d) $f'(x) = 0$

10) Regula Falsi method requires _____ initial approximation to find the root.
- a) 1 b) 2
- c) 3 d) 4

B) Write True/False. **06**
1) A Gauss-Seidel method will always converge on the solution.
2) The n[th] degree polynomial has n real or complex roots.
3) The Bisection method is guaranteed to converge if $|f'(x)| > 1$.
4) LU decomposition is more efficient than Gauss elimination when solving for the inverse of a matrix.
5) If a matrix is multiplied by its inverse, the result will be the identity matrix.
6) The positive root of the equation $x^3 - 4x - 9 = 0$ using Regula Falsi method and correct to 4 decimal places is 2,7065.

Q.2 Answer the following. **16**

a) Round of the number 96.7280 to four significant figures and compute percentage and relative error.

b) Construct a formula for Newton-Raphson method.

c) Define eigen values and eigen vectors.

d) Find the largest eigen value of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by using Rayleigh's power method.

Q.3 Answer the following.

a) Describe rate of convergence of secant method. **08**

b) Reduce the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ to the tridiagonal from. **08**

Q.4 Answer the following.

a) Find all the eigen values of the matrix $\begin{bmatrix} 4 & 6 & 10 \\ 3 & 10 & 13 \\ -2 & -6 & -8 \end{bmatrix}$ **08**

b) Find a real root of the equation $x^4 - 4x - 9 = 0$ by bisection method, correct upto three decimal places. **08**

Q.5 Answer the following.

a) Solve the following system of equations **08**
$2x + y + z = 5, 3x + 5y + 2z = 15, 2x + y + 4z = 8$
by using Gauss-Seidel method.

b) Write a note on Euler's modified method. **08**

Q.6 Answer the following.

a) Explain the construction of Gauss elimination method. **10**

b) Estimate $y(0.1)$ and $y(0.2)$ with $h = 0.1$ for the initial value problem $\frac{dy}{dx} = 2xy^2, y(0) = 1$ Using Runge-Kutta method. **06**

Q.7 Answer the following.

a) Find a real root of the equation $x^3 = x - 10 = 0$ by method of False position, correct upto three decimal places. **08**

b) Solve the following system of equations **08**
$3x + 2y + z = 9, x + 2y + 3z = 6, 3x + y + 2z = 8$
by using LU decomposition method.

Seat No.	

Set P

M.Sc. (Mathematics) (Semester - IV) (New/Old) (CBCS) Examination: March/April - 2025
Probability Theory (MSC15410)

Day & Date: Tuesday, 27-May-2025 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If for two independent events A and $B, P(A) = 0.2, P(B) = 0.4$, then $P(A \cup B) =$ _____.
a) 0.68 b) 0.55
c) 0.52 d) 0.68

2) If $x \in A$ implies $x \in B$, then _____.
a) $A \subset B$ b) $B \subset A$
c) $A = B$ d) All of these

3) A class $\mathcal{F}$ is said to be closed under finite intersection, if $A, B \in \mathcal{F}$ f implies _____.
a) $A \cap B \in \mathcal{F}$, for all $A, B \in \mathcal{F}$ b) $A^c \in \mathcal{F}, B^c \in \mathcal{F}$
c) both (b) and (b) d) none of these

4) If F is a σ-field, then which of the following is not always correct?
a) F is a field
b) F is a class closed under countable unions.
c) F is a class closed under complementation.
d) F is a minimal sigma field.

5) A class F is said to be monotone class, if _____.
a) It is a field
b) If it is closed under monotone operations
c) Both (a) and (b)
d) Either (a) and (b)

6) Distribution function of a random variable is always _____.
a) Non-negative b) right continuous
c) Monotone non-decreasing d) All of the above

7) A simple function can take _____ values.
a) Finitely many b) Infinitely many
c) Uncountably many d) None of these

8) Monotonic sequence of sets _____.
a) Always converges
b) Converges, only if it is bounded above
c) Converges, only if it is bounded below
d) Converges, only if it is bounded

9) Which of the following is the weakest mode of convergence?
a) convergence in r^{th} mean
b) convergence in probability
c) convergence in distribution
d) convergence in almost sure

10) If events A, B and C are mutually independent, then which of the following is not correct?
a) A and B are pairwise independent.
b) A and C are pairwise independent.
c) B^c and C are independent
d) All are correct

B) Fill in the blanks. **06**

1) If $F(.)$ is a distribution function for some random variable, then $lim_{x\to-\infty}F(x) =$ _____.
2) A class closed under complementation and finite union is called as _____.
3) The σ- field generated by the intervals of the type$(-\infty, x), x \in R$ is called _____.
4) If P is a probability measure defined on $(\Omega, \mathcal{A})$, then $P(\Omega) =$ _____.
5) Expectation of a random variable X exists, if and only if _____ exists.
6) If A is empty set, then $P(A) =$ ..., where $P(.)$ is a probability measure.

Q.2 Answer the following. **16**

a) Define mixture of two probability measures. Show that mixture is also a probability measure.
b) Prove or disprove: Arbitrary intersection of fields is a field.
c) Discuss σ-field induced by r.v. X.
d) Define conditional probability measure. Show that it is also a probability measure.

Q.3 Answer the following.

a) Discuss limit superior and limit inferior of a sequence of sets. Find the same for sequence $\{A_n\}$, where $A_n = \left(0,3 + \frac{(-1)^n}{n}\right), n \in N$ **08**
b) Define field and σ-field. Show that there exist classes which are field but not σ-field. **08**

Q.4 Answer the following.

a) Prove that collection of sets whose inverse images belong to a σ-field, is a also a σ-field. **08**

b) Define expectation of simple random variable. If X and Y are simple random variables, prove the following: **08**

i) $E(X+Y) = E(X) + E(Y)$

ii) $E(cX) = cE(X)$, where c is real number

iii) If $X > 0$ a.s., then $E(X) > 0$.

Q.5 Answer the following.

a) Prove that if $\{B_n\}$ converges to B, then $P(B_n)$ also converges to $P(B)$. **08**

b) Define characteristic function of a random variable. Prove any three properties of characteristic function. **08**

Q.6 Answer the following.

a) State and prove Borel-Cantelli lemma. **08**

b) Prove that expectation of a random variable X exists, if and only if $E|X|$ exists. **08**

Q.7 Answer the following.

a) Define, explain and illustrate the concept of limit superior and limit inferior of a sequence of sets. **08**

b) Prove that any random variable can be expresses as a limit of sequence of simple random variables. **08**

Seat No.	

Set **P**

M.Sc. (Mathematics) (Semester - II) (New) (NEP CBCS) Examination: March/April - 2025
Algebra - II (MSC15201)

Day & Date: Wednesday, 14-May-2025 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figures to the right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) Every automorphism of every field E leaves _____ elements of E fixed.
a) at least one b) at least two
c) two d) one

2) Two finite field of same order are _____.
a) homomorphic b) isomorphic
c) not isomorphic d) none of these

3) Characteristic of an integral domain is _____.
a) either zero or prime number
b) always prime number
c) always zero
d) never zero

4) A field K is regarded as a vector space over _____ of K.
a) any subset b) any subfield
c) any subring d) any subgroup

5) A field C of complex numbers is _____ extension of field R of real numbers.
a) finite b) simple
c) algebraic d) All of these

6) $O(G(Q(\sqrt{2}), Q))$ is _____.
a) equal to 1 b) equal to 2
c) less than or equal to 1 d) less than or equal to 2

7) If a & b are algebraic over F of degree m & n respectively then $a+b$ is algebraic of degree _____.
a) $m+n$ b) atmost $m+n$
c) mn d) atmost mn

8) If characteristic of F is zero and $f(x) \in F[x]$ is irreducible then $f(x)$ has _____ roots.
a) multiple b) distinct
c) imaginary d) real

B) Write True/False. **04**

1) Every complex number is algebraic over R.
2) It is not possible to find an extension of finite field.
3) Fixed field of $G(K, F)$ if contained in F.
4) π is algebraic over R.

Q.2 Answer the following. (Any Six) **12**

a) Check whether $\sqrt{5} + 2^{1/3}$ is algebraic over Q or not.
b) Find degree and basis of $Q(2^{1/3}, i)$, over Q.
c) Write short note on elementary symmetric functions.
d) Prove or disprove: Doubling the cube is impossible.
e) Construct a field with 9 elements.
f) Prove that R is not normal extension of Q.
g) Define:
i) Separable element ii) Perfect fields
h) Define: Algebraic element and its degree

Q.3 Answer the following. (Any three) **12**

a) With usual notations Prove or disprove that: $Q\left(\sqrt{3} + \sqrt{7}\right) = Q(\sqrt{3}, \sqrt{7})$
b) If $a \in K$ be algebraic over F and $p(x)$ be minimal polynomial for a over F then prove that $p(x)$ is irreducible over F.
c) Check whether $\sqrt{5 - \sqrt{11}}$ is algebraic over Q or not.
d) If $f(x) \in f[x]$ be of degree $n \geq 1$ then prove that there is a finite extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

Q.4 Answer the following. (Any two) **12**

a) Prove that: Any finite extension of a field of characteristic zero is a simple extension.
b) If K is a field and $\sigma_{1,}\sigma_2 \dots \sigma_n$ are n distinct automorphisms of K then prove that it is impossible to find the elements $a_{1,}a_2 \dots .. a_n$ not all zero in K such that $a_1\sigma_1(u) + a_2\sigma_2(u) + \cdots + a_n\sigma_n(u) = 0$ for all $u \in K$.
c) If L is a finite extension of K and K is finite extension of F then prove that L is finite extension of F.

Q.5 Answer the following. (Any two) **12**

a) Prove that: The polynomial $f(x) \in F[x]$ has a multiple root iff $f(x)$ and $f'(x)$ have a nontrivial common factor.
b) If K be an extension of F and $a \in K$ be algebraic over F then prove that $F(a)$ is isomorphic to $\frac{F[x]}{V}$ where V is an ideal of $F[x]$ generated by the minimal polynomial for a over F.
c) Find Galois group of $x^2 - 7$ over Q.