Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
STATISTICS
Distribution Theory (2329101)

Day & Date: Friday, 10-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **08**

1) Let X be distributed as $U(0,\theta)$.Then distribution of $Y = X/\theta$ is _____.
   a) $U(0,1)$ b) $U(0,\theta)$
   c) $U(0,1/\theta)$ d) $Exp(Mean\ \theta)$
2) For which of the following distribution, $E(X)$ does not exist?
   a) Normal b) Uniform
   c) Cauchy d) Exponential
3) Which of the following is a scale family?
   a) $U(0,\theta)$ b) $N(0,\sigma^2)$
   c) $Exp(\theta)$ d) All the above
4) For $X > 0$, which of the following is not true?
   a) $E[X^2] \geq [E(X)]^2$ b) $E[1/X] \geq 1/E(X)$
   c) $E[\sqrt{X}] \geq \sqrt{E(X)}$ d) $E[\log X] \leq \log[E(X)]$
5) A random variable X is symmetric about point α then ______.
   a) $f(\alpha + x) = f(\alpha - x)$ b) $f(\alpha + x) = f(x - \alpha)$
   c) $f(\alpha + x) = -f(\alpha - x)$ d) None of these
6) Let $F(x)$ denotes the distribution function of random variable $X.$ Then which of the following is not true?
   a) $0 \leq F(x) \leq \infty$ b) $F(x_1) \leq F(x_2)$ if $x_1 < x_2$
   c) $F(-\infty) = 0$ d) $F(+\infty) = 1$
7) Let X and Y be independent random variables each having uniform distribution on $[-1, 2]$. Then Var $(X + Y)$ is equal to ______.
   a) $5/2$ b) $3/2$
   c) 4 d) 6
8) If X and Y are two independent random variables then ______.
   a) $E(X\,Y) = E(X)\,E(Y)$ b) $Cov(X,Y) = 0$
   c) $\rho(X,Y) = 0$ d) All the above

B) Fill in the blanks. **04**

1) Suppose X is $U(0,1)$ random variable then $Y = -\log X$ has _____ distribution.
2) If Z is standard normal variate then variance of Z^2 is ______.
3) Let X be a $N(\mu,\sigma^2)$ variable. Then distribution of e^x is ______.
4) Let X has $B(1,p)$ distribution. The distribution of $Y = 1 - X$ is ______.

Q.2 Answer the following. (Any Six) **12**

a) State any two properties of the symmetric random variable.
b) Define convolution of two random variables.
c) Define scale family.
d) State Jensen's inequality.
e) Define probability generating function (PGF) of random variable X.
f) Define distribution function of bivariate random variate (X, Y).
g) Define a bivariate Poisson distribution.
h) Write the joint p.d.f. of any two order statistics.

Q.3 Answer the following. (Any Three) **12**

a) Let X be $U(0, \theta)$, where θ is an integer greater than one. Find the distribution of $Y = [X]$.
b) Derive the pdf of largest order statistic based on a random sample of size n from a continuous distribution with $pdf\ f(x)$ and $cdf\ F(x)$.
c) If F_1 and F_2 are distribution functions and $0 < \alpha < 1$, show that $F = \alpha F_1 + (1 - \alpha)F_2$ is a distribution function.
d) Define location family of distributions. Examine which of the following are in location family.
1) $X \sim N(\theta - 1)$
2) $X \sim Exp(\theta, 1)$

Q.4 Answer the following. (Any Two) **12**

a) State and prove Holder's inequality.
b) Let X is a non-negative continuous random with distribution function $F(x)$. If $E(X)$ exist then show that $E(X) = \int_0^\infty [1 - F(u)]du$
c) Let X has *Poisson* (λ) distribution. Obtain the PGF of X. Hence obtain its mean and variance.

Q.5 Answer the following. (Any Two) **12**

a) For a multinomial distribution with k cells, obtain the expression for correlation between i^{th} and j^{th} components random variables. Comment on the result.
b) Let (X, Y) has $\text{BVN}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain the marginal distributions of Y.
c) Let(X, Y) be a bivariate random variable with joint pdf given by

$$f(x, y) = \begin{cases} 4x(1 - y), 0 < x < 1, 0 < y < 1 \\ 0, \text{ elsewhere} \end{cases}$$

Find marginal distributions of X and Y.

SLR-HR-2

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
STATISTICS
Estimation Theory (2329102)

Day & Date: Monday, 13-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) An estimator $\hat{\theta}$ is said to be unbiased estimator of θ if _____.
   a) $E[\hat{\theta}] = \theta$
   b) $\hat{\theta} = E[\theta]$
   c) $[E(\hat{\theta})]^2 = \theta$
   d) $E[\hat{\theta}] = \theta^2$

2) A statistic $T(X)$ for θ is said to be ancillary if _____.
   a) $T(X)$ is independent of θ
   b) $T(X)$ is dependent of θ
   c) The distribution of $T(X)$ is independent of θ
   d) The distribution of $T(X)$ is depends of θ

3) Posterior distribution is the _____.
   a) joint distribution of X and $\theta.$
   b) distribution of parameter $\theta.$
   c) conditional distribution of X given $\theta.$
   d) conditional distribution of θ given X

4) The denominator of Cramer-Rao inequality gives _____.
   a) lower bound
   b) upper bound
   c) amount of information
   d) None of the above

5) If T_n is consistent estimator of θ then $\phi(T_n)$ is consistent estimator of $\phi(\theta)$ if _____.
   a) ϕ is linear function
   b) ϕ is continuous function
   c) ϕ is differentiable function
   d) None of these

6) Let X_1, X_2 is a random sample of size 2 from $U(0, \theta), \theta > 0$, MLE of θ is ______.
   a) $X_1 + X_2$
   b) $X_1 X_2$
   c) $\max\{X_1, X_2\}$
   d) $\min\{X_1, X_2\}$

7) If T_n is an unbiased estimator of $\theta,$ then Cramer-Rao inequality provides a lower bound on _____.
   a) $Var(T_n)$
   b) $E(T_n)$
   c) $\max(T_n)$
   d) $\min(T_n)$

8) If T_1 is sufficient statistic for θ and T_2 is an unbiased estimator of $\theta,$ then an improved estimator of θ in terms of its efficiency is _____.
   a) $E(T_1 T_2)$
   b) $E(T_1 + T_2)$
   c) $E(T_1/T_2)$
   d) $E(T_2/T_1)$

B) Fill in the blanks. **04**

1) Let $X_1, X_2, \ldots, X_n$ is a random sample of size n from $U(0, \theta)$ distribution then unbiased estimator θ is _____.
2) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(0, \sigma^2)$ Then sufficient statistic for σ^2 is _____.
3) Bayes estimator of a parameter under squared error loss function is_____.
4) Le $I(\theta)$be the Fisher information on $\theta,$ supplied by the sample. If T is an unbiased estimator of $\Psi(\theta)$, then Cramer-Rao lower bound for the variance of T is _____.

Q.2 Answer the following. (Any Six) **12**

a) Define a minimal sufficient statistic.
b) Define consistent asymptotically normal (CAN) estimator.
c) State Basu's theorem.
d) State any two small sample properties of MLE.
e) State Rao-Blackwell theorem.
f) Define uniformly minimum variance unbiased estimator (UMVUE).
g) State invariance property of consistent estimator.
h) Define Power series family of distributions.

Q.3 Answer the following. (Any Three) **12**

a) Show that $B(n, \theta)$ distribution belong to power series family.
b) Let random variable X has Poisson (θ) distribution. Show that distribution of X is complete.
c) Obtain a sufficient statistic for θ based on n iid observations from $N(\theta, 1)$ distribution.
d) Show that if there are two consistent estimators then we can construct infinitely many consistent estimators.

Q.4 Answer the following. (Any Two) **12**

a) State and prove Cramer-Rao inequality with necessary regularity conditions.
b) Describe a method of moments for estimation.
c) Let $X_1, X_2, \ldots, X_n$ be iid from $U(0, \theta)$. Obtain moment estimator and likelihood estimator of θ.

Q.5 Answer the following. (Any Two) **12**

a) State and prove Lehmann-Scheffe theorem.
b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from exponential distribution with $pdf\ f(x, \theta) = \theta e^{-x\theta}, x \geq 0, \theta, > 0$. Find UMVUE of (i) θ and (ii) $(1/\theta)$.
c) Let $X_1, X_2, \ldots, X_n$ be iid from $U(0, \theta)$ random variables. Show that $X_{(n)}$ is biased and consistent estimator for θ.

SLR-HR-3

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP) (CBCS) Examination: March/April-2024
STATISTICS
Statistical Mathematics (2329107)

Day & Date: Wednesday, 15-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **08**

1) Monotonic increasing bounded above sequence is ______.
   a) Always divergent b) Always convergent
   c) May or may not converge d) Oscillatory

2) The limit of sequence $S_n = \frac{1}{n}, n \epsilon N$ is __________.
   a) 1 b) 0
   c) 10 d) 2

3) A superset of uncountable set is ______.
   a) Always Countable
   b) Always Uncountable
   c) May or may not be countable
   d) None of these

4) A series of positive terms ________.
   a) Always converges b) Always diverges
   c) May or may not converge d) None of the above

5) The $\alpha\,(.)$ function in R-S integral is _____.
   a) Always non negative
   b) Always monotonic non-increasing
   c) Always monotonic non-decreasing
   d) Always constant

6) A vector space is closed under ______.
   a) Vector addition & scalar multiplication
   b) Vector addition & vector product
   c) Scalar multiplication and vector product
   d) None of these

7) If v_1, v_2, v_3 are three vectors such that $4v_1 + 2v_2 + v_3 = 0$, then __.
   a) v_1, v_2, v_3 are linearly dependent vectors
   b) v_1, v_2, v_3 are linearly independent vectors
   c) Need to verify other linear combinations to check independence.
   d) None of these

8) If A is a non-empty subset of B and B is set of independent vectors, then the vectors in A are-
   a) independent vectors
   b) May or may not be independent vectors
   c) dependent vectors
   d) None of these

B) Fill in the blanks. **04**

1) If number of rows is less than number of columns, then the matrix is called as __________.
2) The rank of identity matrix of order 4 is _________.
3) In the system of linear equations AX=b with unique solution, the matrix A is ______.
4) _____ is equal to the maximum number of linearly independent row vectors in a matrix.

Q.2 Answer the following. (Any Six) **12**

a) Define partial sum sequence of a series.
b) Define a symmetric matrix.
c) Define span of a set of vectors.
d) Define comparison test of convergence.
e) Define convergence of a sequence.
f) Define monotonic sequence.
g) Define supremum of a set.
h) Define a bounded set.

Q.3 Answer the following. (Any Three) **12**

a) Show that every convergent sequence is a Cauchy sequence.
b) Prove: Product of two diagonal matrices is again a diagonal matrix of same order.
c) Define a vector space stating all ten essential properties.
d) Show that rank of a matrix is unaltered by pre-multiplication with a non-singular matrix.

Q.4 Answer the following. (Any Two) **12**

a) Discuss the convergence of a geometric series with common ratio r.
b) Define a bounded sequence. Show that a convergent sequence is always bounded. Is every bounded sequence convergent? Justify.
c) Discuss in detail Riemann integration.

Q.5 Answer the following. (Any Two) **12**

a) Prove or disprove: Every square matrix can be written as sum of symmetric and skew symmetric matrix.
b) Define inverse of a matrix. Show that it is unique.
c) Show that the following system of equations is consistent. Also find solution for the same.

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x + 4y + 7z = 30$$

SLR-HR-6

Seat No.	

Set P

M.Sc. (Semester - I) (New) (NEP CBCS) Examination: March/April-2024
STATISTICS
Research Methodology in Statistics (2329103)

Day & Date: Friday, 17-05-2024 Max. Marks: 60
Time: 03:00 PM To 05:30 PM

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. 08

1) A research study undertaken to portray accurately the characteristics of a particular individual, situation or a group is termed as _____.
   a) exploratory research study b) descriptive research studies
   c) diagnostic research studies d) All of the above

2) The _____ research relies on experience or observation alone, often without due regard for system and theory.
   a) Conceptual b) Empirical
   c) Pure d) Applied

3) Redman and Mory define research as a _____.
   a) scientific and systematic search for pertinent information on a specific topic
   b) a search for knowledge.
   c) systematized effort to gain new knowledge
   d) a movement from the known to the unknown

4) The _____ relates to the conditions under which the observations are to be made.
   a) observational design b) sampling design
   c) statistical design d) operational design

5) Which one of the following is not an example of non-sampling error?
   a) measurement error
   b) refusal by a unit to respond
   c) editing error
   d) error due to selecting only a part of the population as sample

6) If n units are selected in a sample from N population units, the sampling fraction is given as _____.
   a) $\frac{1}{n}$ b) $\frac{N}{n}$
   c) $\frac{n}{N}$ d) $\frac{2}{n}$

7) Non-response in surveys means _____.
   a) Non-availability of respondents
   b) Non-return of questionnaire by respondents
   c) Refusal to give information by respondents
   d) All the above

8) A large city is subdivided into 150 non-overlapping blocks. Five blocks are selected at random and completely enumerated. This procedure is known as _____.
   a) Systematic sampling b) Stratified sampling
   c) Cluster sampling d) Partial census

B) Fill in the blanks. **04**

1) Gathering knowledge for knowledge's sake is termed _____ research.
2) The research is carried on over several time-periods is called as _____.
3) If a heterogeneous population can be easily divided into sub populations with relatively small variability between the subpopulations then appropriate sampling design is _____.
4) The probability of a specified unit being included in the sample under SRSWOR is _____.

Q.2 Answer the following. (Any Six) **12**

a) Define systematic sampling.
b) Define quota sampling.
c) What is meant by literature survey?
d) Define snowball sampling.
e) Define treatments.
f) Define judgement sampling.
g) Define empirical research.
h) What do you mean by non-response error?

Q.3 Answer the following. (Any Three) **12**

a) Describe objectives of research.
b) List down the steps involved in report writing.
c) Discuss sampling and non-sampling errors.
d) Prove: With usual notations, the bias of ratio estimator $\hat{R} = \frac{\bar{y}}{\bar{x}}$ is $B(\hat{R}) = \frac{-cov(\hat{R},\bar{x})}{\bar{X}}$

Q.4 Answer the following. (Any Two) **12**

a) Explain different types of research studies.
b) Obtain Des Raj estimator for population mean for PPSWOR method.
c) Prove: With usual notations, in PPSWR sampling, an unbiased estimator of population total is $\hat{Y} = \frac{1}{n}\sum_{i=1}^{n}\frac{yi}{pi}$ with $var(\hat{Y}) = \frac{1}{n}\sum_{i=1}^{N}p_i\left(\frac{yi}{pi} - Y\right)^2$

Q.5 Answer the following. (Any Two) **12**

a) Discuss research designs.
b) Discuss the meaning of the research.
c) Explain cumulative total method for unequal probability sampling.

SLR-HR-7

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April - 2024
STATISTICS
Real Analysis (MSC16101)

Day & Date: Friday, 10-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. No. 1 and Q. No 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The limit points of $\{1+(-1)^n, n\epsilon N\}$ are ______.
   a) 1, 0 b) 0, 2
   c) 1, 1 d) 2, 1

2) Monotonic bounded sequence is always ______.
   a) convergent b) divergent
   c) oscillatory d) may or may not be convergent

3) The set of limit points of finite set is ______.
   a) finite b) empty
   c) non empty d) infinite

4) A geometric series with common ratio r converges, if ______.
   a) $|r| < 1$ b) $|r| > 1$
   c) $r = 1$ d) None of these

5) Intersection of two closed sets is always _____.
   a) closed b) not closed
   c) open d) None of these

6) Set of all even natural numbers is ______.
   a) uncountable b) finite
   c) countable d) none of these

7) A sequence $\left\{\frac{1}{5n}, n\epsilon N\right\}$ is ______.
   a) convergent b) bounded
   c) having unique limit point d) all of these

8) A series $\sum\frac{1}{n^4}$ is ______ series.
   a) convergent b) divergent
   c) oscillatory d) none of these

9) A compact set is _______.
   a) closed set b) bonded set
   c) closed and bounded set d) none of these

10) The function $f(x) = x^2$ is ______.
   a) continuous b) discontinuous
   c) uniformly continuous d) none of these

B) State whether the following statements are True or False. **06**

1) A limit point of a set is always a member of the set.
2) A sequence converges to more than one point.
3) A subset of countable set is always countable.
4) Every differentiable function is continuous.
5) The function $f(x) = |x|$ is discontinuous.
6) The set of interior points for set (1,2) is (1,2)

Q.2 Answer the following. **16**

a) Prove that a set of integers is countable set.
b) Prove or disprove: Cauchy sequence is convergent sequence.
c) Write short note on the following:
1) Countable and uncountable sets
2) Bounded and unbounded sets.

d) Define and illustrate:
1) Greatest lower bound (Supremum)
2) Lowest upper bound (Infimum)

Q.3 Answer the following.

a) Show that: The set of the real numbers in [0,1] is uncountable.
b) Define closed and open set. Prove that, finite union of open sets is open.

Q.4 Answer the following.

a) Test the convergence of following series: **08**
1) $\sum \frac{n!}{n^n}$
2) $\sum \frac{n^2(n+1)^2}{(n+1)!}$

b) Explain limit superior and limit inferior of a sequence with application. **08**

Q.5 Answer the following.

a) State Leibnitz rule and its one application. **08**
b) Explain how to calculate Riemann integration of a continuous function. **08**

Q.6 Answer the following.

a) Optimize $f(x,y) = 6 - x^2 - y^2$ with the constraint $x + y - 2 = 0$ **08**
b) Define geometric series and verify its convergence for different values of common ratio. **08**

Q.7 Answer the following.

a) Find liminf, limsup and limit of following sequence **08**

$$S_n = 1 + \left[\frac{(-1)^n}{n}\right], n \epsilon N$$

b) Prove that a set is closed, if and only if its compliment is open. **08**

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Linear Algebra & Liner Models (MSC16102)

Day & Date: Monday, 13-05-2024 Max. Marks: 80
Time: 3:00 PM To 6:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) Let S be a vector space. Then the smallest spanning set for the vector space S is called as _____.
   a) Superset of S b) Span of S
   c) Subset of S d) Basis of S

2) The characteristic polynomial of matrix A is $\lambda^4 - 1$. Then, $A^4 =$ _____.
   a) cannot be known b) I
   c) A d) λA

3) Pre-multiplying by identity matrix will _____.
   a) Increase rank of given matrix
   b) decrease rank of given matrix
   c) Not change rank of given matrix
   d) Nothing can be said.

4) The eigen values of 2×2 matrix A are 2 and 7. Then _____.
   a) $|A| = 9$ b) $|A| = 14$
   c) Trace $(A) = 14$ d) Trace $(A) = 2$

5) Elementary transformation _____.
   a) Increases rank of matrix b) Decreases rank of matrix
   c) Do not alter rank of matrix d) Doubles the rank

6) If $A^{-1} = A'$, then matrix A is called as _____.
   a) Square matrix b) Orthogonal matrix
   c) Symmetric matrix d) None of these

7) The matrix with only one column is called as _____.
   a) Column matrix b) Row matrix
   c) Square matrix d) None of these

8) A set of vectors containing a null vector is _____.
   a) Not necessarily dependent
   b) Necessarily dependent
   c) Necessarily independent
   d) A vector space

9) The quadratic form $3{X_1}^2 + 2{X_2}^2$ is _____.
   a) positive definite b) negative definite
   c) positive semi definite d) negative semi definite

10) Caley-Hamilton theorem can be used to _____.
   a) obtain inverse of a matrix
   b) determine definiteness of a quadratic form
   c) characteristic roots of a matrix
   d) None of these

B) Fill in the blanks. **06**

1) If AB is invertible then $(AB)^{-1} =$ _____.
2) The rank of identity matrix of order 4 is _____.
3) If number of columns is greater than number of rows, then the matrix is called as _____.
4) If determinant of a square matrix is zero, then such matrix is called as _____.
5) _____ is the maximum number of linearly independent row vectors in a matrix.
6) M is negative definite matrix if and only if all of its Eigen values are _____.

Q.2 Answer the following **16**

a) Prove: In any vector space $V, \alpha.\underline{0} = \underline{0}$ for every scalar α.
b) Define a vector space stating all ten essential properties.
c) Define-
   i) Upper triangular matrix
   ii) Lower triangular matrix
d) Write a short note on
   i) Trace of a matrix
   ii) Determinant of a matrix

Q.3 Answer the following.

a) Define subspace. State the conditions needed to verify whether a subset of a vector space is a subspace. **08**
b) Determine whether following set is set of linearly dependent vectors. **08**

$$S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

Q.4 Answer the following.

a) Define norm of a vector. Explain Gram-Schmidt orthogonalisation process in detail. **08**
b) Define diagonal matrix **08**
Prove:
   i) Sum of two diagonal matrices is again a diagonal matrix of same order.
   ii) Product of two diagonal matrices is again a diagonal matrix of same order.

Q.5 Answer the following.

a) Define rank of a matrix. Reduce the following matrix to a row-reduced form and hence determine its rank. **08**

$$A = \begin{bmatrix} 1 & 3 & 8 \\ 5 & 2 & 1 \\ 7 & 6 & 1 \end{bmatrix}$$

b) Find the solution, if exists, for the following system: **08**

$$x + 2y + z = 0$$
$$2x + 3y + 3z = 0$$
$$-3x + 10y + 2z = 0$$

Q.6 Answer the following.

a) Define Moore-Penrose inverse. State and prove its properties. **08**

b) Prove or disprove: A linear parametric function $P'\beta$ is estimable if and only $P' \in R(X) \equiv R(X'X)$ **08**

Q.7 Answer the following.

a) Define generalized inverse (G-inverse) of a matrix. Show that G is g-inverse of A if and only if $AGA = A$. **08**

b) Show that rank of a matrix is unaltered by multiplication with a non-singular matrix. **08**

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Distribution Theory (MSC16103)

Day & Date: Wednesday, 15-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Let X and Y be iid $U(0,1)$ random variables then which of the following statement is correct?
   a) $X+Y$ is $U(0,2)$ b) $X-Y$ is $U(-1,1)$
   c) $1-Y$ is $U(-1,1)$ d) $1-X$ is $U(0,1)$

2) For which of the following distribution, $E(X)$ does not exist?
   a) Normal b) Uniform
   c) Cauchy d) Exponential

3) The pdf of the first order statistic in $f(x;\theta)=\theta e^{-\theta x}, x>0$ is _____.
   a) Exponential b) Uniform
   c) Beta d) none of these

4) Which of the following is a scale family?
   a) $U(0,\theta)$ b) $N(0,\sigma^2)$
   c) $Exp(\theta)$ d) all the above

5) If X is symmetric about α then $(X-\alpha)$ is symmetric about _____.
   a) α b) $1-\alpha$
   c) 0 d) 1

6) Let X and Y be independent random variables each having uniform distribution on $[-1,2]$. Then $Var(X+Y)$ is equal to _____.
   a) $5/2$ b) $3/2$
   c) 4 d) 6

7) Let X and Y be two iid random variables with pdf $f(x)=2e^{-2x}, x\geq 0$. The distribution of $Z=X-Y$ is _____.
   a) Laplace b) exponential
   c) beta d) Cauchy

8) Let $X_1, X_2, \dots, X_k$ is a multinomial random variate then $Cov(X_i, X_j), i=j=1,2\dots,k, i\neq j$ is _____.
   a) $n\,p_i$ b) np_ip_j
   c) $-np_ip_j$ d) $n^2p_ip_j$

9) The MGF of normal variable X is $M_X(t)=e^{3\,t+8\,t^2}$ then mean and variance of X are _____.
   a) $\mu=3, \sigma^2=8$ b) $\mu=3, \sigma^2=16$
   c) $\mu=8, \sigma^2=3$ d) None of these

10) If a random variable X has standard exponential distribution then _____.
   a) $E(X) = 2Var(X)$ b) $Var(X) = 2\,E(X)$
   c) $E(X) = Var(X)$ d) None of these

B) Fill in the blanks: **06**

1) If Z is standard normal variate then variance of Z^2 is _____.
2) Let X and Y are *iid* $N(0,1)$ variates. The distribution of $Z = Y/X$ is _____.
3) If $\mu_1' = 2, \mu_2' = 8$ and $\mu_3 = 3$ then value of μ_3' is ______.
4) The mean of probability density function

   $f(x) = \begin{cases} 12x^2(1-x), 0 \le x < 1 \\ 0, otherwise \end{cases}$ is _____.
5) Let $\underline{X} = (X_1, X_2, \dots, X_k)$ is a multinomial random variate with parameters

   $n, p_1, p_2, \dots, p_k, \sum_{i=1}^{k} p_i = 1.$ Then marginal distribution of X_1 is _____.
6) Let X be a $N(\mu, \sigma^2)$ variable. Then distribution of e^x is _____.

Q.2 Answer the following. **16**

a) Define location-scale family. Give one example of the same.
b) Let X has $U(0,\theta)$ distribution. Find the distribution of $Y = X/\theta.$
c) Let F be a distribution function of random variable X. Define $G(x) = [F(x)]^n, n$ is positive integer. Examine $G(x)$ to be a distribution function.
d) Derive the pdf of smallest order statistic based on random sample of size n from a continuous distribution.

Q.3 Answer the following.

a) Define distribution function of bivariate random variate (X, Y). State and prove its important properties. **08**
b) Define probability generating function (PGF) of a random variable. Let X has geometric distribution with pmf **08**
$P(X = x) = p\,q^x, x = 0,1,2\dots \quad 0 < p < 1,\ q = 1-p$
Obtain the PGF of X and hence mean and variance of X.

Q.4 Answer the following.

a) Define power series distribution. Obtain its moment generating function. Show that Poisson distribution is power series distribution. **08**
b) Let X is a non-negative continuous random with distribution function $F(x)$. **08**
If $E(X)$ exist then shown that $E(X) = \int_0^\infty [1 - F(u)]du.$

Q.5 Answer the following.

a) Define multinomial distribution. Obtain its MGF. Hence or otherwise obtain its variance-covariance matrix. **08**
b) State Jensen's inequality. Using Jensen's inequality, derive the inequality between AM, HM and GM. **08**

Q.6 Answer the following.

a) State and prove Holder's inequality. **08**

b) Let (X, Y) be a bivariate random variable with joint *pdf* given by **08**

$$f(x, y) = \begin{cases} 4x(1-y), 0 < x < 1, 0 < y < 1 \\ 0, elsewhere \end{cases}$$

Find:

i) Marginal distributions of X and Y

ii) Conditional distribution of X given $Y = y$

Q.7 Answer the following.

a) Let (X, Y) has $BVN\ (\mu_1, \mu_2, \sigma_1^2, \sigma_2^2\ \rho)$. Obtain the marginal distribution of X. **08**

b) Let $Y_1 < Y_2 <$……$< Y_n$ be the order statistics corresponding to n observations from a distribution with probability density function **08**

$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

show that the k^{th} order statistics Y_k has Beta distribution of first kind with parameters k and $n - k + 1$.

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Estimation Theory (MSC16104)

Day & Date: Friday, 17-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The MLE of parameter θ is a statistic which _____.
   a) is sufficient for parameter for θ
   b) maximizes the likelihood function L
   c) is a solution of $\frac{\partial \log L}{\partial \theta} = 0$
   d) is always unbiased

2) A sufficient statistic contains all the information which is contained in _____.
   a) population b) sample
   c) parameter d) none of the above

3) If T is an unbiased estimator of θ then T^2 is _____.
   a) unbiased estimator for θ^2
   b) biased estimator for θ^2
   c) unbiased estimator for $(\theta^2 + 1)$
   d) biased estimator for $(\theta^2 + 1)$

4) If T_n is an estimator of θ, then Cramer-Rao inequality provides a lower bound on _____.
   a) $Var(T_n)$ b) $E(T_n)$
   c) $Max(T_n)$ d) $Min(T_n)$

5) Posterior distribution is the _____.
   a) Joint distribution of X and θ
   b) Conditional distribution of X given θ
   c) Conditional distribution of θ given X
   d) none of these

6) A statistic $T(X)$ for θ is said to be ancillary if ______.
   a) $T(X)$ is independent of θ
   b) $T(X)$ is dependent on θ
   c) The distribution of $T(X)$ is independent of θ
   d) The distribution of $T(X)$ is depends on θ

7) Bhattacharya bound is the generalization of the _____.
   a) Rao-Blackwell theorem
   b) Cramer-Rao inequality
   c) Neyman-Pearson lemma
   d) Chapman-Robbins-Kiefer bound

8) Which of the following statements is / are correct?
   i) UMVUE is always unique if it exists.
   ii) UMVUE is provided by C-R lower bound only
   Select the correct answer using the code given below:
   a) i) only
   b) ii) only
   c) Both i) and ii)
   d) Neither i) nor ii)

9) Jeffrey's prior is given by _____.
   a) $\pi(\theta) \propto I_X(\theta)$
   b) $\pi(\theta) \propto \frac{1}{I_X(\theta)}$
   c) $\pi(\theta) \propto \sqrt{I_X(\theta)}$
   d) None of these

10) Let X_1, X_2 is a random sample of size 2 from *Poisson* $(\lambda), \lambda > 0$, moment estimator of λ is _____.
   a) $X_1 + X_2$
   b) $\frac{X_1 + X_2}{2}$
   c) $2X_1 - X_2$
   d) $\frac{(X_1 - X_2)^2}{2}$

B) Fill in the blanks. **06**

1) Based on random sample of size n from $N(\mu, 1), \mu \epsilon R$ population, MLE of μ is _____.
2) If T is an unbiased estimator for θ then $\phi(T)$ is unbiased for $\phi(T)$ when ϕ is a _____ function.
3) Bayes estimator of a parameter under absolute error loss function is — of posterior distribution.
4) The estimator of λ for exponential distribution with pdf $f(x, \lambda) = \lambda e^{-\lambda x}, 0 \leq x < \infty$ by method of moments is ______.
5) If a prior p.d.f. $\pi(\theta)$ contains no information about θ then it is called _____ prior.
6) Mean squared error (MSE) of an estimator T of θ is expressed as _____.

Q.2 Answer the following. **16**

a) Define:
   i) Complete sufficient statistic
   ii) Ancillary statistic

b) State and prove Basu's theorem. Give its one application

c) Define a maximum likelihood estimator for a parameter θ,and state the properties of estimator.

d) Define Fisher information in a single observation. Find the same for $B(n, \theta)$ distribution, when n is known.

Q.3 Answer the following.

a) Define Pitman family of distributions. Show that the following distributions belong to Pitman family, **08**
   i) $U\,(0, \theta)$
   ii) Exponential with location θ

b) Define complete family of distributions. Let random variable X has $N(\theta, 1)$ distribution. Show that family of X is complete. **08**

Q.4 Answer the following.

a) State and establish Chapman-Robins-Kiefer inequality. **08**

b) Derive a Cramer-Rao lower bound for an unbiased estimator of Poisson mean θ based on random sample of size n from Poisson distribution. **08**

Q.5 Answer the following.

a) Define UMVUE. State and prove Rao-Blackwell theorem. **08**

b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $U(0, \theta), \theta > 0$. Find UMVUE of **08**

i) θ

ii) θ^2

iii) $(1/\theta)$

Q.6 Answer the following.

a) Define moment estimator. Describe a method of moments for estimation and give an example for it. **08**

b) Obtain MLE of (μ, σ^2) based on a random sample of size n from $N(\mu, \sigma^2)$ distribution. **08**

Q.7 Answer the following.

a) Define prior and posterior distributions. Illustrate with one example for each of them. **08**

b) Let $X_1, X_2, \dots, X_n$ be a random sample from Poisson (λ). For estimating λ using quadratic error loss function, prior distribution of λ is $\pi(\lambda) = e^{-\lambda}, \lambda > 0$. Derive Bayes estimator of $\lambda \; and \; \Psi(\lambda) = e^{-\lambda}$ **08**

SLR-HR-11

Seat No.	

Set P

M.Sc. (Semester - I) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Statistical Computing (MSC16108)

Day & Date: Monday, 20-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) Let X be a continuous random variable, then its c. d. f. $F_x(x)$ follows _____.
   a) $U(0,1)$ b) Exp (1)
   c) $N(0,1)$ d) Cauchy $(0,1)$

2) The formula for generating random numbers from Linear Congruential method is _____.
   a) $X_{i+1} = (aX_i + b)(mod\ m)$ b) $X_{i+1} = (aX_i + b)/m)$
   c) $X_{i+1} = (aX_i + b) \times m$ d) $X_{i+1} = (aX_i + b)^m$

3) In Simpson's $\frac{3^{th}}{8}$ rule the numbers of intervals must be _____.
   a) odd b) even
   c) multiple of 3 only d) at least 6

4) To obtain one observation from bi-variate exponential distribution, we need to draw _____ exponential random numbers.
   a) One b) Two
   c) Three d) Four

5) Which of the following numerical methods uses approximate root to solve $f(x) = 0$?
   a) Bisection method b) Trapezoidal rule
   c) Secant method d) Newton – Raphson

6) In Bootstrap method _____ sampling method is used.
   a) SRSWOR b) SRSWR
   c) Stratified sampling d) Systematic sampling

7) In the direct search algorithm _____ are considered.
   a) Partial differentiation b) numerical integration
   c) corner points d) Centre points

8) The steepest ascent method is used to find _____ of the given function.
   a) minimum b) maximum
   c) nominal level d) mean

9) The two point Gauss - Legendre quadrature formula is _____.

a) $$\int_{-1}^{1} f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) + E(I)$$

b) $$\int_{-1}^{1} f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) - f\left(\frac{1}{\sqrt{3}}\right) + E(I)$$

c) $$\int_{-1}^{1} f(x)dx \cong f\left(\frac{-1}{\sqrt{3}}\right) + f(0) + f\left(\frac{1}{\sqrt{3}}\right) + E(I)$$

d) $$\int_{-1}^{1} f(x)dx \cong \frac{\pi}{2} f\left(\frac{-1}{\sqrt{2}}\right) + \frac{\pi}{2} f\left(\frac{1}{\sqrt{2}}\right) + E(I)$$

10) Jackknife estimator was introduced by _____.

a) Efron b) Quenouille
c) Fisher d) Jackknife

B) Fill in the blanks. **06**

1) _____ gradient search method is used to find the minimum of the given function.

2) If $U \sim U(0,1)$ then, $Z = \sum_{i=1}^{12} U_i - 6$ follows _____ distribution.

3) Jackknife technique is also known as _____.

4) In EM algorithm 'M' stands for _____.

5) To generate random numbers between 0 and 1 using linear congruential method, we have to use _____ formula.

6) If we have use Simpson's $1/3^{rd}$ and $3/8^{th}$ both approximations, then we have to take at least ______ sub intervals.

Q.2 Answer the following. **16**

a) Write a short note on search algorithm

b) Explain Jackknife technique as a bias reduction technique.

c) Describe Regula Falsi method.

d) Explain acceptance rejection method of random number generation. Write its algorithm.

Q.3 Answer the following.

a) State and prove the result to generate random numbers from $N(\mu, \sigma^2)$ distribution using Box-Muller transformation. Also write the algorithm. **08**

b) What is convolution of statistical distribution? State and prove the result of convolution for Poisson distribution. **08**

Q.4 Answer the following.

a) Explain Monte Carlo integration technique. **08**

b) Explain the Bisection method for finding solution to the equation $f(x) = 0$ **08**

Q.5 Answer the following.

a) Let $X_1, X_2, \ldots X_n$ be a random sample of size n from $N(\mu, \sigma^2)$, where μ is known. Obtain the jackknife estimator of σ^2 **08**

b) Obtain an algorithm to estimate universal constant e using Monte Carlo method. **08**

Q.6 Answer the following.

a) Explain Bootstrap technique and write its advantages. **08**

b) Describe gradient search method. Write algorithm for steepest descent and steepest ascent method. **08**

Q.7 Answer the following.

a) Explain theory of importance sampling with application to reduce Monte Carlo error. **08**

b) State and prove the result to generate n random numbers from Poisson (λ) distribution. **08**

SLR-HR-13

Seat No.	

Set P

M.Sc. (Semester - II) (New) (NEP CBCS) Examination: March/April-2024
STATISTICS
Stochastic Processes (2329201)

Day & Date: Thursday, 09-05-2024 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All Questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) Recurrent state is also called as _____.
   a) Persistent b) Transient
   c) Aperiodic d) None of these

2) For a non-null recurrent state 'i', the mean recurrent time is _____.
   a) $< \infty$ b) ∞
   c) 0 d) 1

3) The period of an aperiodic state is _____.
   a) 0 b) 1
   c) 2 d) 3

4) In a stochastic process, the variable under study is height of a student on n^{th} day. This is example of _____.
   a) Discrete time discrete state space stochastic process
   b) Discrete time continuous state space stochastic process
   c) Continuous time discrete state space stochastic process
   d) Continuous time continuous state space stochastic process

5) To find n-step transition probabilities, _____ are used.
   a) Newton equations
   b) Sterling's equations
   c) Chapman-Kolmogorov equations
   d) Lebesgue equations

6) If $\{N(t)\}$ is a counting process, then $N(0) =$ _____.
   a) 0 b) 1
   c) 10 d) 2.71

7) If states i and j are communicating states, then _____.
   a) state i leads to state j b) state j leads to state i
   c) either (a) or (b) d) both (a) and (b)

8) Branching process is an example of _____.
   a) Discrete time discrete state space stochastic process
   b) Discrete time continuous state space stochastic process
   c) Continuous time discrete state space stochastic process
   d) Continuous time continuous state space stochastic process

B) Fill in the blanks. **04**

1) The row sum of every row of a transition probability matrix (TPM) is always _____.
2) If probability 'p' of positive jump is 0.5 for a random walk, then it is called as _____.
3) A non-null recurrent aperiodic state is also called as _____.
4) If $\{N(t)\}$ is a Poisson process, then the inter-arrival times follow _____ distribution.

Q.2 Answer the following. (Any Six) **12**

a) Define State space.
b) Define period of a state.
c) Define communicating class.
d) Define counting process.
e) Let $\{X_n\}$ be a stochastic process with state space $= \{1,2,3\}$ and initial distribution [1/2,1/4,1/4]' and tpm P as

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/6 & 5/6 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

Then find $P(X_1 = 2)$
f) Define mean recurrent time of a state.
g) Define TPM of a stochastic process.
h) State properties of TPM of a stochastic process.

Q.3 Answer the following. (Any Three) **12**

a) Describe Poisson Process. State postulates of this process.
b) Define and illustrate Persistent and transient state.
c) Discuss stationary distribution of a Markov chain.
d) Explain the concept of first return and probability of ultimate return to a state.

Q.4 Answer the following. (Any Two) **12**

a) State and prove class property of periodicity.
b) Describe birth and death process and obtain its Kolmogorov differential equations.
c) If $\{N(t)\}$ is a Poisson process, then for $s < t$, obtain the distribution of $N(s)$, if it is already known that $N(t) = k$.

Q.5 Answer the following. (Any Two) **12**

a) Establish the equivalence between two definitions of Poisson process.
b) Verify the states of random walk model for persistency as well as for periodicity.
c) Give classification of Stochastic processes according to state space and time domain.

SLR-HR-14

Seat No.	

Set P

M.Sc. (Semester - II) (New) (NEP CBCS) Examination: March/April-2024
STATISTICS
Theory of Testing of Hypotheses (2329202)

Day & Date: Saturday, 11-05-2024 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 08

1) A size α test is said to be unbiased if ______.
   a) It has maximum power in the class of all size α tests.
   b) size and power are equal.
   c) power is smaller than size
   d) size of the test does not exceed its power

2) Family of Cauchy $(1, \theta)$ distribution ______.
   a) does not have MLR property
   b) belong to one parameter exponential family
   c) has mean θ
   d) has MLR property

3) In likelihood ratio test, under some regularity conditions on $f(x, \theta)$, the random variable $-2 \log \lambda(x)$ (where $\lambda(x)$ is a likelihood ratio) is asymptotically distributed as ______.
   a) normal b) exponential
   c) chi-square d) F distribution

4) The variance stabilizing transformation for Poisson population is ______.
   a) square root b) $\tanh^{-1}$
   c) Sin^{-1} d) logarithmic

5) Homogeneity of several population variances can be tested by _____.
   a) LRT b) Chi-square test
   c) Bartlett test d) Rao's test

6) The approximate distribution of Kruskal-Wallis test statistic is ______.
   a) standard normal b) Chi-square
   c) binomial d) None of these

7) Based on random sample of size n from $N(\theta, 1)$ distribution, the pivotal quantity for construction of confidence interval for θ is _____.
   a) $(\bar{X} - \theta)/\sqrt{n}$ b) $(\bar{X} - \theta)/n$
   c) $\sqrt{n}\,(\bar{X} - \theta)$ d) $n(\bar{X} - \theta)$

8) For testing simple hypothesis H_0 against simple alternative H_1, let power of two MP tests at levels α_1 and α_2 be $(1 - \beta_1)$ and $(1 - \beta_2)$ respectively. Then always _____.
   a) $\beta_1 \geq \beta_2$ b) $\beta_1 \leq \beta_2$
   c) $\beta_1 = \beta_2$ d) $(\beta_1/\beta_2) = (\alpha_1/\alpha_2)$

B) Fill in the blanks. **04**

1) Level of significance is the probability of _____ error.
2) In testing independence in a 3×4 contingency table, the number of degrees of freedom in χ^2 distribution is _____.
3) A test function which takes either value 0 or 1 is called ____ test function.
4) Generalized NP lemma is used to construct ____ tests.

Q.2 Answer the following. (Any Six) **12**

a) Explain simple and composite hypotheses with suitable example.
b) Define level of significance and size of a test.
c) Show that one parameter exponential family has monotone likelihood ratio.
d) Define pivotal quantity. Give an example.
e) Define confidence set and UMA confidence set of level $(1-\alpha)$.
f) What is goodness of fit test? Give its application.
g) State the generalized Neyman-Pearson lemma.
h) Define similar test and test having Neyman structure.

Q.3 Answer the following. (Any Three) **12**

a) Define most powerful (MP) test. Show that MP test need not be unique using suitable example.
b) Describe variance stabilization transformation for a Poisson population.
c) Write a note on: Test for independence of attributes.
d) A sample of size one is taken from Poisson distribution with parameter λ . To test the hypothesis $H_0\colon \lambda = 1$ against $H_1\colon \lambda = 2$, consider the test

$$\phi(x) = \begin{cases} 1, if\ x > 3 \\ 0, otherwise \end{cases}$$

Find the probability of type I error and power of the test.

Q.4 Answer the following. (Any Two) **12**

a) Define monotone likelihood ratio (MLR) property of a family of distributions. Explain the use of MLR in the construction of UMP test with the help of suitable example.
b) Obtain a most powerful test of size α for testing $H_0 : \sigma = \sigma_0$ against $H_1\colon \sigma = \sigma_1 (> \sigma_0)$ based on a random sample of size n from $N(\mu, \sigma^2)$, where μ is known.
c) Describe Kruskal Wallis test for analyzing data in one-way classification.

Q.5 Answer the following. (Any Two) **12**

a) Define likelihood ratio test. Show that LRT for testing simple hypothesis against simple alternative is equivalent to MP test.
b) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\theta, 1)$ distribution. Obtain LRT for testing $H_0\colon \theta = \theta_0$ against $H_1\colon \theta \neq \theta_0$.
c) Let $X_1, X_2, \dots, X_n$ be a random sample of size n from $N(\mu, \sigma^2)$population, when μ is known. Obtain $100(1-\alpha)\%$ confidence interval for σ^2.

SLR-HR-15

Seat No.	

Set P

M.Sc. (Semester - II) (New) (NEP CBCS) Examination: March/April-2024
STATISTICS
Probability Theory (2329207)

Day & Date: Tuesday, 14-05-2024 Max. Marks: 60
Time: 11:00 AM To 01:30 PM

Instructions: 1) All questions are compulsory.
2) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **08**

1) if $x \in A$ implies $x \in B$, then ______.
   a) $A \subset B$ b) $B \subset A$
   c) $A = B$ d) all of these

2) Convergence in probability always implies _____.
   a) Convergence in distribution b) Convergence in mean
   c) Convergence almost sure d) None of these

3) A mapping X is said to be a random variable if _____.
   a) X is one-to-one b) X is many-to-one
   c) X is linear d) None of these

4) If X_1 and X_2 two are independent random variables, then $\varphi_{X_1+X_2}(t) =$ ______.
   a) $\varphi_{X_1}(t).\varphi_{X_2}(t)$ b) $\varphi_{X_1}(t) + \varphi_{X_2}(t)$
   c) $\varphi_{X_1}(t) - \varphi_{X_2}(t)$ d) None of these

5) The indicator function of a random variable is a _____ function.
   a) Simple b) Elementary
   c) Infinite d) None of these

6) If for two independent events A and $B, P(A) = 0.2, P(B) = 0.6$, then $P(A \cup B) =$ _____.
   a) 0.8 b) 0.78
   c) 0.12 d) 0.68

7) If F is a σ -field, then which of the following is not correct?
   a) F is a field.
   b) F is a class closed under countable unions.
   c) F is a minimal sigma field.
   d) F is a class closed under complementation.

8) If F_1 and F_2 are two fields defined on subsets of Ω, then which of the following is/are always a field?
   a) $F_1 \cup F_2$ b) $F_1 \cap F_2$
   c) both (a) and (b) d) neither (a) nor (b)

B) Fill in the blanks. **04**

1) If P is a probability measure defined on $(\Omega, \mathcal{A})$, then $P(\Omega)$= _____.
2) A _____ function is a countable linear combination of indicators of set.
3) The σ - field generated by the intervals of the type $(-\infty, x), x \in R$ is called _____.
4) The collection of all subsets of Ω is called as _____.

Q.2 Answer the following. (Any Six) **12**

a) Define σ- field.
b) Define Lebesgue measure.
c) Define almost sure convergence.
d) State Kolmogorov's three series theorem for almost sure convergence.
e) Define characteristic function of a random variable.
f) Define elementary random variable.
g) Prove or disprove: Union of two fields is a field.
h) Prove: If $P(.)$ is a probability measure, then $P(\Phi) = 0$.

Q.3 Answer the following. (Any Three) **12**

a) Define conditional probability measure. Show that it is also a probability measure.
b) Prove that inverse mapping preserves all set relations.
c) Define mixture of two probability measures. Show that mixture is also a probability measure.
d) Write a note on Lebesgue-Stieltje's measure.

Q.4 Answer the following. (Any Two) **12**

a) State and prove monotone convergence theorem.
b) Prove that probability measure is a continuous measure.
c) Discuss limit superior and limit inferior of a sequence of sets. Find the same For sequence $\{A_n\}$ where $A_n = \left(0, 3 + \frac{(-1)^n}{n}\right), n \in N$

Q.5 Answer the following. (Any Two) **12**

a) Prove that collection of sets whose inverse images belong to a σ-field, is a also a σ-field.
b) Discuss convergence in probability and convergence in distribution.
c) Prove any three properties of characteristic function.

SLR-HR-18

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Probability Theory (MSC16201)

Day & Date: Thursday, 09-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The universal set has ______ probability.
   a) Zero b) One
   c) 0.5 d) None of these

2) Which of the following is not correct?
   a) Empty set always has probability zero
   b) Set with probability zero is always an empty set
   c) Sample has always has probability one
   d) None of these

3) If $\{A_n\}$ is increasing sequence of sets, then the sequence $\{A_n^C\}$ is _____.
   a) Decreasing b) Increasing
   c) Need more information d) None of these

4) A class F is said to be closed under finite intersection, if $A, B \in \mathfrak{F}$ is implies.
   a) $A \cap B \in \mathfrak{F}$, for all $A, B \in \mathcal{F}$ b) $A^C \in \mathfrak{F}, B^C \in \mathfrak{F}$
   c) both (a) and (b) d) None of these

5) If F_1 and F_2 are two fields, then ______ is a field.
   a) $F_1 \cap F_2$ b) $F_1 \cup F_2$
   c) Both (a) and (b) d) Neither (a) nor (b)

6) The sequence of sets $\{(o, n), n = 1,2,3, \dots\}$ is _____.
   a) Convergent b) Divergent
   c) Oscillatory d) None of these

7) If $A \subset B$, then $P(A) \dots . P(B)$.
   a) $<$ b) $\leq$
   c) $=$ d) $>$

8) If for events A and B, $A \cup B = \Omega$ then these events are called as ______.
   a) exhaustive b) Exclusive
   c) both (a) and (b) d) Complementary

9) Convergence in probability always implies ______.
   a) Convergence in distribution b) Convergence in mean
   c) Convergence almost sure d) None of these

10) Expectation of a simple non-negative random variable satisfies _____.
   a) Linearity property b) Scale preserving property
   c) Non-negativity property d) All of these

B) Fill in the blanks. **06**

1) A _____ function is a countable linear combination of indicators of set.
2) If P is a probability measure defined on $(\Omega, \mathbb{A})$, then $P(\Omega) =$ ______.
3) Convergence in probability always implies ______.
4) The largest field of subsets of Ω is called as ______.
5) The indicator function of a random variable is a ______ function.
6) If for two independent events A and B, $P(A) = 0.2$, $P(B) = 0.6$, then $P(AUB) =$ ______.

Q.2 Answer the following. **16**

a) Discuss Lebesgue measure.
b) Define $\lim\inf$ of sequence of sets $\{A_n\}$
c) Discuss Indicator function.
d) Describe mixture of probability measures.

Q.3 Answer the following.

a) Define $\sigma - field.$ Prove that an arbitrary intersection of $\sigma - fields$ is also $\sigma - field$. **08**
b) Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence. **08**

Q.4 Answer the following.

a) Define $\lim\inf$ and $\lim\sup$ of a sequence of sets. With usual notations show that **08**
$\overline{\lim}(A_n \cup B_n) = \overline{\lim}\, A_n \cup \overline{\lim}\, B_n$.
b) Define limit of sequence of sets. Prove or disprove: If $\lim A_n$ exists then $\lim A_n^C$ also exists. **08**

Q.5 Answer the following.

a) Discuss the convergence in probability and convergence in distribution. **08**
b) Define: **08**
1) Weak law of large numbers
2) Strong law of large numbers
3) Central limit theorem

Q.6 Answer the following.

a) Define a field. Examine for the class of finite or co-finite sets to be a field. **08**
b) Prove that $P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$. **08**

Q.7 Answer the following.

a) Prove that inverse mapping preserves all the set relations. **08**
b) Define Pairwise and mutual independence of events. State the relationship between them. **08**

Seat No.	

Set **P**

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Stochastic Processes (MSC16202)

Day & Date: Saturday, 11-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct options. **10**

1) The state space and time domain for branching process are _____ respectively.
   a) discrete and discrete
   b) discrete and continuous
   c) continuous and discrete
   d) continuous and continuous

2) A Markov chain is completely specified by _____ and TPM.
   a) States b) State space
   c) Initial distribution d) None of these

3) For a non-null recurrent state 'i', the mean recurrent time is _____.
   a) $< \infty$ b) ∞
   c) 0 d) 1

4) Pure birth process is also called as _____.
   a) Yule-Furry process b) Poisson process
   c) Martingale process d) None of the above

5) Suppose $\{X_n, n \geq 0\}$ be a markov chain, then state j is transient iff _____.
   a) $\sum p_{jj}^{(n)} = 1$ b) $\sum p_{jj}^{(n)} = \infty$
   c) $\sum p_{jj}^{(n)} < \infty$ d) $\sum p_{jj}^{(n)} < 1$

6) In a Branching process if $E(X_1) = m$, then $E(X_n) =$_____.
   a) N b) m^n
   c) n^m d) None of these

7) Addition of two independent Poisson processes is _____.
   a) a Poisson process
   b) may or may not be Poisson process
   c) a Bessel process
   d) None of these

8) The state space and time domain for random walk model are _____ respectively.
   a) discrete and discrete b) discrete and continuous
   c) continuous and discrete d) continuous and continuous

9) The process $\{X(t), t > 0\}$, where $X(t)$ = number of particles in a room at time t, is an example of _____ stochastic process.
   a) discrete time continuous state space.
   b) discrete time discrete state space
   c) continuous time continuous state space
   d) continuous time discrete state space

10) In Markov analysis, the likelihood that any system will change from one state to the next is revealed by the _____.
   a) identity matrix
   b) transition-elasticities
   c) matrix of state probabilities
   d) matrix of transition probabilities.

B) Fill in the blanks. **06**

1) If period of a state is one, then the state is called as _____.
2) If X_n denotes number of active cases of COVID on n^{th} day, then $\{X_n\}$ is _____ time discrete state space stochastic process.
3) A state I is said to be accessible from state j, if _____.
4) If $\{N(t)\}$ is a Poisson process, then the inter-arrival times follow _____.
5) Number of accidents because of high speed of vehicle by time $t(> 0)$ is an example of _____ time, _____ state space stochastic process.
6) If the probability of ultimate first return $F_{ii} < 1$ then the state i is _____.

Q.2 Answer the following. **16**

a) Write a note on Stochastic process.
b) Write a note on first return probability for a state.
c) Discuss in detail null recurrent state.
d) Write a note on Poisson process.

Q.3 Answer the following.

a) Prove or disprove: Periodicity is a class property. **08**
b) Prove that a state j of a Markov chain is recurrent if and only if $\sum p_{jj(n)} = \infty$ **08**

Q.4 Answer the following.

a) Prove that communication is an equivalence relation. **08**
b) A Markov chain with state space $S = \{1,2,3\}$ has tpm $\begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$ It is known that the process has started with the state $X_0 = 2$. **08**
   1) $P(X_1 = 2)$
   2) $P(X_2 = 3)$
   3) $P(X_0 = 1)$
   4) $P(X_3 = 2/X_1 = 1)$

Q.5 Answer the following.

a) Write down the algorithm for the simulation of Poisson process and branching process. **08**
b) If $\{N(t)\}$ is a Poisson process, then for $s < t$, obtain the distribution of $N(s)$, if it is already known that $N(t) = k$. **08**

Q.6 Answer the following.

a) Define stochastic process. Prove that, Markov chain is completely specified by one step t.p.m. and initial Distribution. **08**

b) Calculate the extinction probability for branching process. **08**

Q.7 Answer the following.

a) State Markov property for stochastic process. State and prove Chapman-Kolmogorov equation for Markov chain. **08**

b) Discuss stationary distribution of a Markov chain in detail. Illustrate with the help of example. **08**

SLR-HR-20

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April-2024
STATISTICS
Theory of Testing of Hypotheses (MSC16203)

Day & Date: Tuesday, 14-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) A UMP test _____.
   a) is biased test b) is an unbiased test
   c) always exist d) none of these

2) For testing simple null against simple alternative hypothesis which of the following statements is most appropriate?
   a) UMP level α test exists
   b) UMPU level α test exists
   c) UMP invariant test exists
   d) Most powerful level α test exists

3) Let $H_0: \mu = 5, \mu$ is the mean of normal population from which sample is taken
   H_1: Population follows a standard normal distribution.
   a) both H_0 and H_1 are simple
   b) H_0 is simple and H_1 is composite
   c) H_0 is composite and H_1 is simple
   d) both H_0 and H_1 are composite

4) Uniform distribution $U(0, \theta)$ _____.
   a) has a MLR property.
   b) belongs to one parameter exponential family.
   c) both (a) and (b)
   d) neither (a) nor (b)

5) The test $\phi(x) \equiv \alpha,\ \forall x \in X$ is _____.
   a) UMP b) MP
   c) unbiased d) biased

6) Let $X_1, X_2, \ldots, X_n$ are iid with $N(\theta, 1)$. Let $H_0: \theta = \theta_0$ and $H_1: \theta \neq \theta_0$. For any $\alpha,\ 0 < \alpha < 1$, _____.
   a) there exists a UMP level α test.
   b) there does not exist a UMP level α test.
   c) there exists a test with one sided.
   d) none of these

7) The non-parametric test for goodness of fit of a distribution is _____.
   a) Run test b) Kolmogorov-Smirnov test
   c) Median test d) Sign test

8) In likelihood ratio test, under some regularity conditions on $f(x,\theta)$, the random variable $-2\log\lambda(x)$ (where $\lambda(x)$ is a likelihood ratio) is asymptotically distributed as _____.
   a) chi-square b) exponential
   c) normal d) F distribution

9) In a Chi-square test, the contingency table has 4 rows and 4 columns. What is the number of degrees of freedom?
   a) 3 b) 4
   c) 8 d) 9

10) Based on random sample of size n from $N(\theta,1)$ distribution, the pivotal quantity for construction of confidence interval for θ is _____.
   a) $(\bar{X}-\theta)/\sqrt{n}$ b) $(\bar{X}-\theta)/n$
   c) $\sqrt{n}(\bar{X}-\theta)$ d) $n(\bar{X}-\theta)$

B) Fill in the blanks. **06**

1) The power of a test is related to the probability of _____ error.
2) For testing simple versus simple hypotheses MP and LRT tests are _____.
3) If there are 10 symbols of two types, equal in number, the maximum possible number of runs is _____.
4) The approximate distribution of Kruskal-Wallis test statistic is _____.
5) The range of Kendall's rank correlation τ is _____ to _____.
6) Generalized NP lemma is used to construct _____ tests.

Q.2 Answer the following. **16**

a) Explain probabilities of type I and type II errors.

b) Define
   i) Similar test and
   ii) test having Neyman structure

c) Describe a goodness of fit test based on chi-square distribution.

d) Describe Wilcoxon's signed-rank test for one sample problem.

Q.3 Answer the following

a) Define MP test. Prove that power of MP test for testing simple hypothesis against simple alternative is greater than its size. **08**

b) Let X be $B(3,p)$. Let $H_0: p = 1/4$ and $H_1: p = 1/2$. Consider the test function **08**

$$\phi(x) = \begin{cases} 0.3, & \text{if } x = 0 \\ 0.2, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the power and probability of type I error of the test function.

Q.4 Answer the following

a) Define monotone likelihood ratio (MLR) of probability distributions. Show that exponential distribution with mean θ Possesses MLR property. **08**

b) To test $H_0: \theta = 0$ against $H_1: \theta = 1$ for a single observation from the distribution **08**

$f(x,\theta) = \frac{2(x+\theta)}{1+2\theta}, 0 < x < 1$ is used. Find MP test of level α and its power.

Q.5 Answer the following

a) Explain: **08**

i) UMP test

ii) Unbiased test

iii) UMP unbiased test

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $U(0, \theta)$ distribution. Obtain UMP level α test for testing $H_0: \theta \leq \theta_0$ against $H_1: \theta > \theta_0$. **08**

Q.6 Answer the following.

a) Define shortest length confidence interval. Explain the method of finding shortest length confidence interval for a real parameter. **08**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\theta, 1)$. Obtain shortest length confidence interval for θ. **08**

Q.7 Answer the following.

a) Describe Mann-Whitney U test. **08**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta, \sigma^2)$; σ^2 is unknown. Derive LRT to test $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$. **08**

SLR-HR-21

Seat No.	

Set P

M.Sc. (Semester - II) (Old) (CBCS) Examination: March/April – 2024
STATISTICS
Sampling Theory (MSC16206)

Day & Date: Thursday, 16-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) Probability of including a specified unit in a sample of size n out of population N units is ______.
   a) $\frac{1}{N}$ b) $\frac{\eta}{N}$
   c) $\frac{1}{\eta!}$ d) $\frac{1}{\binom{N}{\eta}}$

2) In which situation two stage sampling is better than simple random sampling?
   a) When the elements in the same stage are positively correlated.
   b) When the elements in the same stage are negatively correlated.
   c) When the elements in the same stage are uncorrelated.
   d) None of the above

3) Under proportional allocation, the size of sample from each stratum depends on: ______.
   a) Total sample size b) Size of the strata
   c) Population size d) All the above

4) In usual notations, Horwitz-Thompson estimator of population total is, ____.
   a) $\sum_{i=1}^{n} \frac{aiyi}{\pi_i}$ b) $\sum_{i=1}^{N} \frac{aiYi}{\pi_i}$
   c) $\sum_{i=1}^{n} \frac{yi}{\pi_i}$ d) $\sum_{i=1}^{n} \frac{yi}{n\pi_i}$

5) In systematic random sampling, if the sampling interval is k and the population size $N = nk$ then $V(\bar{y}_{sy})$ is given by ______.
   a) $\frac{N-n}{nN} S_{wst}^2[1 + (n-1)\rho_{wst}]$
   b) $\frac{N-n}{nN} S_{wst}^2[1 - (n-1)\rho_{wst}]$
   c) $\frac{N+n}{nN} S_{wst}^2[1 + (n-1)\rho_{wst}]$
   d) $\frac{N-n}{nN} S_{wst}^2[(n-1)\rho_{wst} - 1]$

6) Sampling error can be reduced by ______.
   a) Choosing a proper probability sampling.
   b) Selecting a sample of adequate size.
   c) Using suitable estimator
   d) All the above

7) In cluster sampling in usual notations, $N = 10, n = 3, M = 30, S^2 = 6$, $\rho = 0.4$, variance of sample mean per element is ______.
   a) 0.6512 b) 0.6152
   c) 0.6215 d) 0.6145

8) Regression estimator is equally efficient to ratio estimator if ______.
   a) $R = \rho \frac{S_Y}{S_X}$ b) $R = \rho \frac{S_X}{S_Y}$
   c) $R = \rho \frac{S_Y^2}{S_X^2}$ d) $R = \rho \frac{S_X^2}{S_Y^2}$

9) If the cost per unit of survey for all units is same then the $V(\bar{y}_{st})$ under Neyman allocation is,
   a) $\frac{1}{n}\left[\sum_{h=1}^{k} W_h S_h\right]^2 - \frac{1}{N}\sum_{h=1}^{k} W_h S_h^2$
   b) $\frac{1}{N}\left[\sum_{h=1}^{k} W_h S_h^2\right] - \frac{1}{n}\sum_{h=1}^{k} W_h S_h$
   c) $\frac{1}{N}\left[\sum_{h=1}^{k} W_h S_h\right]^2 - \frac{1}{N}\sum_{h=1}^{k} W_h S_h^2$
   d) $\frac{1}{N}\left[\sum_{h=1}^{k} W_h S_h^2\right] - \frac{1}{n}\sum_{h=1}^{k} W_h S_h^2$

10) Suppose that, in cluster sampling S_w^2, represents the variance within the clusters and s_b^2 between the clusters then the relation between and is ____.
   a) $S_w^2 = S_b^2$ b) $S_w^2 \geq S_b^2$
   c) $S_w^2 \leq S_b^2$ d) $S_w^2 \neq S_b^2$

B) Fill in the blanks. **06**

1) Determination of sample size for each stratum subject to the cost constrain is known as _____ allocation.
2) Efficiency of cluster sampling ______ as the cluster size decreases.
3) Under probability proportional to size sampling, a unit has ______ chance of being included in the sample than a unit smaller to it.
4) If information is not available on certain items of questionnaire, then it is called as ______.
5) In case of double sampling, an unbiased estimator of population mean is ______.
6) Hartley-Ross unbiased type estimator of population mean is, ______.

Q.2 Answer the following **16**

a) If a simple random sample without replacement of size n clusters is drawn from the population of N clusters with size of i^{th} cluster M_i, then derive an unbiased estimator of population mean with its variance.

b) Show that variance of an unbiased estimator of population mean in proportional allocation is larger than that of in optimal allocation.

c) Explain how variance of an unbiased estimator of population mean is estimated using inter-penetrating systematic sample. Also explain need of inter-penetrating systematic sample.

d) In case of probability proportional to size sampling show that,

1) $\sum_{i=1}^{N} \pi_i = n$

2) $\sum_{j\neq i=1}^{N} \pi_j = n - \pi_i$

3) $\sum_{j\neq i=1}^{N} \pi_{ij} = (n-1)\pi_i$

4) $\sum_{i=1}^{N}\sum_{j<i}^{N} \pi_{ij} = \frac{n(n-1)}{2}$

where π_i and π_{ij} are inclusion probabilities.

Q.3 Answer the following.

a) Suppose there are two strata with relative sizes $W_1 = \frac{N_1}{N}$ and $W_2 = \frac{N_2}{N}$ and sample sizes n_1, n_2. If the mean square errors for both the strata, S_1 and S_2 are equal then for a given cost function, $C = c_1 n_1 + c_2 n_2$, prove or disprove, (ignoring finite population correction) **08**

$$\left[\frac{Var_{prop}}{Var_{opt}}\right] = \frac{W_1 c_1 + W_2 c_2}{[W_1\sqrt{c_1} + W_2\sqrt{c_2}]^2}$$

where Var_{prop} and Var_{opt} are variances of an unbiased estimator of population mean under proportional and optimal allocation.

b) If the cost function is of the form, $C = C_0 + \sum_{h=1}^{k} t_h \ln(\sqrt{n_h})$ where C_0 and t_h are known numbers. In order to minimize the variance of an unbiased estimator of population mean, $\bar{y}_{st}$, in stratified random sampling with k strata for fixed cost, examine whether the condition on h^{th} stratum size, n_h, is $n_h \propto \frac{W_h S_h^2}{t_h}$ with justification. **08**

Q.4 Answer the following.

a) In systematic sampling, in usual notations, prove that mean of systematic sample $(\bar{y}_{sys})$ is an unbiased estimator of population mean. Show that **06**

$$var(\bar{y}_{sys}) = \frac{N-1}{N}S^2 - \frac{n-1}{n}S_{wsy}^2$$

where S_{wsy}^2 is variance among units belonging to the same systematic sample and S^2 is variance for entire population. Derive the condition when systematic sampling is more precise than simple random sampling without replacement.

b) Show that for populations with linear trend and $N = nk$, **10**

$$V_{st}: V_{sy}: V_r :: \frac{1}{n} : 1 : \frac{nk+1}{k+1}$$

where V_{st}, V_{sy}, V_r stands for variance of estimated sample mean for stratified, systematic and simple random sampling.

Q.5 Answer the following.

a) State Horwitz-Thompson estimator of population total. Show that it is an unbiased estimator and obtain its variance. Obtain Yates-Grundy form of variance of Horwitz-Thompson estimator of population total. **08**

b) Derive the condition when the Hansen-Hurwitz estimator, in case of probability proportional to size (PPS) sample drawn with replacement is more precise than simple random sampling with replacement. **08**

Q.6 Answer the following.

a) Define a ratio estimator of population mean when a simple random sample of size n is drawn from a population of N units without replacement. Derive its approximate mean square error. **06**

b) Suppose a preliminary random sample of size n' is selected from the population of N units without replacement and then stratified to k strata. A second random sample of size $n_h = g_h n'_h$ is drawn without replacement from h^{th} stratum of size n'_h; g_h is fixed and $0 < g_h \leq 1$. Show that $\bar{y}_{std} = \sum_{h=1}^{k} W_h \bar{y}_h$ is an unbiased estimator of population mean where $w_h = \frac{n_h}{n'}$ and $\bar{y}_h$ is sample mean of h^{th} stratum. Also obtain variance of $\bar{y}_{std}$. **10**

Q.7 Answer the following.

a) Suppose a simple random sample of size n' is selected without replacement from population of N units, of which only n'_1 were responded. Another simple random sample of size $n_2 = \frac{n'_2}{k}$; $k > 1$ is drawn without replacement from non-respondents. If $\bar{y}_1$ and $\bar{y}_2{}'$ are the sample means of units which respond at the first and second attempts respectively, then **06**

1) Prove that $\bar{y} = \frac{n_1\bar{y}_1 + n_2\bar{y}'_2}{n}$ is an unbiased estimator of population mean.
2) Obtain $V(\bar{y})$.

b) In case of two stage sampling when n primary stage units and m secondary stage units are drawn from the population using simple random sampling without replacement at both stages, obtain the expressions for m and n so that variance of the estimator of population mean per element is minimized for fixed total cost of the form $C = nC_1 + nmC_2$ **10**

SLR-HR-23

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
STATISTICS
Asymptotic Inference (MSC16301)

Day & Date: Friday, 10-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative **10**

1) For Cauchy distribution with location θ, the consistent estimator of θ is _____.
   a) sample mean b) sample median
   c) sample variance d) None of these

2) Consider the following statements:
   i) Joint consistency implies marginal consistency.
   ii) Marginal consistency implies joint consistency.
   Which of the above statements is / are true?
   a) only i) b) only ii)
   c) both i) and ii) d) neither i) nor ii)

3) Let T_n be an unbiased and consistent estimator of θ. Then T_n^2 as an estimator of θ^2 is _____.
   a) biased and consistent
   b) unbiased and consistent
   c) unbiased and inconsistent
   d) biased and inconsistent

4) In case of $N(\mu, \sigma^2), \mu \in R, \sigma^2 > 0$, the MLE of σ^2 is _____.
   a) unbiased and consistent
   b) biased and consistent
   c) unbiased and not consistent
   d) biased and not consistent

5) An estimator $\bar{X}_n$ based on sample of size n from $B(1, \theta)$ is _____.
   a) unbiased b) consistent
   c) CAN d) All the above

6) The family $\{U(0, \theta), 0 > 0\}$ is _____.
   a) one parameter exponential family
   b) Cramer family
   c) both a and b
   d) neither a nor b

7) For sufficiently large sample size, the likelihood equation admits _____.
   a) unique solution
   b) consistent solution
   c) both a and b
   d) neither a nor b

8) Let $X_1, X_2, \dots, X_n$ be iid from Poisson (θ) and $\bar{X}_n$ is CAN for θ. Then CAN estimator of $P_\theta(X = 0)$ is _____.
   a) $\bar{X}_n$ b) $e^{-\bar{X}_n}$
   c) $\bar{X}_n\, e^{-\bar{X}_n}$ d) None of these

9) The asymptotic distribution of LRT statistic is _____.
   a) Normal b) t
   c) chi-square d) F

10) Let $X_1, X_2, \dots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Let T_1 represent the moment estimator and T_2 represent the MLE of σ^2. Then which of the following is true?
   a) $T_1 = T_2$ b) $T_1 \neq T_2$
   c) $Var\,(T_1) > Var(T_2)$ d) $Var\,(T_1) < Var(T_2)$

B) Fill in the blanks. **06**

1) If T_n is consistent estimator of θ then $\phi(T_n)$is consistent estimator of $\phi(\theta)$ if ϕ is _____ function.
2) Consistency of an estimator is a _____sample property only.
3) In testing independence in a 3×4 contingency table, the number of degrees of freedom in χ^2 distribution is _____.
4) The asymptotic distribution of Wald's statistic is ______
5) The variance stabilizing transformation for Poisson population is _____.
6) Cramer family is _____ than exponential family.

Q.2 Answer the following. **16**

a) Define
   i) Consistent estimator
   ii) BAN estimator

b) Give an example of consistent estimator which is not MLE.

c) Describe Wald's test score test. State its asymptotic distributions

d) Let $X_1, X_2, \dots, X_n$ be $iid\ B(1, \theta)$. Show that sample mean $\bar{X}_n$ is consistent for θ.

Q.3 Answer the following

a) Define a joint and marginal consistency for a vector parameter. Show that joint consistency is equivalent to marginal consistency. **08**

b) Let $X_1, X_2, \dots, X_n$ be iid from exponential distribution with mean θ. Obtain consistent estimator for median. Hence, obtain consistent estimator for mean of the distribution. **08**

Q.4 Answer the following.

a) Define CAN estimator for a real parameter θ. State and prove invariance property for a CAN estimator. **08**

b) Let $X_1, X_2, \dots, X_n$ be $iid\ U\{\theta, 1), \theta > 0$. Show that $2\bar{X}_n$ is CAN for θ but $X_{(n)}$ is not CAN for θ. **08**

Q.5 Answer the following.

a) State Cramer regularity conditions in one parameter set up. Give an example of distribution which satisfies Cramer regularity conditions. Justify your answer. **08**

b) Let $X_1, X_2, \dots, X_n$ be $iid\ N(\theta, 1)$. Let $\phi(\theta) = \theta^2$. Obtain CAN estimator for $\phi(\theta)$ **08**

Q.6 Answer the following.

a) Explain variance stabilizing transformations and illustrate their use in large sample confidence intervals. **08**

b) Obtain the variance stabilizing transformation for exponential distribution with mean θ. Using the same, obtain $100(1-\alpha)\%$ confidence interval for θ. **08**

Q.7 Answer the following.

a) Define likelihood ratio test (LRT). Derive its asymptotic distribution. **08**

b) Let $X_1, X_2, \dots, X_n$ be iid $N(\theta_1, \theta_2)$. Obtain moment estimator of (θ_1, θ_2). Show that it is CAN. Obtain its asymptotic variance-covariance matrix. **08**

SLR-HR-24

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
STATISTICS
Multivariate Analysis (MSC16302)

Day & Date: Monday, 13-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1and 2 are compulsory.
2) Attempt any Three questions from Q. No. 3 to Q. No. 7
3) Figures to the right indicate full marks.

Q.1 A) Choose Correct Alternative. **10**

1) Generalised variance is _______ of covariance matrix.
   a) trace+ determinant b) Trace
   c) Determinant d) None of these

2) A principal component analysis was run and the following eigen values were obtained: 9.24, 3.08, 1.85. How many components would you retain so that 50% of the variation present in the old variables will be explained?
   a) 1 b) 2
   c) 3 d) 0

3) Wishart distribution is multivariate extension of _____.
   a) normal distribution b) chi-square distribution
   c) t-distribution d) F-distribution

4) Principal component analysis is a multivariate method that reduces _______.
   a) heterogeneity of data b) dimensions of data
   c) skewness of data d) multicollinearity of data

5) Let A has $W_p(m, \Sigma)$ and $a \in R^p$ which is independently distributed A with $a'\Sigma\, a \neq 0$. Then distribution of $\frac{a' \mathrm{A}\, a}{a'\Sigma\, a}$ is ________.
   a) χ^2_m b) χ^2_p
   c) χ^2_{m-p} d) χ^2_{m-p+1}

6) The mean vector of $(X_1 + X_2\, , X_1 - X_2)$ is $(10, 0)$ then mean vector of $(X_1\, , 2X_1 - X_2)$ is ________.
   a) $(5, 10)$ b) $(10, 0)$
   c) $(10,5)$ d) $(5, 5)$

7) If $\underline{X} = (X_1, X_2, \ldots, X_p)' \sim N_p\, (\underline{\mu}, \Sigma)$, then any linear combination of X_i's follows ______
   a) Wishart distribution b) multivariate normal distribution
   c) univariate normal d) None of these

8) Characteristic function of $W_p(n, \Sigma)$ distribution is ________.
   a) $\left|I_p - ik\Sigma\right|^n$ b) $\left|I_p - ik\Sigma\right|^{-(n/2)}$
   c) $\left|I_p - ik\Sigma\right|^{-n}$ d) $\left|I_p - ik\Sigma\right|^{(n/2)}$

9) To classify a given multivariate observation to either of two populations, we use _______.
   a) Principle components analysis
   b) Discriminant analysis
   c) Cluster analysis
   d) None of these

10) Principal components are _______.
   a) Orthogonal
   b) Uncorrelated
   c) both (a) and (b)
   d) None of these

B) Fill in the blanks. **06**

1) The measure of association between two sets of variables is called ______.
2) Marginal distribution of any single variable from multivariate normal vector follows _______.
3) The ____ distribution is a multivariate generalization of chi-square distribution.
4) Let A has $W_p(n, \Sigma)$ distribution then $E(A) =$ ______.
5) Finding the hidden factors responsible for observed variables is called _____.
6) While applying ______ clustering algorithm, the distance between two clusters is taken to be the largest distance between observations from two clusters.

Q.2 Answer the following. **16**

a) What do you mean by distance matrix? Explain with the help of illustration.
b) Discuss the independence of two normal vectors.
c) Find maximum likelihood estimator for $\underline{\mu}$ based on a random sample from multivariate normal distribution $N_p(\underline{\mu}, \Sigma)$.
d) Write a note on singular and non-singular multivariate normal distribution.

Q.3 Answer the following.

a) With usual notations, derive the density of multivariate normal distribution. **08**
b) Obtain the moment generating function of multivariate normal distribution. **08**

Q.4 Answer the following.

a) What is meant by discriminant analysis? Obtain the classification rule for the case of two populations with densities $N_p(\mu_1, \Sigma)$ and $N_p(\mu_2, \Sigma)$. **08**
b) What is meant by clustering? Explain agglomerative clustering in detail. **08**

Q.5 Answer the following.

a) Describe principal components analysis in detail. **08**
b) What is meant by canonical correlation? Explain in detail. **08**

Q.6 Answer the following.

a) Define multivariate normal distribution. Show that every linear combination of components of a normal vector follows univariate normal distribution. **08**
b) Describe- **08**
   1) Single linkage
   2) Average linkage

   Illustrate with the help of an example.

Q.7 Answer the following.

a) Find maximum likelihood estimator of Σ based on a random sample from multivariate normal distribution $N_p(\mu, \Sigma)$. **08**
b) Describe Wishart distribution. State and prove additive property of Wishart distribution. **08**

SLR-HR-25

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
STATISTICS
Planning and Analysis of Industrial Experiments (MSC16303)

Day & Date: Wednesday, 15-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

1) The total treatments to be conducted in half fraction of 2^7 experiments are ______.
   a) 128 b) 64
   c) 32 d) 16

2) RBD is a ____design.
   a) Connected b) Balanced
   c) Orthogonal d) All the above

3) In a single replicate design, error has _____ degrees of freedom.
   a) 0(zero) b) 1
   c) 2 d) 3

4) In a 2^5 factorial experiments, there are ___ three factor effects.
   a) 10 b) 5
   c) 6 d) 2

5) In a 2^5 experiment in four block, the generalized interaction of ABCD and CDE is _____.
   a) ABC b) ABE
   c) ABD d) CDE

6) In a 2^3 experiment, the contrast corresponding to main effect B is ___.
   a) (a) + (b) + (c) + (ab) - (ac) - (bc) - (abc) - (1)
   b) (b) + (ab) + (bc) + (abcd) - (1) - (a) - (c) - (ac)
   c) (b) + (a) + (c) - (1) - (abc) - (ac) - (bc) + (ab)
   d) (a) + (b) + (c) + (abc) - (ac) - (bc) - (ac) - (1)

7) In 2^3 factorial experiment 8 treatment combinations are grouped into 2 blocks of size 4 as follows.
   B_1: c ac bc abc
   B_2: (1) a b ab
   The confounded factorial effect is - _________
   a) AC b) BC
   c) B d) C

8) In a BIBD, if number of treatments is equal to the number of plots in a block, then BIBD is - ________
   a) reduces to CRD b) reduces to RBD
   c) reduces to LSD d) None of these

9) In one-way ANOVA model with v treatments, which of the following is not assumption of errors?
   a) errors are uncorrelated
   b) errors have constant variance
   c) errors have mean zero
   d) errors have binomial distribution

10) Smaller the experimental error _____ efficient the design.
   a) less
   b) more
   c) equally
   d) none of these

B) Fill in the blanks. **06**

1) If the number of levels of different factors are equal then factorial experiment is called _____ factorial experiment.
2) In an orthogonal block design, the BLUE of estimable treatment contrast and BLUE of estimable block contrasts are _______.
3) In confounding _____ is/are reduced.
4) A design in which main effects are confounded with 2-way interactions is resolution ____design.
5) The rank of estimation space in one-way ANOVA with v treatment is ______.
6) In general, in total confounding _______ effect is selected as generator.

Q.2 Answer the following. (4X4) **16**

a) Define two-way classification model. Obtain the least square estimates of parameters of the same model.
b) Define total confounding. Illustrate using example.
c) Describe graphical representation of main effects and interaction effects.
d) Write lay out of 3^3 factorial experiment in single replicate.

Q.3 Answer the following.

a) Derive the test for testing treatments in two-way classification without interaction. **08**
b) Obtain half-fraction of 2^5 experiments. Write its complete alias structure. **08**

Q.4 Answer the following.

a) Obtain the reduced normal equations of general block design. **08**
b) Describe analysis of general 2^n full factorial experiment. **08**

Q.5 Answer the following.

a) Derive the expression for least square estimates of parameters in two-way classification with interaction equal observations per call. **08**
b) Define resolution of design and minimum aberration design. Illustrate both. **08**

Q.6 Answer the following.

a) Write 2^6 experiments in two blocks. Explain analysis of confounded experiments. **08**
b) Describe analysis of single replicate design. **08**

Q.7 Answer the following.

a) Define: i) Orthogonal block design **08**
ii) Balanced block design
b) State and prove properties of $Q = T - NK^{-\delta}B$ **08**

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2024
STATISTICS
Regression Analysis (MSC16306)

Day & Date: Friday, 17-05-2024 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) In simple linear regression model $Y = \beta_0 + \beta_1 X + \varepsilon$, β_0 and β_1 are respectively _____.
   a) slope and intercept b) intercept and error
   c) error and slope d) intercept and slope

2) In a regression analysis, the variable that is being predicted is _____.
   a) the dependent variable b) the independent variable
   c) usually denoted by X d) none of these

3) In a multiple linear regression model with $\varepsilon \sim N(0, \sigma^2 I)$, the distribution of residual vector e is _____.
   a) $N(0, H\sigma^2)$ b) $N(0, (I - H)\sigma^2)$
   c) $N(0, \sigma^2 I)$ d) $N(0, (X'X)^{-1}\sigma^2)$

4) To test significance of an individual regression coefficient in multiple linear regression model _____ is used.
   a) F test b) Z test
   c) t test d) χ^2 test

5) Normal probability plot is used to _____.
   a) verify the normality assumption of errors
   b) assess the independence of errors
   c) verify that errors are uncorrelated
   d) none of these

6) Which of the following is true about coefficient of determination (R^2)?
   a) $-1 \leq R^2 \leq 1$ b) $0 \leq R^2 < \infty$
   c) $0 \leq R^2 \leq 1$ d) $R^2 > 1$

7) Autocorrelation is concerned with _____.
   a) correlation among regressor variables
   b) correlation among response and regressor variables
   c) correlation among disturbance terms
   d) correlation between disturbance term and response variable

8) The joint points of pieces in polynomial fitting are usually called _____.
   a) residuals b) errors
   c) splines d) knots

9) If a response variable in a GLM follows Poisson distribution, then _____ link function is suitable.
   a) θ b) $\log\theta$
   c) $-\log\theta$ d) $\log\left(\frac{\theta}{1-\theta}\right)$

10) In logistic regression 'logit' transformation is defined as _____.
   a) $ln\left(\frac{\pi(x)}{1-\pi(x)}\right)$ b) $ln(1-\pi x)$
   c) $ln(\pi(x))$ d) $ln\left(\frac{1-\pi(x)}{\pi(x)}\right)$

B) Fill in the blanks: **06**

1) In a simple linear regression model, the distribution of response variable is _____.
2) In a multiple linear regression model with $\varepsilon \sim N(0, \sigma^2 I)$, Var $(\hat{\beta})$ is _____.
3) $Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon$ is a polynomial regression model in _____ variables.
4) Durbin-Watson test is widely used to detect the presence of _____.
5) The difference between the observed value Y_i and corresponding fitted value $\hat{Y}_i$ is called _____.
6) The model $Y = \beta_0 X^{\beta 1}$ can be linearized by using _____ transformation.

Q.2 Answer the following **16**

a) In a multiple linear regression model, obtain the variance-covariance matrix of residual vector.
b) Define hat matrix. State and prove properties of hat matrix.
c) Discuss the detection of multicollinearity by examination of correlation matrix.
d) Explain the concept of non-linear regression model

Q.3 Answer the following.

a) Discuss the concept of multicollinearity with suitable example. Describe eigen value analysis of matrix $X'X$ method for detection of multicollinearity. **08**
b) Describe backward elimination methods of subset selection in linear regression. **08**

Q.4 Answer the following.

a) Explain the problem of autocorrelation. Discuss Cochrane-Orcutt method of parameter estimation. **08**
b) Discuss Box-Cox power transformation. Explain the procedure of computing λ, the parameter of power transformation. **08**

Q.5 Answer the following.

a) Define multiple linear regression model and obtain the least squares estimates of its parameters. **08**
b) Discuss confidence interval for regression coefficient and prediction interval for future observation in the context of multiple regression. **08**

Q.6 Answer the following.

a) Write in detail about the use of orthogonal polynomials in regression analysis, and how we choose its appropriate degree for a given data set. **08**
b) Discuss least squares method for estimation of parameters for non-linear regression model. **08**

Q.7 Answer the following.

a) What is the logistic regression model? Give a real life situation when this model is appropriate. Obtain MLE of the regression parameters of the model with single covariate. **08**

b) Obtain the weighted least squares estimator of the parameters involved in generalized linear model. **08**

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
STATISTICS
Data Mining (MSC16401)

Day & Date: Thursday, 09-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. No.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) _____ is the process of fixing or removing incorrect, corrupted, incorrectly formatted, duplicate, or incomplete data within a dataset.
a) Data transformation b) Data cleaning
c) Data fitting d) Data exploration

2) The class label of training tuples is not known and the number or set of classes to be learned is also not known in advance. Then it is known as:
a) Self learning b) Unsupervised learning
c) Supervised learning d) None of these

3) The process through which we can transform continuous variables, models or functions into a discrete form is called as _____.
a) Data Monitorization b) Data Discretization
c) Data Assessment d) Data Cleaning

4) The part of the entire data, which is used for building the model is called as _____.
a) Training data b) Testing data
c) Irrelevant data d) Residual data

5) _____ maps data into predefined groups.
a) Regression b) Time series analysis
c) Prediction d) Classification

6) Which of the following is the not a type of clustering?
a) k-means b) Hierarchical
c) Non-hierarchical d) Splitting

7) Which one is example of case based learning?
a) Decision Tree b) k-Nearest neighbor
c) Genetic algorithm d) Neural networks

8) Which of the following is not supervised learning?
a) k-nearest neighbor b) Clustering
c) Decision Tree d) Naive Bayesian

9) Data Mining is also called as _____.
   a) Data discovery from knowledge
   b) Datalogy
   c) Knowledge discovery from data
   d) Data chronology

10) With the help of _____, the model can predict the output on the basis of prior experiences.
   a) unsupervised learning
   b) supervised learning
   c) Semi-supervised learning
   d) Clustering

B) Fill in the blanks. **06**

1) With usual notations, the ratio (TP+TN)/ (P+N) is called as _____.
2) The ratio of TP/P is called as _____.
3) The human brain consists of a network of _____.
4) Fraction of transactions that contain an itemset is _____.
5) Task of inferring a model from unlabeled training data is called _____ learning.
6) Data used to verify performance of the built model is called _____.

Q.2 Answer the following. **16**

a) Discuss Tanh activation function in detail.
b) Discuss backpropagation with respect to ANN.
c) What is the problem of imbalanced data? Describe in detail.
d) Discuss advantages and disadvantages of unsupervised learning.

Q.3 Answer the following.

a) What are the different metrics for evaluating classifier performance? Explain in detail. **08**
b) Write the algorithm of ANN. **08**

Q.4 Answer the following.

a) What is meant by unsupervised learning? Also explain market basket analysis. **08**
b) Discuss different methods of calculating the distances between observations in the context of clustering. **08**

Q.5 Answer the following.

a) Explain SVM classifier in detail. **08**
b) Explain agglomerative clustering in detail. **08**

Q.6 Answer the following.

a) Describe supervised learning. Also explain kNN classifier. **08**
b) Discuss logistic regression as a classifier. **08**

Q.7 Answer the following.

a) Discuss: **08**

i) Sigmoid activation function
ii) Leaky ReLU activation function

b) Describe: **08**

i) Accuracy of a model
ii) Precision of a model

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
STATISTICS
Industrial Statistics (MSC16402)

Day & Date: Saturday, 11-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) The type of inspection that classifies items as being good or defective is _____.
   a) variable inspection b) attribute inspection
   c) fixed inspection d) All the above

2) Type I error occurs when _____.
   a) a good lot is rejected
   b) a bad lot is accepted
   c) the number of defectives is very large
   d) the population is worse than the AQL

3) Which of the following is useful in searching the root cause of a problem?
   a) Ishekawa diagram b) Control chart
   c) Pareto chart d) Defect concentration diagram

4) For a single sampling plan with sample size n, the ASN of the plan is _____.
   a) less than n b) equal to n
   c) greater than n d) not depend on n

5) In demerit system, the unit will not fail in service but has minor defects in finish or appearance is classified as _____ defects.
   a) class A b) class B
   c) class C d) class D

6) Normality assumption of population data values is made for _____ index.
   a) C_p b) C_{pk}
   c) C_{pm} d) All the above

7) The capability index C_{pk} involves _____ parameters(s) to be estimated
   a) only μ b) only σ
   c) Both μ and σ d) None of the above

8) The ASN of a double sampling plan reduces to that of a single sampling plan if probability of making a decision on the basis of first sample is_____.
   a) 1 b) 0.5
   c) 0.75 d) 0

9) Tabular method is used to implement _____ chart.
   a) Moving average b) EWMA
   c) CUSUM d) CRL

10) When acceptance quality levels are very small, the sample size required by attribute sampling plan is_____.
   a) very small b) very large
   c) moderate to large d) None of the above

B) Fill in the blanks. **06**

1) Usually 3-sigma limits are called_____.
2) The performance measure of c charts is based on the assumption that the occurrence of nonconformities follows _____ distribution.
3) The concept of Six-Sigma was developed by _____ company.
4) To identify potential relationship between two variables _____ SPC tool is used.
5) For a variable sampling plan, the distribution of quality characteristic is assumed to be _____.
6) EWMA chart is better than Shewhart $\bar{X}$ chart for detecting _____ shifts in process mean.

Q.2 Answer the following. **16**

a) Explain chance causes and assignable causes of variation.
b) Describe a double sampling plan for attributes.
c) Discuss the use of Ishikawa diagram in the relation to process control.
d) Define type I and type II errors relative to control charts.

Q.3 Answer the following.

a) Define ARL and OC function of a control chart. Obtain the same for $\bar{X}$ chart assuming normality of process with known standards. **08**
b) Discuss the p chart for monitoring fraction nonconforming when sample size is (i) fixed and (ii) variable. **08**

Q.4 Answer the following.

a) Define process capability indices C_p and C_{pk}. Derive the relationship between them. **08**
b) Explain the method of estimation of capability indices C_p and C_{pk}. **08**

Q.5 Answer the following

a) Discuss the development and operation of demerit control chart when standards are not given. **08**
b) What is CUSUM chart? Explain construction and operation of tabular CUSUM for monitoring process mean. **08**

Q.6 Answer the following.

a) Describe sampling inspection plan by variables when lower specification limit is given and the standard deviation is known. **08**
b) Stating the underlying assumptions, explain the construction and operation of sequential sampling plan for attributes. **08**

Q.7 Answer the following.

a) Describe the development and operation of Hotelling's T^2 chart to monitor process mean vector. **08**
b) Explain the basic concept of six-sigma methodology. Also, explain the benefits of implementing the same. **08**

SLR-HR-30

Seat No.	

Set P

M.Sc. (Semester-IV) (New) (CBCS) Examination: March/April-2024
STATISTICS
Reliability and Survival Analysis (MSC16403)

Day & Date: Tuesday, 14-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) As the number of components n increases, the reliability of parallel system _____.
   a) increases b) decreases
   c) remains unchanged d) nothing can be said

2) In series system of five components, the entire system will fail if ______.
   a) any two components fail
   b) any three components fail
   c) any one of the components fail
   d) any four components fail

3) If $\phi(x)$ is a structure function then dual of $\phi(x)$ is ______.
   a) $1 - \phi(x)$ b) $1 - \phi(1 - x)$
   c) $\phi(1 - x)$ d) none of these

4) The i^{th} component of a system is relevant if_____.
   a) $\phi(1_i, \underline{x}) = 1 \; and \; \phi(0_i, \underline{x}) = 1$
   b) $\phi(1_i, \underline{x}) = 0 \; and \; \phi(0_i, \underline{x}) = 0$
   c) $\phi(1_i, \underline{x}) = 1 \; and \; \phi(0_i, \underline{x}) = 0$
   d) $\phi(1_i, \underline{x}) = 0 \; and \; \phi(0_i, \underline{x}) = 1$

5) A life time distribution F having finite mean is said to be $NBUE$ for $t \geq 0$, if
   a) $\mu_t \leq \mu_0$ b) $\mu_t \geq \mu_0$
   c) $\mu_t = \mu_0$ d) None of the above

6) MTTF is _____.
   a) maximum time to failure b) minimum time to failure
   c) median time to failure d) mean time to failure

7) Censoring technique is used for reducing _____.
   a) time of experiment b) cost of experiment
   c) number of failures d) none of the above

8) The TTT transform of an IFR distribution is _____.
   a) convex b) concave
   c) linear d) neither concave nor convex

9) A sequence of (2x2) contingency tables is used in _____.
   a) Gehan's test b) Mantel-Haenzel test
   c) Log-rank test d) Mann-Whitney test

10) To obtain confidence band for survival function ______ statistic is used.
   a) Kolmogorov-Smirnov   b) Chi-square
   c) Wilcoxon   d) None of the above

B) Fill in the blanks. **06**

1) Parallel system of n components has ______ minimal path sets.
2) For a series system of two independent components each having reliability 0.5 then the reliability of system is ______.
3) The distribution of structure function $\phi(\underline{x})$ is ______.
4) The number of failures is fixed in ______ censoring.
5) For a distribution with finite variance, the degree of estimability of variance is ______.
6) The hazard function ranges between ______ and ______.

Q.2 Answer the following. **16**

a) Define reliability of a system. Obtain the reliability of parallel system of n independent components.

b) Define:
   i) Structure function
   ii) Coherent structure

   Illustrate giving one example each.

c) Describe Type-II censoring with one illustration.

d) Obtain the nonparametric estimator of survival function based on complete data.

Q.3 Answer the following. (8+8) **16**

a) Define dual of a structure function. Obtain the dual of k-out-of- n system.

b) If $X_1, X_2, \ldots., X_n$ are associated state variables of coherent system then prove that

$$\prod_{i=1}^{n} P(X_i = 1) \le P(\phi(X) = 1) \le \coprod_{i=1}^{n} P(X_i = 1)$$

Q.4 Answer the following. (8+8) **16**

a) Define IFR and IFRA classes of distributions. Prove that $IFR \subset IFRA$.

b) If failure time of an item has the distribution.

$f(t) = \frac{\lambda^\alpha}{\Gamma\alpha} t^{\alpha-1} e^{-\lambda t}, t > 0,\ \lambda, \alpha > 0.$ Examine whether it belongs to IFR or DFR.

Q.5 Answer the following. (8+8) **16**

a) Define star shaped function. Prove that $F \in IFRA$ if and only if - $log\ R(t)$ is star shaped.

b) Discuss maximum likelihood estimation of parameters of a Weibull distribution based on complete data.

Q.6 Answer the following. (8+8) **16**

a) Describe Type-I censoring. Obtain MLE of mean of exponential distribution under Type I censoring.

b) Describe actuarial method of estimation of survival function, with suitable illustration.

Q.7 Answer the following. (8+8) **16**

a) Describe Mantel's technique of computing Gehan's statistics for a two-sample problem for testing equality of two life distributions.

b) Describe Kaplan-Meier estimator and derive an expression for the same.

SLR-HR-31

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
STATISTICS
Optimization Techniques (MSC16404)

Day & Date: Thursday, 16-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any Three questions from Q.No.3 to Q.No.7.
3) Figures to the right indicate full marks.

Q.1 A) Choose correct alternative. 10

1) The dual has an unbounded solution, primal has ______.
   a) an unbounded solution b) an infeasible solution
   c) a feasible d) one of these

2) When maximin and minimax values of the game are same, then _____.
   a) there is saddle point b) solution does not exist
   c) strategies are mixed d) None of these

3) Simplex method is applicable to those LPPs that starts with _____.
   a) an infeasible solution
   b) a feasible solution
   c) an infeasible but not optimum
   d) a feasible but not optimum

4) The following are useful in integer programming _____.
   a) branch and bound method b) cutting plane method
   c) Both a) and b) d) Neither a) nor b)

5) In standard form LPP _____.
   a) The constraints are strict equations
   b) The constraints are inequalities of ≤ type
   c) The constraints are inequalities of ≥ type
   d) None of these

6) Branch bound method divides the feasible solution space into smaller parts by _____.
   a) Enumerating b) Branching
   c) Bounding d) All of these

7) i^{th} constraint in the primal is an equality iff i^{th} dual variable is _____.
   a) Unrestricted in sign
   b) restricted to less than zero
   c) restricted to greater than zero
   d) None of these

8) Objective function in a general LPP is _____.
   a) quadratic function b) linear function
   c) non- linear function d) constant

9) For a given LPP, if Z is objective function, then _____.
   a) $\text{Max}\, Z = -\text{Min}\, Z$ b) $\text{Min}\, Z = \text{Max}\,(-Z)$
   c) $\text{Min}\, Z = -\text{Min}\, Z$ d) None of these

10) Which of the following is not correct?
   a) A feasible solution of an LPP is independent of the objective function
   b) A feasible region of an LPP must be convex set
   c) The feasible region is also termed as solution space
   d) It is not possible to obtain feasible solution of an LPP by graphical method.

B) Fill in the blanks: **06**

1) If the players select the same strategy each time, then it is referred as _____.
2) The basic feasible solution of L.P.P is said to be _____, if at least one basic variable is zero.
3) For maximization LPP, the objective function coefficient for an artificial variable is ______.
4) In dual simplex method the starting basic solution is always _____.
5) The competitors of game are known as ______.
6) The optimal solution to an LPP exist at ______ point.

Q.2 Answer the following. **16**

a) Describe two persons zero sum game.
b) Write a short note on two phase method.
c) Define: solution, feasible solution, optimal solution, and basic solution of an LPP.
d) Explain the terms: Pure, mixed, optimum strategies.

Q.3 Answer the following.

a) Solve following integer programming problem **08**
$Maximize\ Z = 7x_1 + 9x_2$, subject to constraints
$-x_1 + 3x_2 < 6, 7x_1 + x_2 \leq 35, x_2 \leq 7,\ x_1, x_2 \geq 0$ and integers.

b) Obtain the range of change in b_i values to maintain feasibility of the optimal solution. **08**

Q.4 Answer the following.

a) Solve following game **08**

Player B

Player A $\begin{pmatrix} 4 & 2 & 4 \\ 2 & 4 & 0 \\ 4 & 0 & 8 \end{pmatrix}$

b) 1) Write down graphical procedure to solve LLP. **08**

2) Use graphical method to solve the following LPP.

$\text{Max } Z = 3x_1 + 5x_2$

Subject to $x_1 \leq 4,$

$x_2 \leq 6,$

$3x_1 + 2x_2 \leq 18,$

$x_1, x_2 \geq 0$

Q.5 Answer the following.

a) Write down simplex algorithm to solve LPP. **08**

b) Write a note on Big-M method. **08**

Q.6 Answer the following.

a) Solve following linear programming problem **08**

$$Maximize\ Z = 5x_1 + 3x_2,$$

$$Subject\ to,$$

$$2x_1 + x_2 \leq 1,$$

$$x_1 + 4x_2 \geq 6,$$

$$x_1, x_2 \geq 0$$

b) Give the general rules for converting any primal into its dual with example. **08**

Q.7 Answer the following.

a) Explain Wolfe's method to solve quadratic programming problem. **08**

b) Use dynamic programming to solve the following LPP **08**

$$\text{Max } Z = 3x_1 + 5x_2$$

$$Subject\ to,$$

$$x_1 \leq 4, x_2 \leq 6, 3x_1 + 2x_2 \leq 18, x_1, x_2 \geq 0$$

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2024
STATISTICS
Time Series Analysis (MSC16407)

Day & Date: Saturday, 18-05-2024 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The time series $\{X_t\}$ is always ordered w.r.t._____.
   a) Size of X b) time t
   c) Both (a) and (b) d) None of these

2) Let time series $\{X_t\}$ be an IID noise, then $E(X_t) =$ _____.
   a) 0 b) 1
   c) 2 d) 3

3) Let $\{X_t\}$ be a weakly stationary time series, then _____.
   a) $E(X_t)$ is constant w.r.t. t
   b) Covariance unction is constant w.r.t. t
   c) Both (a) and (b)
   d) None of these

4) The mean of white noise process is _____.
   a) Always zero b) Always constant
   c) Always one d) None of these

5) Turning point test is used for testing _____.
   a) trend in the given series
   b) seasonality in the given series
   c) average value of the given series
   d) None of these

6) Irregular variations in the time series data is also called as _____.
   a) Random noise component
   b) Trend component
   c) Seasonal variation component
   d) None of the above

7) The approach of applying difference operator to time series data was developed by _____.
   a) Box and Jenkin b) Cox and Box
   c) Fisher and Neyman d) Sam and Ayyar

8) The process $X_t = \phi_1 X_{t-1} + Z_t$ where $\{Z_t\} \sim WN(0, \sigma^2)$ is causal process if _____.
   a) $|\phi_1| < 1$ b) $|\phi_1| = 1$
   c) $|\phi_1| > 1$ d) $|\phi_1| < 1.5$

9) The sample autocorrelation follows _____ distribution.
   a) Student's t b) Chi-square
   c) Asymptotic Normal d) F distribution

10) For applying moving average filter of order q, the q is taken to be _____.
   a) 0 b) Odd
   c) Even d) None of the above

B) Fill in the blanks. **06**
1) The additive model of time series is given by _____.
2) In exponential smoothing, recent observations get weights _____ than the older observations.
3) The moving average filter is also called as _____ filter.
4) The Spencer's moving average filter considers _____ number of successive observations at a time.
5) The time series data with removal of seasonal component is called as _____.
6) ACF of white noise for time lag greater than 2 is _____.

Q.2 Answer the following. **16**
a) Define the following time series models:
   i) White Noise
   ii) IID Noise
b) What do you mean by auto-covariance function.
c) Describe causality of a time series model.
d) Define sample ACF.

Q.3 Answer the following.
a) How do you check causality of a time series model. Also illustrate with an example. **08**
b) How do you check invertibility of a time series model. Also illustrate with an example. **08**

Q.4 Answer the following.
a) Explain the following tests w.r.t. time series: **08**
   i) Turning point test
   ii) Difference sign – test
b) State and prove elementary properties of auto-covariance function. **08**

Q.5 Answer the following.
a) Describe the diagnostic checking methods in time series analysis. **08**
b) Explain moving average as a method of estimation and elimination of trend. **08**

Q.6 Answer the following.
a) Explain moving average time series model of order 1. Also obtain its auto-covariance function. **08**
b) Describe single exponential smoothing. **08**

Q.7 Answer the following.
a) Describe ARCH and GARCH models in detail. **08**
b) Define MA(q) process. Obtain PACF of MA(q) process. **08**