SLR-SR-1

Seat No.	

Set **P**

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
STATISTICS
Real Analysis (MSC16101)

Day & Date: Wednesday, 19-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Set of real numbers is _____.
   a) Countable b) Uncountable
   c) Both a and b d) neither a) nor b)

2) A set A is said to be countable if there exists a function $f: A \rightarrow N$ such that _____.
   a) f is bijective. b) f is surjective
   c) f is identity function d) None of the

3) A convergent sequence has only _____ limit(s).
   a) One b) Two
   c) Three d) none of these

4) A sequence $\{(-1)^n, n \in N\}$ is _____.
   a) unbonded b) convergent
   c) divergent d) bounded

5) A sequence of real numbers is Cauchy iff _____.
   a) it is bounded
   b) it is convergent
   c) it is positive term sequence
   d) it is convergent but not bounded

6) Series is convergent if its sequence of partial sum is _____.
   a) convergent b) divergent
   c) bounded d) unbounded

7) The closed set includes all of its _____ points.
   a) limit b) interior
   c) member d) none of these

8) A point c is said to be extremum point of function f, if _____.
   a) $f'(c) = 0$ b) $f(c) = 0$
   c) $f'(c) \neq 0$ d) none of these

9) The function $f(x) = |x|$ is _____.
   a) continuous b) discontinuous
   c) differentiable d) none of these

10) If sequence is bounded and decreasing, then it _____.
   a) converges to its supremum b) converges to its infimum
   c) divergent sequence d) None of these

B) Fill in the blanks. **06**

1) The set of all limit points of a set is called _____ set.
2) Finite union of open set is _____.
3) A set is open if and only if its compliment is _____.
4) Arbitrary intersection of closed sets always _____.
5) Every bounded sequence has _____.
6) The minimum value of the function $f(x) = x^2$ is _____.

Q.2 Answer the following **16**

a) Define and illustrate:
  i) Closed Set
  ii) Open Set

b) Prove that: Convergent sequence is bounded.

c) State the following:
  i) Taylor's theorem
  ii) Heine-Borel theorem.
  iii) Bolzano-Weierstrass theorem for set
  iv) Bolzano-Weierstrass theorem for sequence

d) Prove that: Finite intersection of closed sets is closed.

Q.3 Answer the following

a) Check whether $f(x) = x^2$ is Reimann integrable over (0,1). If so, find the integral. **08**

b) Prove that: Set of rational number is countable. **08**

Q.4 Answer the following

a) Define convergent sequence. Prove that every monotonic non-decreasing bounded above sequence is convergent. **08**

b) Describe Lagrange's method of undetermined multipliers. **08**

Q.5 Answer the following.

a) Define Cauchy sequence. Prove that Cauchy sequence is convergent. **08**

b) Test the convergence of **08**
  i) $\int_0^1 \frac{1}{\sqrt{x}} dx$
  ii) $\int_0^1 \frac{1}{1-x} dx$

Q.6 Answer the following.

a) Define any four tests for convergent. **08**

b) Prove that the series $\frac{1}{n^p}$ diverges for $p \leq 1$ and converges for $p > 1$ **08**

Q.7 Answer the following.

a) Define countable set. Prove that countable union of countable sets is countable. **08**

b) State Taylor's theorem. Find the power series expansion for the following functions: **08**
  i) $f(x) = \cos x$
  ii) $f(x) = \tan x$

SLR-SR-2

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
STATISTICS
Linear Algebra & Liner Models (MSC16102)

Day & Date: Thursday, 20-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

1) The smallest sub-space containing finite set of vectors (S) is _____.
   a) Superclass of S b) Span of S
   c) Subset of S d) Basis of S

2) What is the dimension of the vector space R^2 over the field R?
   a) 1 b) Infinite
   c) 2 d) 4

3) If number of column is less than number of rows, then the matrix is called as _____.
   a) Horizontal matrix b) Vertical matrix
   c) Row matrix d) Column matrix

4) If A is symmetric matrix, then which of the following are symmetric matrices?
   a) A' b) A + I
   c) A + 0 d) All of these

5) If A is a 4 x 4 matrix with rank 3, then determinant of A is _____.
   a) 0 b) 1
   c) 4 d) 6

6) Rank of matrix A is ______.
   a) Number of independent rows in A
   b) Dimension of the row space of A
   c) Dimension of the column space of A
   d) All of the above

7) Which of the following property does not hold for matrix multiplication?
   a) Commutative b) Associative
   c) Distributive d) All of these

8) For non-homogenous system of equations Ax=b with k unknowns, unique solution exists if _____.
   a) Rank [A:b] > rank(A) b) Rank [A:b] = rank(A) = k
   c) Rank [A:b] = rank(A) < k d) All of the above

9) ______ is equal to the maximum number of linearly independent row vectors in a matrix.
   a) Row matrix b) Row rank of a matrix
   c) Linear matrix d) Term matrix

10) For a Gauss-Markov model $Y = X\beta + \varepsilon$,

a) $\mathrm{Cov}(\varepsilon_i \varepsilon_j) = 0, \text{if } i \neq j$ b) $\mathrm{Cov}(\varepsilon_i \varepsilon_j) < 0, \text{if } i \neq j$

c) $\mathrm{Cov}(\varepsilon_i \varepsilon_j) > 0, \text{if } i \neq j$ d) None of these

B) Fill in the blanks **06**

1) The rank of identity matrix of order 4 is _____.
2) Multiplication of a matrix with a scalar constant is called ______.
3) If all the elements below the diagonal are zero, then such matrix is called as _____.
4) The eigen values of 2 x 2 matrix A are 2 and 7, then |A| = ______.
5) _____ is equal to the maximum number of linearly independent column vectors in a matrix.
6) If A is non-singular matrix of order 4, then its rank is _____.

Q.2 Answer the following **16**

a) Write a note on basis of a vector space.

b) Define diagonal matrix. Show that product of two diagonal matrices is a again a diagonal matrix.

c) Write a note on trace of a matrix.

d) Define additive inverse of a vector. Show that additive inverse of any vector in a vector space is unique.

Q.3 Answer the following

a) Determine whether $S = \{(x, y, z) \in R^3 / z = 0\}$ is a vector space under regular addition and scalar multiplication. **08**

b) Show that the rank of a product of two matrices cannot exceed the rank of either matrix. **08**

Q.4 Answer the following

a) Define rank of a matrix. Also find the rank of following matrix: **08**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

b) If A and B are two matrices of order $n \times n$, then prove that $\rho(AB) \geq \rho(A) + \rho(B) - n$ **08**

Q.5 Answer the following

a) Describe system of equations in detail. **08**

b) Define G-inverse of a matrix. Show that G is a g-inverse of matrix A, if and only if AGA=A. **08**

Q.6 Answer the following

a) State and prove necessary and sufficient condition for estimability of linear parametric functions. **08**

b) Prove: If G is g-inverse of A, then G_1=GAG is also a g-inverse of A. **08**

Q.7 Answer the following

a) Find G-inverse of the below matrix: **08**

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 9 & 6 \end{bmatrix}$$

b) Describe row space and column space of a matrix. Based on these spaces, define row rank and column rank of the matrix. **08**

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
STATISTICS
Distribution Theory (MSC16103)

Day & Date: Friday, 21-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1and 2 are compulsory.
2) Attempt any Three questions from Q.3 to Q.7
3) Figures to the right indicate full marks.

Q.1 A) Choose Correct Alternative. **10**

1) A random variable X is symmetric about point α then _____.
a) $f(\alpha + x) = f(\alpha - x)$ b) $f(\alpha + x) = f(x - \alpha)$
c) $f(\alpha + x) = -f(\alpha + x)$ d) none of these

2) Let X and Y are independent random variables with $N(0,1)$. The distribution of $(Y/X)^2$ is _____.
a) Normal b) χ^2
c) F d) t

3) Let X and Y be two independent Poisson random variates with means 1 and 2 respectively then variance of $(2X + 3K)$ is _____.
a) 5 b) 8
c) 16 d) 22

4) The PGF of Poisson distribution with mean λ is given by _____.
a) $e^{-\lambda(1-s)}$ b) $e^{-\lambda(s-1)}$
c) $e^{\lambda(e^s-1)}$ d) $e^{\lambda(e^s+1)}$

5) Let (X, Y) bivariate normal $BVN(1,2,16,25,3/4)$, then $E(Y/X = 7)$ is _____.
a) 1 b) 2
c) 4 d) $91/16$

6) The mean of first order statistic in $U(0,1)$ distribution is _____.
a) $1/n$ b) $1/(n + 1)$
c) $1/(n - 1)$ d) $n/(n^2 - 1)$

7) If X and Y are two independent random variables then _____.
a) $E(X\,Y) = E(X)\,E(Y)$ b) $Cov\,(Y, F) = 0$
c) $\rho\,(Y, F) = 0$ d) all the above

8) Which of the following is not a scale family?
a) $U(0,1)$ b) $U(0, \theta)$
c) $N(0, \sigma^2)$ d) $Exp(\theta)$

9) If the distribution function of two-dimensional random variates X and Y is denoted by $F(x, y)$, then _____.
a) $-1 \le F(x, y) \le 1$ b) $0 \le F(x, y) \le 1$
c) $-\infty \le F(x, y) \le \infty$ d) $0 \le F(x, y) \le \infty$

10) If $M_X(t)$ denotes MGF of random variable X. If $Z = a\,X$ then $M_z(t)$ is _____.
a) $aM_X(t)$ b) $aM_X(at)$
c) $M_X(at)$ d) $aM_X(t/a)$

B) Fill in the blanks: **06**

1) If Z is standard normal variate, then mean of Z^2 is _____.
2) The pdf of random variable X is $f(x) = 2\,x, 0 < x < 1$ then $P(X = 0.5)$ is _____.
3) Let X be distributed as Exp (Mean θ) .Then distribution of $Y = X/\theta$ is _____.
4) Let $f(x, y) = 4xy, 0 \leq x \leq 1,\ 0 \leq y \leq 1$ be the joint pdf of (X, T). Then marginal distribution of X is _____.
5) Negative binomial distribution $NB(x: r, p)$ for $r = 1$ reduces to _____distribution.
6) The variance of continuous uniform distribution over $(0, b)$ is ____.

Q.2 Answer the following. **16**

1) Define location family. Give one example of the same.
2) Let X has $N(0,1)$ distribution. Obtain the pdf of $Y = |X|$.
3) Define convolution of distribution functions and give one example.
4) Derive the pdf of largest order statistic based on random sample of size n from a continuous distribution.

Q.3 Answer the following.

a) Let (X, Y) be a discrete bivariate random vector. Define **08**
   i) Joint p.m.f of (X, Y)
   ii) Marginal p.m.f. of X and marginal p.m.f. of Y
   iii) Independence of X and Y
   iv) Covariance (X, Y)

b) Define probability generating function (PGF) of a random variable. Let X has $B(n, p)$ distribution. Obtain the PGF of X. Hence obtain its mean and variance. **08**

Q.4 Answer the following.

a) Define power series distribution. Show that Geometric distribution is power series distribution. Obtain MGF of geometric distribution using MGF of power series distribution. **08**

b) Let X is a non-negative random variable with pmf $P(X = x) = P_x$, $x = 1,2, \ldots$ then show that $E(X) = \sum_{x=1}^{\infty} P[X \geq x]$ **08**

Q.5 Answer the following.

a) Define multinomial distribution. Obtain the MGF of multinomial distribution with k cells. Hence show that pmf of i^{th} component X_i is $B(n, p_i)$. **08**

b) If X is symmetric about α then prove that $E(X) = a$ and Median $(X) = a$. **08**

Q.6 Answer the following.

a) State and prove Jensen's inequality **08**

b) Let X and Y are jointly distributed with pdf **08**

$$f(x, y) = \begin{cases} k(x + 2y), 0 < x < 2, 0 < y < 1 \\ 0, otherwise \end{cases}$$

Find marginal distributions of X and Y.

Q.7 Answer the following

a) Let (X, Y) has $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, p)$. Obtain the marginal distributions of Y. **08**

b) If $X_1, X_2, \ldots, X_n$ *are* random observations from exponential distribution with mean θ. Obtain the *pdf* of r^{th} order statistic $X_{(r)}$. **08**

SLR-SR-4

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
STATISTICS
Estimation Theory (MSC16104)

Day & Date: Saturday, 22-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Let $X_1, X2, \ldots, X_n$ be iid from $B\ (1, \theta)$. Then $\bar{X}$ is _____.
   a) sufficient statistic b) unbiased estimator
   c) complete sufficient statistic d) all the above

2) An estimator $\hat{\theta}$ is said to be unbiased estimator of θ if _____.
   a) $E[\hat{\theta}] = \theta$ b) $\hat{\theta} = E[\theta]$
   c) $\left[E(\hat{\theta})\right]^2 = \theta$ d) $E[\hat{\theta}] = \theta^2$

3) If T_n is an estimator of θ, then Cramer-Rao inequality provides a lower bound on
   a) $E(T_n)$ b) $Var(T_n)$
   c) $Max(T_n)$ d) $Min(T_n)$

4) Let $X_1, X_2, \ldots, X_n$ is a random sample from $U\ (0, \theta),\ \theta > 0$. The MLE of θ is _____.
   a) $\bar{X}$ b) $X_{(1)}$
   c) $X_{(n)}$ d) sample median

5) A sufficient statistic contains all the information which is contained in ____.
   a) population b) sample
   c) parameter d) none of the above

6) Conditional distribution of random variable θ given $X = x$ is called _____.
   a) Posterior distribution b) Prior distribution
   c) Loss function d) Bayes risk

7) Let $T(X)$ is a complete sufficient statistic and $A(X)$ is ancillary statistic, then which one of the following statements is correct?
   a) $T(X)$ and $A(X)$ are distributionally dependent
   b) $T(X)$ and $A(X)$ are functionally dependent
   c) $T(X)$ and $A(X)$ are statistically independent
   d) none of the above

8) Bayes estimator of a parameter under squared error loss function is _____.
   a) posterior mean b) posterior median
   c) posterior mode d) posterior variance

9) Which of the following statements is / are correct?
   1) UMVUE is always unique if it exists.
   2) UMVUE is provided by C-R lower bound only
   Select the correct answer using the code given below:
   a) Both 1 and 2
   b) Neither 1 nor 2
   c) 2 only
   d) 1 only

10) Let $X_1, X_2, \ldots, X_n$ *be iid* $N\ (\mu, \sigma^2)$ and $\hat{\mu}$ and $\hat{\sigma}^2$are the MLE of μ and σ^2 respectively. Consider the following statements:
   a) $\hat{\mu}$ is an unbiased estimator of μ
   b) $\hat{\sigma}^2$ is an unbiased estimator of σ^2
   Which of the above statements is / are correct?
   a) 1 only
   b) 2 only
   c) both 1 and 2
   d) neither 1 nor 2

B) Fill in the blanks. **06**

1) Let (X_1, X_2) denote a random sample of size 2 from $B(2, \theta), 0 < \theta < 1$ distribution. The sufficient statistic for θ is given by _____.
2) Let X_1, X_2 is a random sample of size 2 from $N\ (0, \sigma^2)$. Moment estimator of σ^2 is _____.
3) The estimate of λ for the exponential distribution $f\ (x, \lambda) = \lambda\, e^{-\lambda x}, 0 \le x < \infty$ by method of moments is _____.
4) Bhattacharya bound is the generalization of the _____.
5) Let T_n be an unbiased estimator of θ . Then $3\, T_n + 4$ is estimator of $3\theta + 4$
6) Let $X_1, X_2, \ldots, X_n$ *is* a random sample of size n from $U(0, \theta)$ distribution then is unbiased estimator of θ is _____.

Q.2 Answer the following.

a) Let random variable X has *Poisson* (θ) distribution. Show that distribution of X is complete. **04**

b) State and prove Basu's theorem **04**

c) Define maximum likelihood estimator (MLE). State any two small sample properties of MLE **04**

d) Define power series distribution. Obtain sufficient statistic for power series distribution. **04**

Q.3 Answer the following.

a) Define sufficient statistic and minimal sufficient statistic. Explain the method of constructing minimal sufficient statistic. **08**

b) Define a one parameter exponential family of distributions. Obtain a minimal sufficient statistic for this family. **08**

Q.4 Answer the following.

a) Describe the method of scoring for obtaining maximum likelihood estimator of a parametric function. **08**

b) Obtain MLE of (μ, σ^2) based on a random sample of size n from $N(\mu, \sigma^2)$ distribution **08**

Q.5 Answer the following

a) State and prove Cramer-Rao inequality with necessary regularity conditions. **08**

b) Define Fisher information. Let X be Bernoulli random variable with parameter θ. Obtain Fisher information $I_X\ (\theta)$ **08**

Q.6 Answer the following

a) State and prove Rao-Blackwell theorem. Let $X_1, X_2, \ldots, X_n$ $(n \geq 2)$ be a random sample from $B(1, \theta)$ distribution. Obtain UMVUE of $\psi(\theta) = \theta(1-\theta)$. **08**

b) Let $X_1, X_2, \ldots, X_n$ be iid Poisson (λ) random variables. Show that $T = \bar{X}^2 - \bar{X}$ is biased estimator of λ^2. Find its bias and hence unbiased estimator of λ^2. **08**

Q.7 Answer the following

a) Explain the following with one illustration each. **08**

1) Conjugate Family
2) Conjugate Priors
3) Non-informative Priors
4) Bayes estimator

b) Suppose an observation is taken on random variable X which yielded a value 2. The density of X is **08**

$$f(x/\theta) = \begin{cases} (1/\theta), & 0 < x < \theta \\ 0, \text{otherwise} \end{cases}$$

Suppose that, prior distribution of θ has density

$$\pi(\theta) = \begin{cases} (3/\theta^4), & \theta > 1 \\ 0, \text{otherwise} \end{cases}$$

For squared error loss function, show that Bayes estimate of θ is $8/3$

SLR-SR-5

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
STATISTICS
Statistical Computing (MSC16108)

Day & Date: Sunday, 23-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) Rate of convergence to correct root is very high for _____ method.
   a) Newton-Raphson b) Bisection
   c) Euler's d) Regula - Falsi

2) To generate single random number from bivariate Poisson, we need _____ independent univariate Poisson.
   a) Three b) Two
   c) One d) Four

3) When applying Simpson's $1/3^{rd}$ rule the number of sub-intervals should be _____.
   a) odd b) even
   c) prime d) multiple of five

4) If U ~ U(0,1) then X = 2 + U follows _____ distribution.
   a) Gamma b) U(2, 3)
   c) U(3, 2) d) U(0, 1)

5) If X ~ Gamma(n) then Y= 2X follows Chi-square distribution with _____ d.f.
   a) n b) n - 1
   c) 2n d) n + 1

6) Which of the following method is used for generating random numbers from a statistical distribution?
   a) Acceptance-Rejection b) Inverse transformation
   c) Monte Carlo simulation d) All the above

7) Simpson's $1/3^{rd}$ rule is obtained by taking n = _____ in Newton's general formula for numerical integration.
   a) 0 b) 2
   c) 1 d) -1

8) _____ is generalization of geometric distribution.
   a) Negative Binomial b) Bivariate Poisson
   c) Gamma d) None of these

9) Random numbers from normal distribution can be generated using _____.
   a) CLT method b) Box - Muller method
   c) Chi-square method d) all the above

10) In false position method we choose the two points x_0 and x_1 such that $f(x_0)$ and $f(x_1)$ are ____.
    a) Opposite sign b) same sign
    c) Zero d) None of these

B) Fill in the blanks **06**

1) If U ~ U(0,1) then X = - 0.51n(1-U) follows _____.
2) Convolution theorem gives distribution of _____ random variates.
3) _____ in 1979 introduced the bootstrap method.
4) To generate bootstrap sample _____ sampling method is used.
5) In EM algorithm 'E' stands for _____.
6) EM algorithm is used to obtain _____.

Q.2 Answer the following **16**

a) What is convolution of random variable? Obtain formula for convolution of continuous random variable.

b) Write an algorithm for generating random numbers from N(μ, σ^2) using Box-Muller transformation

c) Describe bisection method of finding solution to the equation $f(x) = 0$

d) Write a short note on Markov Chain Monte Carlo methods.

Q.3 Answer the following **16**

a) Let X ~ U (0, 1) and Y ~ U (0, 1). Define Z = X + Y, obtain the distribution of Z using convolution theorem.

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from the displaced exponential distribution with pdf $e^{-(x-\theta)} I_{(\theta,\infty)}(x)$. Show that, the jackknife estimator is unbiased estimator of θ.

Q.4 Answer the following **16**

a) State and prove the result for generating random numbers from Binomial distribution.

b) What is Acceptance - Rejection method? Illustrate with an example.

Q.5 Answer the following **16**

a) Obtain an EM algorithm for estimating parameters for mixed normal model.

b) State and prove the result for generating random numbers from χ^2_{2n}

Q.6 Answer the following **16**

a) Write in detail numerical methods for single and double integration.

b) Describe linear congruential method of random number generation. Illustrate with example.

Q.7 Answer the following **16**

a) What are bias reduction techniques? Explain Jack - Knife estimation.

b) Write algorithms for generating random numbers from bivariate exponential and bivariate Poisson distribution.

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
STATISTICS
Probability Theory (MSC16201)

Day & Date: Wednesday, 19-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) If $\{A_n\}$ is decreasing sequence of sets, then the sequence $\{A_n^c\}$ is ____.
   a) Decreasing b) Increasing
   c) Need more information d) None of these

2) Monotonic sequence of sets _____.
   a) Always converges
   b) Converges, only if it is bounded above
   c) Converges, only if it is bounded below
   d) Converges, only if it is bounded

3) Which of the following is the weakest mode of convergence?
   a) convergence in r^{th} mean b) convergence in probability
   c) convergence in distribution d) convergence in almost sure

4) Which of the following is a Borel set?
   a) $(0, x), x$ is a real number b) $\{x\}$
   c) $[x, x + 1]$ d) All of these

5) Expectation of a simple non-negative random variable satisfies _____
   a) Linearity property b) Scale preserving property
   c) Non-negativity property d) All of these

6) Probability measure is continuous from _____.
   a) Above b) Below
   c) Both (a) and (b) d) Either above or below

7) If F_1 and F_2 are two fields defined on subsets of Ω, then which of the following is/are always a field?
   a) $F_1 \cup F_2$ b) $F_1 \cap F_2$
   c) both (a) and (b) d) neither (a) nor (b)

8) A simple function can take _____ values.
   a) Finitely many b) Infinitely many
   c) Uncountably many d) None of these

9) Distribution function of a random variable is always _____.
   a) Non-negative
   b) right continuous
   c) Monotone non-decreasing
   d) All of the above

10) If events A, B and C are mutually independent, then which of the following is not correct?
    a) A and B are pairwise independent
    b) A and C are pairwise independent
    c) B^c and C are independent
    d) All are correct

B) Fill in the blanks. **06**

1) Lebesgue measure of a singleton set {k} is _____.
2) Expectation of a random variable X exists, if and only if _____ exists.
3) If $F(.)$ is a distribution function for some random variable, then $\lim_{n\to\infty} F(x) =$_____
4) Convergence in distribution is also called as _____.
5) If Ω contains 2 elements, then the largest field of subsets of Ω contains _____ sets
6) The σ - field generated by the intervals of the type $(-\infty, x)$,$x \in R$ is called _____.

Q.2 Answer the following **16**

a) Define the characteristic function of a r.v. and find the same for exponential distribution.
b) Write a note on Lebesgue-Stieltje's measure.
c) Discuss σ - field induced by r.v. X.
d) Prove or disprove: Mapping preserves all set relations.

Q.3 Answer the following.

a) Prove that probability measure is a continuous measure. **08**
b) Discuss limit superior and limit inferior of a sequence of sets. Find the same for sequence $\{An\}$, where $An = \ (0, 3\ + \frac{(-1)^n}{n}), n \in N$ **08**

Q.4 Answer the following.

a) State and prove Fatou's lemma. **08**
b) Define convergence in probability and convergence in distribution. Also prove that convergence in probability implies convergence in distribution. **08**

Q.5 Answer the following.

a) State and prove Yule-Slutsky results. **08**
b) Prove that any random variable can be expresses as a limit of sequence of simple random variables. **08**

Q.6 Answer the following.

a) Define, explain and illustrate the concept of limit superior and limit inferior of a sequence of sets. **08**
b) State and prove monotone convergence theorem. **08**

Q.7 Answer the following.

a) Prove or disprove: Convergence in distribution implies convergence in probability. **08**
b) Define characteristic function of a random variable. Prove any three properties of characteristic function. **08**

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
STATISTICS
Stochastic Processes (MSC16202)

Day & Date: Sunday, 23-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct options. **10**

1) If $\{N(t)\}$ is a Poisson process with parameter λ, then var$(N(t))$ = _____.
   a) λ b) λt
   c) t d) λ^2

2) A matrix can be a TPM, if _____.
   a) It is a square matrix
   b) If it contains all non-negative entries
   c) If each row sum is one
   d) All of these

3) If the probability of ultimate first return $F_{ii} < 1$, then the state i is _____.
   a) persistent state b) Transient state
   c) Aperiodic state d) None of these

4) If states i and j are communicating states, then _____.
   a) state i leads to state j b) state j leads to state i
   c) either (a) or (b) d) both (a) and (b)

5) Which of the following is always true?
   a) column sum of TPM is always one
   b) Recurrent state is also called as persistent state
   c) Every absorbing state is transient
   d) All of these

6) Which of the following are class properties?
   a) Persistency b) Periodicity
   c) Transientness d) all of these

7) For a symmetric random walk, probability ‘p’ of positive jump is _____.
   a) 0.25 b) 0.5
   c) 1 d) 0

8) State space of stochastic process can be _____.
   a) Discrete b) Continuous
   c) Neither(a) nor (b) d) Both(a) and (b)

9) For a null recurrent state ‘í’, the mean recurrent time is _____.
   a) $< \infty$ b) ∞
   c) 0 d) 1

10) The collection of all possible states of a stochastic process is called as _____.
    a) State Space
    b) Time Space
    c) Chain space
    d) all of these

B) Fill in the blanks **06**

1) A Markov chain is completely specified by _____ and _____.
2) For a TPM of a irreducible Markov chain, the row sum is _____.
3) A branching process is an example of _____ state, _____ time space stochastic process.
4) Persistent state is also called as _____.
5) The probability of escape from a closed class is _____.
6) A non-null recurrent aperiodic state is called as _____.

Q.2 Answer the following **16**

a) Write a short note on Mean recurrent time of a state.
b) Define and explain Markov property.
c) State and illustrate:
   1) State space
   2) Stochastic Process
   3) TPM
d) State and prove Chapman-Kolmogorov equations.

Q.3 Answer the following

a) Classify the states of random walk model. **08**
b) Discuss the classification of stochastic processes according to state space and index set. **08**

Q.4 Answer the following

a) Let $\{X_n, n \geq 0\}$ be a Markov chain with state space $S = \{0,1,2\}$,t.p.m. **08**
$P = \begin{bmatrix} 0.6 & 0 & 0.4 \\ 0 & 0.6 & 0.4 \\ 0.4 & 0 & 0.6 \end{bmatrix}$ and initial distribution (0.5,0.5,0).
Compute
i) $P(X_2 = 1, X_0 = 1)$
ii) $E(X_2)$
b) Describe birth and death process and obtain its Kolmogorov differential equations. **08**

Q.5 Answer the following

a) Discuss Gambler's ruin problem in detail. **08**
b) Show that recurrence is a class property. **08**

Q.6 Answer the following

a) Prove that a state j of a Markov chain is recurrent if and only if $\sum p_{jj}^{(n)} = \infty$ **08**
b) State and prove class property of periodicity. **08**

Q.7 Answer the following

a) Establish the equivalence between two definitions of Poisson process. **08**
b) Define branching process. Derive expression for the mean of the population size at n^{th} generation. **08**

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
STATISTICS
Theory of Testing of Hypotheses (MSC16203)

Day & Date: Tuesday, 25-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) For testing a simple hypothesis against simple alternative, which of the following values of α and β is not correct?
   a) $\alpha = 0.05, \quad \beta = 0.80$ b) $\alpha = 0.01, \quad \beta = 0.70$
   c) $\alpha = 0.05, \quad \beta = 0.98$ d) $\alpha = 0.05, \quad \beta = 0.5$

2) In order to obtain a most powerful test, we ______.
   a) minimize the level of significance
   b) minimize the power
   c) minimize the level of significance and fix the power
   d) fix the level of significance and maximize the power

3) Let X has a $B(n,p)$ distribution. Then a simple hypothesis will be _____.
   a) $H_0: p = 1/2$ b) $H_0: p \leq 1/2$
   c) $H_0: p \geq 1/2$ d) $H_0: p \neq 1/2$

4) Family of $Cauchy\ (1,\theta)$ distribution _____.
   a) has MLR property
   b) belong to one parameter exponential family
   c) has mean θ
   d) does not have MLR property

5) If λ is the likelihood ratio test statistic, which one of the following has got its asymptotic distribution as χ^2distribution?
   a) $\log_e(1/\lambda^2)$ b) $\log_e(1/\lambda)$
   c) $\log_e(\lambda^2)$ d) $\log_e(\lambda)$

6) Based on single observation X from $U(0,1)$ distribution for testing $H_0: \theta = 1$ against $H_1: \theta \neq 1$ _____.
   a) no UMP test exist
   b) UMP test exist
   c) UMPU test exist which is not UMP
   d) every test that exist is biased

7) If $\emptyset(x) \equiv \alpha,\ \forall x \in X$ then ____.
   a) $\emptyset(x)$ is MP test.
   b) $\emptyset(x)$ is not a valid test function
   c) power of $\emptyset(x)$ is α
   d) $\emptyset(x)$ is biased test

8) Test with Neyman-Structure is _____.
   a) similar test b) subset of similar test
   c) not a subset of similar test d) none of these

9) For an $(r \times c)$ contingency table the number of degrees of freedom equals _____.
   a) rc b) $r + c$
   c) $(r - 1) + (c - 1)$ d) $(r - 1)(c - 1)$

10) Which of the following tests is analogous of χ^2 test of goodness of fit?
   a) Mann-Whitney U test b) Kolmogorov-Smirnov test
   c) Wilcoxon signed-rank test d) median test

B) Fill in the blanks. **06**

1) Area of critical region depends on size of _____ error.
2) If there are 10 symbols of two types, equal in number, the minimum possible number of runs is _____.
3) The distribution of statistic used in sign test is _____.
4) Probability of rejecting the false hypothesis is _____.
5) The statistic H in Kruskal-Wallis test is approximately distributed as _____.
6) When ranking combined data in Wilcoxon signed-rank test, the data that receives rank of 1 is the _____ value.

Q.2 Answer the following. **16**

a) Define simple hypothesis and composite hypothesis. Give on example for each.
b) Define MLR property of a family of distributions. Give an example of a distribution which does not have MLR property.
c) Explain the run test to test randomness.
d) Explain the likelihood ratio (LR) test for testing hypothesis.

Q.3 Answer the following

a) State Neyman Pearson lemma. Show that power of MP test given by N-P lemma is at least its size. **08**
b) A sample of size one is taken from Poisson distribution with parameter λ. To test the hypothesis **08**
$H_0: \lambda = 1$ against $H_1: \lambda = 2$, consider the test

$$\emptyset(x) = \begin{cases} 1, \text{if } x > 3 \\ 0, \text{otherwise} \end{cases}$$

Find the probability of type I error and power of the test.

Q.4 Answer the following

a) Define most powerful (MP) test. Show that MP test need not be unique using suitable example. **08**
b) To test $H_0: \theta = 1$ against $H_1: \theta = 0$ for a single observation from the distribution $f(x, \theta) = (2\theta x + 1 - \theta), 0 < x < 1$ is used. Find MP test of level α and its power. **08**

Q.5 Answer the following

a) Show that for a family having MLR property, there exists UMP test for testing one sided hypothesis against one sided alternative. **08**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta, 1)$. Obtain UMP level α test for testing $H_0: \theta \le \theta_0$ against $H_1: \theta > \theta_0$. **08**

Q.6 Answer the following.

a) Describe a chi-square test for goodness of fit. **08**

b) Let $X_1, X_2, \dots., X_n$ be a random sample of size n from $U(0, \theta)$. Obtain shortest length confidence interval for θ. **08**

Q.7 Answer the following.

a) Describe Kolmogorov-Smirnov test. **08**

b) Let $X_1, X_2, \dots., X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Develop LRT for $H_0: \sigma^2 = \sigma_0^2$ against $H_1: \sigma^2 \neq \sigma_0^2$, where both μ and σ^2 are unknown. **08**

SLR-SR-10

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
STATISTICS
Sampling Theory (MSC16206)

Day & Date: Thursday, 27-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative: **10**

1) In simple random sampling with replacement, the same sampling unit may be included in the sample _____.
   a) only once b) only twice
   c) more than once d) none of these

2) If a population contains periodic type of variation then most suitable type of sampling is _____.
   a) Cluster sampling b) Systematic sampling
   c) Stratified sampling d) Simple random sampling

3) If heterogeneous population can be easily divided into subpopulations with relatively small variability between the subpopulations, then appropriate sampling design is _____.
   a) Stratified b) Two-stage
   c) Systematic d) Cluster

4) In systematic sample of size n from a population of size $N = nk$, where k is given, the probability that a specified unit is included in the sample is _____.
   a) $1/N$ b) $1/k$
   c) n/N d) $1/n$

5) What is the effect of increasing sample size on sampling error?
   a) it reduces sampling error
   b) it increases sampling error
   c) it has no effect on sampling error
   d) none of the above

6) Hansen-Hurwitz technique is used to deal with _____.
   a) sampling errors b) non-sampling errors
   c) non-response errors d) none of the above

7) Which of the following estimators is generally biased?
   a) Ratio estimator
   b) Difference estimator
   c) Hansen-Hurwitz estimator
   d) Horvitz-Thompson estimator

8) If 100 students are selected out of 500, and 25 students are then selected from the selected 100 students. The procedure adopted is _____.
   a) Stratified b) Cluster
   c) Systematic d) Two-stage

9) The error committed in presenting data are categorized as _____.
   a) sampling error b) non-sampling error
   c) margin of error d) none of these

10) Ratio estimator $\hat{R}$ is unbiased if _____.
   a) $\rho(\hat{R}, \bar{x}) < 0$ b) $\rho(\hat{R}, \bar{x}) > 0$
   c) $\rho(\hat{R}, \bar{x}) = 1$ d) $\rho(\hat{R}, \bar{x}) = 0$

B) Fill in the blanks. **06**

1) If n units are selected in a sample from N population units then sampling fraction is _____.
2) Variance of proportional allocation is always _____ that of optimum allocation.
3) A large city is subdivided into 150 non-overlapping blocks. Five blocks are selected at random and completely enumerated. This procedure is known as _____.
4) Stratified sampling is more precise than systematic sampling if serial correlation coefficients are _____.
5) In systematic sampling with $k = N / n$, then k is called _____.
6) If a larger units have more probability of their inclusion in the sample, the sampling is known as _____.

Q.2 Answer the following.

a) Describe simple random sampling without replacement (SRSWOR). Give a procedure to select such samples. **04**

b) Define linear regression estimator for population mean. Is it unbiased? **04**

c) Describe the cumulative total method for drawing PPSWR samples. **04**

d) What is double sampling? Explain any one practical situation where double sampling is appropriate. **04**

Q.3 Answer the following.

a) What are the basic principles of sample survey? Give advantages of sampling method over census method. **08**

b) In SRSWR, suggest an unbiased estimator of the population mean and derive its sampling variance. **08**

Q.4 Answer the following

a) Describe stratified random sampling. Explain various sample allocation criteria in stratified sampling. **08**

b) Explain the concept of systematic sampling. Derive the sampling variance of unbiased estimator of population mean under the linear systematic sampling. **08**

Q.5 Answer the following.

a) Define Horvitz-Thompson estimator for population total. Show that it is unbiased. Obtain its variance. **08**

b) Define Des Raj’s ordered estimator for population mean on the basis of a sample of size 2 and show that it is unbiased. **08**

Q.6 Answer the following.

a) Explain cluster sampling and clearly specify the advantages of the scheme. When this method is better than SRSWOR? **08**

b) Define PPSWR sampling design. Obtain an unbiased estimator of the population mean and its variance when a PPSWR sample of size n is drawn from a population of size N. **08**

Q.7 Answer the following.

a) Explain ratio method of estimation. Assuming SRSWOR, derive approximate expression for bias of ratio estimator. **08**

b) What is the problem of non-response? Discuss Hansen-Hurwitz techniques of tackling this problem giving all the details. **08**

SLR-SR-12

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
STATISTICS
Asymptotic Inference (MSC16301)

Day & Date: Monday, 10-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative **10**

1) An estimator T_n of parameter θ is strongly consistent for θ means
   a) $\lim_{n\to\infty} P_\theta\{|T_n-\theta|>\varepsilon\}=1$
   b) $\lim_{n\to\infty} P_\theta\{|T_n-\theta|>\varepsilon\}=0$
   c) $P_\theta\left\{\lim_{n\to\infty} T_n=\theta\right\}=1$, for all $\theta \in \Theta$
   d) None of these

2) Consider the statements:
   X) Joint consistency implies marginal consistency.
   Y) Weak consistency implies strong consistency.
   Consider the statements:
   a) Only X b) Only Y
   c) both X and Y d) neither X nor Y

3) In case of $N(\mu,\sigma^2),\ \mu \in R, \sigma^2 > 0$, the MLE of σ^2 is _____.
   a) unbiased and consistent
   b) unbiased but not consistent
   c) asymptotically unbiased and not consistent
   d) asymptotically unbiased and consistent

4) For an estimator to be CAN _____.
   a) unbiasedness of estimator is necessary
   b) consistency of estimator is necessary
   c) both a) and b)
   d) neither a) nor b)

5) Sample distribution function at a given point is _____for the population function at the same point.
   a) consistent b) CAN
   c) both a) and b) d) neither a) nor b)

6) In a random sample of size n from *Poisson* (λ) distribution, MLE of λ was reported to be 2. The variance of the asymptotic normal distribution of case of $\sqrt{n}(e^{-\bar{X}_n} - e^{-\lambda})$ is estimated by _____.
   a) $4e^{-4}$ b) $2e^{-4}$
   c) $2e^{-2}$ d) e^{-4}

7) Based on random sample of size n from $N(\theta, 1)$, asymptotic distribution of $n\,\bar{X}_n^2$ is _____.
   a) $\chi_n^2, \theta \in R$
   b) $\chi_1^2, \theta = 0$
   c) $\chi_1^2, \theta \neq 0$
   d) $N(\theta^2, 1), \theta \in R$

8) A sequence of estimators T_n is said to be BAN for θ if _____.
   a) T_n is consistent for θ
   b) T_n is CAN for θ
   c) Asymptotic Var (T_n) = CRLB
   d) all the above

9) For sufficiently large sample size, the likelihood equation admits _____.
   a) unique consistent solution
   b) two consistent solutions
   c) more than two consistent solutions
   d) no consistent solution

10) In LRT, under some regularity conditions on $f(x, \theta)$, the random variable $-2\log\lambda(x)$ [where $\lambda(x)$ is likelihood ratio] is asymptotically distributed as _____.
   a) normal
   b) exponential
   c) chi-square
   d) F-distribution

B) Fill in the blanks. **06**

1) For Cauchy distribution with location θ, the consistent estimator of θ is _____.
2) Let $X_1, X_2, \ldots, X_n$ be iid from Poisson (θ). CAN estimator of $P_\theta(X = 1)$ is ______.
3) Cramer family is _____ than exponential family.
4) To investigate the significance difference between variances of several normally distributed populations _____ test is used.
5) Test based on score functions was proposed by ______.
6) The variance stabilizing transformation for binomial population is _____.

Q.2 Answer the following **16**

a) Define weak consistency and strong consistency.
b) Describe Wald's test. State its asymptotic distribution.
c) State Cramer-Huzurbazar results.
d) Based on random sample of size n from $B(1, p)$ obtain variance stabilizing transformation of the estimator.

Q.3 Answer the following

a) Define consistent estimator. State and prove invariance property of consistent estimator of a real valued parameter θ. **08**
b) Let $X_1, X_2, \ldots, X_n$ be *iid* $N(\theta, 1)$, computing the actual probability show that $\bar{X}_n$ is consistent estimator of θ. **08**

Q.4 Answer the following

a) Define one-parameter (θ) family of distributions. For a one parameter exponential family, prove that maximum likelihood estimator leads to a CAN estimator of θ. **08**
b) Let $X_1, X_2, \ldots, X_n$ *be iid* $U(\theta, 1), \theta > 0$. Show that $2\bar{X}_n$ is CAN for θ but $X_{(n)}$ is not CAN for θ. **08**

Q.5 Answer the following

a) Define joint and marginal consistency for a vector parameter θ. Show that joint consistency is equivalent to marginal consistency. **08**

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $N(\mu, \sigma^2)$. Obtain MLE of (μ, σ^2). Show that it is CAN for (μ, σ^2). Obtain its asymptotic variance covariance matrix. **08**

Q.6 Answer the following

a) Explain variance stabilizing transformations and illustrate their use in large sample confidence intervals. **08**

b) Let $X_1, X_2, \ldots, X_n$ be iid $N(\mu, \sigma^2)$ random variables. Find variance stabilizing transformation for S^2 and obtain $100(1-\alpha)\%$ confidence interval for σ^2 based on the transformation. **08**

Q.7 Answer the following

a) Define likelihood ratio test. Derive the asymptotic null distribution of the likelihood ratio statistic. **08**

b) Let $X_1, X_2, \ldots, X_n$ be *iid* $N(\theta, 1)$. Let $\Psi(\theta) = \theta^2$. Obtain CAN estimator for $\Psi(\theta)$. Discuss its asymptotic distribution at $\theta = 0$. **08**

SLR-SR-13

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
STATISTICS
Multivariate Analysis (MSC16302)

Day & Date: Tuesday, 11-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos.1and 2 are compulsory.
2) Attempt any Three questions from Q. No. 3 to Q. No. 7
3) Figures to the right indicate full marks.

Q.1 A) Choose Correct Alternative. **10**

1) If $\underline{X}$ has $N_p(\mu, \Sigma)$ distribution then moment generating function of vector $\underline{X}$ is _____

a) Exp $\left(t'\mu - \frac{1}{2}t'\Sigma\, t\right)$
b) Exp $\left(t'\mu + \frac{1}{2}t'\Sigma\, t\right)$
c) Exp $\left(t'\mu + \frac{1}{2}t'\Sigma^{-1}t\right)$
d) Exp $\left(t'\mu - \frac{1}{2}t'\Sigma^{-1}t\right)$

2) In factor analysis, if there are k variables and m factors, then _____.

a) $k < m$
b) $m = k$
c) $m < k$
d) None of these

3) Let A has $W_p(n, \Sigma)$ distribution then $E(A^{-1})$ = _____.

a) $\frac{1}{p-1}\Sigma^{-1}$
b) $\frac{1}{n-1}\Sigma^{-1}$
c) Σ^{-1}
d) $\frac{1}{n-p-1}\Sigma^{-1}$

4) Le A has $W_p(n, \Sigma)$ distribution and h be any vector distribution of $\frac{h'\Sigma^{-1}h}{h'A^{-1}h}$ is _____.

a) χ^2_n
b) χ^2_p
c) χ^2_{n-p}
d) χ^2_{n-p+1}

5) Let vector $\underline{Y}$ has $N_p(\mu, \Sigma)$ distribution. For a constant matrix $A_{q\mathrm{X}p}$ and vector $b_{q\mathrm{X}1}$ the distribution of $\underline{X} = A\underline{Y} + b$ is _____.

a) $N_p\left(A_{\mu,} A\Sigma A'\right)$
b) $N_q(A\mu, A\Sigma A')$
c) $N_q(A\mu + b, A\Sigma A')$
d) $N_q(A\mu + b, A\Sigma A')$

6) Factor analysis can be used in which of the following circumstances?

a) To identify underlying dimensions, or factors, that explains the correlations among a set of variables.
b) To identify a new, smaller set of uncorrelated variables to replace the original set of correlated variables in subsequent multivariate analysis.
c) To identify a smaller set of salient variables from a larger set for use in subsequent multivariate analysis.
d) All are correct circumstances.

7) Let $\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_n$ be a random sample of size n from $N_p(\mu, \Sigma)$ distribution. Then MLE of μ is .... for population mean vector.

a) unbiased but not sufficient
b) biased and sufficient
c) unbiased and sufficient
d) biased and not sufficient

8) Which of the following statistical techniques uses two or more independent, metric variables to classify observations into categories of a nominal, dependent variable?
   a) Factor analysis
   b) Multivariate analysis of variance
   c) Cluster analysis
   d) Discriminant analysis

9) The goal of discriminant analysis is to predict a _____.
   a) Numerical variable
   b) Categorical variable
   c) Variable measured on interval scale
   d) All of these

10) Which of the following statistical techniques identifies homogenous subgroups?
   a) Factor analysis
   b) Multivariate analysis of variance
   c) Cluster analysis
   d) Discriminant analysis

B) Fill in the blanks **06**

1) As the distance between two populations decreases, misclassification error _____.
2) Agglomerative clustering uses _____ approach for clustering.
3) Generalized variance is _____ of variance-covariance matrix.
4) A graphical representation of clustering can be done using _____.
5) The maximum variation is explained by _____ principal component.
6) The diagonal elements of variance-covariance matrix are _____.

Q.2 Answer the following. **16**

a) Show that two p-variate normal vectors $\underline{X}_1$and $\underline{X}_2$ are independent if and only if cov $\left(\underline{X}_1, \underline{X}_2\right) = 0$.

b) Write a note on singular and non-singular multivariate normal distribution.

c) Obtain characteristic function of multivariate normal distribution.

d) Find maximum likelihood estimator for $\underline{\mu}$ based on a random sample from multivariate normal distribution $N_P(\underline{\mu}, \Sigma)$

Q.3 Answer the following.

a) Derive the density of multivariate normal distribution. **08**

b) Derive expressions for principle components. Show that total variation explained by principal components is same as total variation in original variables. **08**

Q.4 Answer the following.

a) If $\underline{X} \sim \mathrm{N_p}(\underline{\mu}, \Sigma)$, and random vector $\underline{X}$ be partitioned into two sub – vectors as $\underline{X_{(1)}}$ and $\underline{X_{(2)}}$. Then with usual notations obtain the conditional distribution of $\underline{X_{(2)}}$ given $\underline{X_{(1)}}$ **08**

b) Based on random sample of size n from $N_p(\underline{\mu}, \Sigma)$ distribution, obtain a LRT for testing $H_0: \underline{\mu} = \underline{\mu}_0$ against $H_1: \underline{\mu} = \underline{\mu}_1$. **08**

Q.5 Answer the following.

a) Describe the problem of classification. Derive Fisher's best linear discriminant function. **08**

b) What is meant by discriminant analysis? Obtain the classification rule for the case of two populations with densities $N_p(\underline{\mu_1}, \Sigma)$ and $N_p\left(\underline{\mu_2}, \Sigma\right)$. **08**

Q.6 Answer the following.

a) Discuss hierarchical and non-hierarchical clustering. Discuss k-means clustering in detail. **08**

b) Define: **08**

1) Distance matrix
2) Single linkage
3) Complete linkage
4) Average linkage

Q.7 Answer the following

a) Describe agglomerative clustering in detail. Illustrate with the help of example using single linkage method. **08**

b) Describe Wishart distribution. State and prove additive property of Wishart distribution. **08**

SLR-SR-14

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
STATISTICS
Planning and Analysis of Industrial Experiments (MSC16303)

Day & Date: Wednesday, 12-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) In two-way classification model $Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}; i = 1,2, \dots v. j = 1,2 \dots . n_i$ and assumptions on errors are followed; the error degrees of freedom are _____.
   a) (n - 1)(v- 1) b) (v - 1)(b - 1)
   c) (n - v)(b - v) d) (v - n)(v - b)

2) In a 2^3 factorial experiment with, the contrast due to main effect A is _____.
   a) [(a) + (ab) + (ac) + (abc) - (1) - (b) - (c) - (bc)]
   b) [(a) + (ab) - (ac) + (abc) - (1) + (b) - (c) + (bc)]
   c) [(a) + (ab) + (ac) - (abc) - (1) + (b) + (c) - (bc)]
   d) [(a) + (ab) + (ac) + (abc) - (1) - (b) - (c) + (bc)]

3) In a symmetric BIBD $(\mathrm{v}, \mathrm{b}, \mathrm{r}, \mathrm{k}, \lambda)$ _____.
   a) r = v, b = k b) v = b, r = k
   c) v = 1, b = 2 d) v = 2, b = 1

4) Which of the following is one-way ANOCOVA model with single covariate?
   a) $Y_{ij} = \mu + \alpha_i + \beta_j + \gamma z_{ij} + \epsilon_{ij}$ b) $Y_{ij} = \mu + \alpha_i * \beta_j + z_{ij} + \epsilon_{ij}$
   c) $Y_{ij} = \mu + \alpha_i + \gamma z_{ij} + \epsilon_{ij}$ d) $Y_{ij} = \mu + \alpha_i + \beta_j - z_{ij} + \epsilon_{ij}$

5) In a 2^5 experiment, number of two - factor interaction effects are _____.
   a) 10 b) 05
   c) 31 d) 32

6) In a RBD with 4 treatments and 5 blocks, the error degrees of freedom are _____.
   a) 20 b) 12
   c) 15 d) 16

7) Every balanced design is _____.
   a) Orthogonal b) Disconnected
   c) Complete d) Connected

8) In general block design, C matrix is given by _____.
   a) $R^\delta - NK^{-\delta}N'$ b) $K^\delta - N'^{R^{-\delta}}N$
   c) $R^\delta + NK^{-\delta}N'$ d) $R^\delta \times \left(NK^{-\delta}N'\right)^{-1}$

9) In one - way ANOVA model, the treatment sum of squares is an estimate of _____.
   a) within treatment variation b) between treatment variation
   c) total variation d) none of these

10) In a 3^2 experiment, interaction effect AB has ____ degrees of freedom.
   a) 2 b) 3
   c) 4 d) 9

B) Fill in the blanks **06**

1) Replication, ______ and local control are three basic principles of design of experiments.
2) In 2^3 factorial experiment in 3 replicates the error has _____ degrees of freedom.
3) If all elementary contrasts are estimable then the design is said to be _____.
4) In partial confounding _____ effect is confounded in _____ replicates.
5) The principle block of a 2^4 factorial experiment contains the treatments {(1) ab, ac, bc, ad, bd, cd, abcd} then _____ effect is confounded with block effects.
6) The total number of treatments in 2^6 experiment are _____.

Q.2 Answer the following **16**

a) Define resolution of a design. Illustrate using one example.
b) Define BIBD (v, b, r, k, λ). Write all parametric relationship.
c) In one-way classification, show that elementary contrasts are estimable.
d) Define balanced design. Show that RBD is a balanced design.

Q.3 Answer the following

a) Derive the test for testing the hypothesis $H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_v$ against appropriate alternative in the model $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij}, i = 1,2, ...,$ $v. j = 1,2, ... n_i. \varepsilon_{ij} \sim N(0, \sigma^2)$ **08**

b) Obtain the least square estimates of parameters in the following model. **08**

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, i = 1,2, \dots, v. j = 1,2, \dots b. \varepsilon_{ij} \sim N(0, \sigma^2)$$

Q.4 Answer the following

a) Derive the necessary and sufficient condition for orthogonality of a connected block design and hence show that RBD is connected as well as orthogonal. **08**
b) Prove that dual of a symmetric BIBD is also a symmetric BIBD. **08**

Q.5 Answer the following

a) In a connected block design prove that rank (C) = v -1 and hence rank of estimation space is v + b - 1. **08**
b) Derive ½ fraction of 2^5 experiment and write its consequences. **08**

Q.6 Answer the following

a) Describe tile analysis of 2^k full replicated factorial experiments. **08**
b) State and prove any four properties of BIBD (v, b, r, k, λ). **08**

Q.7 Answer the following

a) In general block design state and prove the properties of C matrix. **08**
b) Derive the test for testing hypothesis of equality of all treatment effects in two-way classification model. **08**

SLR-SR-15

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
STATISTICS
Regression Analysis (MSC16306)

Day & Date: Thursday, 13-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. ***10***

1) In simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$, X and Y are respectively_____.
   a) response variable and regressor variable
   b) predictor variable and response variable
   c) response variable and predictor variable
   d) slope and intercept

2) In a multiple linear regression model with $\mathcal{E} \sim N(0, \sigma^2 I)$, the distribution of response vector Y is _____
   a) $N(X\beta, H\sigma^2)$ b) $N(X\beta, (I - H)\sigma^2)$
   c) $N(X\beta, \sigma^2 I)$ d) $N(X\beta, (X'X)^{-1}\sigma^2)$

3) In usual notations, the residual refers to _____
   a) $\overline{y_1} - \widehat{y_1}$ b) $y_i - \widehat{y_1}$
   c) $\widehat{y_1} - \overline{y_1}$ d) $y_i - \overline{y_1}$

4) Which of the following is true about coefficient of determination (R^2)?
   a) $0 \le R^2 \le 1$
   b) $R^2 = 1$ indicates the best fit of the model.
   c) $R^2 = 0.95$ indicates the model is 95% good
   d) All the above

5) Forward selection procedure starts with the _____ predictor variables in the model.
   a) Without b) Some
   c) All d) none of these

6) Autocorrelation is concerned with_____.
   a) correlation among regressor variables
   b) correlation among response and regressor variables
   c) correlation among disturbance terms
   d) correlation between disturbance term and response variable

7) Cochrane-Orkut method for parameter estimation is used in _____.
   a) the presence of multicollinearity
   b) the presence of autocorrelation
   c) nonparametric regression
   d) nonlinear regression

8) If a response variable in a GLM follows Poisson distribution, then _____ link function is suitable.
   a) θ b) $\log\theta$
   c) $-\log\theta$ d) $\log\left(\frac{\theta}{1-\theta}\right)$

9) In a logistic regression model with single covariate, the odd ratio Ψ is related to the regression coefficient β_1 by _____.
   a) $\Psi = e^{\beta_1}$ b) $\Psi = \beta_1$
   c) $\Psi = 1n\beta_1$ d) $\Psi = e^{\beta 0}$

10) Which one of the following measures is used to test goodness of fit in a generalized linear model?
   a) Deviance b) F-ratio
   c) t-statistic d) none of these

B) Fill in the blanks. **06**

1) In a multiple linear regression model with k regressors, the distribution of (SS_{Res}/σ^2) is ______.
2) The model $Y = \beta_0 e^{\beta_1} X \varepsilon$ can be linearized by using ____ transformation.
3) The sum of residuals weighted by corresponding fitted value is always equal to _____.
4) If eigen values of the matrix $X' X$ are 4.2, 0.3, 1.0 and 0.03. Then condition index is_____.
5) In usual notations, $a(\emptyset)$ for Poisson (λ) family is always equal to _____.
6) If response function is curvilinear then initially _____ polynomial model should be considered.

Q.2 Answer the following **16**

a) Discuss Box-Cox power transformation.
b) With usual notations, show that $Var\left(\hat{Y}\right) = H\sigma^2$
c) Discuss examination of correlation matrix method for detection of multicollinearity.
d) What is logistic regression model? Give one situation where such model is appropriate.

Q.3 Answer the following.

a) Describe multiple linear regression model. Stating the assumptions, obtain mean and variance of least squares estimators of β. **08**
b) Discuss confidence interval for regression coefficient and prediction interval for future observation in the context of multiple regression. **08**

Q.4 Answer the following.

a) Explain the problem of multicollinearity in connection with linear regression model. What are its consequences on least squares estimators? **08**
b) Describe backward elimination method of subset selection in linear regression. **08**

Q.5 Answer the following.

a) Explain the problem of autocorrelation. Discuss Cochrane-Orcutt method of parameter estimation. **08**

b) Explain the following plots: **08**

i) Normal probability plot.

ii) Residual against the fitted values.

Q.6 Answer the following.

a) Give formal structure of GLM. Discuss Nelder-Wedderburn method for parameter estimation in GLM. **08**

b) Explain the non-linear regression model. Discuss linearization parameter estimation method for this model. **08**

Q.7 Answer the following.

a) Define a one parameter natural exponential family. Show that**,** member $N(\mu, \sigma^2), \mu \epsilon R, \sigma^2 > 0$ is member of natural exponential family. **08**

b) Define 'Deviance statistic'. Find it when data comes from Poisson distribution with mean λ. **08**

SLR-SR-17

Seat No.	

Set **P**

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
STATISTICS
Data Mining (MSC16401)

Day & Date: Monday, 10-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternatives from the options. **10**

1) Market-basket problem was formulated by _____.
   a) Agrawal et al. b) Toda et al.
   c) Steve et al. d) Simon et. Al

2) _____ data are noisy and have many missing attribute values.
   a) Discretized b) Real-world
   c) Cleaned d) Transformed

3) Which of the following is the not a type of clustering?
   a) k-means b) Hierarchical
   c) Non-hierarchical d) Splitting

4) The problem of finding hidden structure in unlabeled data is called _____.
   a) supervised learning
   b) unsupervised learning
   c) mixed learning
   d) all of these

5) Bayesian classifiers is _____.
   a) A class of learning algorithm that tries to find an optimum classification of a set of examples using the probabilistic theory
   b) Any mechanism employed by a learning system to constrain the search space of a hypothesis
   c) An approach to the design of learning algorithms that is inspired by the fact that when people encounter new situations, they often explain them by reference to familiar experiences, adapting the explanations to fit the new situation.
   d) None of these

6) In data mining, SVM stands for _____.
   a) Service Vector Machine b) Standard Vector Machine
   c) Standard Vector Method d) Support Vector Machine

7) Each neuron is made up of a number of nerve fibres called
   a) Molecules b) Dendrites
   c) Atoms d) Sigmoid

8) The final output of data mining is ____
   a) Data b) Clean data
   c) Information d) All of these

9) Which of the following tool can be best used for classification?
   a) Linear regression b) Logistic regression
   c) polynomial regression d) All of these

10) Classification of new species to one of the earlier known families of species is _____.
   a) Supervised learning b) Unsupervised learning
   c) Traditional learning d) None of these

B) Fill in the blanks. **06**

1) Task of inferring a model from unlabeled training data is called _____ learning.
2) KNN is an example of _____ Learning method.
3) Looking for combinations of items purchased together is called _____.
4) In data mining, ANN stands for _____.
5) In _____ learning, class labels are provided
6) The part of the entire data, which is used for building the model is called as _____.

Q.2 Answer the following. **16**

a) What is meant by imbalanced data?
b) Differentiate between training data and testing data
c) Discuss, with illustration, the concept of supervised learning.
d) Discuss sensitivity and specificity of a model.

Q.3 Answer the following. **16**

a) Discuss Bayesian classifier. Also explain why it is called as naive classifier.
b) Discuss k-nearest neighbor classifier in detail.

Q.4 Answer the following. **16**

a) Discuss logistic regression classifier in detail.
b) Discuss the working mechanism of ANN.

Q.5 Answer the following. **16**

a) Write down the algorithm for decision tree classifier.
b) Write down the algorithm for Bayesian classifier.

Q.6 Answer the following. **16**

a) Discuss in detail about how the order of features is considered in decision tree with respect to information gain.
b) Discuss the different metrics for Evaluating Classifier Performance.

Q.7 Answer the following. **16**

a) Describe unsupervised learning. Also explain in detail, market basket analysis.
b) Describe – 1) Accuracy of a model 2) Precision of a model

SLR-SR-18

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
STATISTICS
Industrial Statistics (MSC16402)

Day & Date: Wednesday, 12-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) The performance measure of *c* and *u* charts is based on the assumption that the occurrence of nonconformities follows _____ distribution.
   a) Geometric b) Binomial
   c) Poisson d) Normal

2) Usually 3-sigma limits are called _____.
   a) warning limits b) action limits
   c) specification limits d) none of these

3) Quality is inversely proportional to _____.
   a) variability b) Cost
   c) Method d) Time

4) ‘Vital few and trivial many’ is the principle of the _____.
   a) control chart b) Ishekawa diagram
   c) check sheet d) Pareto analysis

5) The type II error occurs when _____.
   a) good lot is rejected
   b) a bad lot is accepted
   c) the number of defectives are very large
   d) the population is worse than the AQL

6) Memory type control charts are developed specifically for detecting _____ shifts efficiently.
   a) Moderate b) Large
   c) Small d) All of these

7) In a demerit system, the unit will cause personal injury or property damage is classified as _____ defect.
   a) class A b) class B
   c) Class C d) Class D

8) The capacity index C_{pk} involves _____ parameter(s) to be estimated.
   a) Only μ b) Only σ
   c) Both μ and σ d) None of these

9) For a centered process _____.
   a) $C_p = C_{pk}$ b) $C_p < C_{pk}$
   c) $c_p > c_{pk}$ d) None of these

10) Designing a single sampling plan for attributes means ______.
    a) finding α and β for given n and c values
    b) finding n and c for given α and β Values
    c) finding n and α for given β and c values
    d) finding α and c for given n and β values

B) Fill in the blanks **06**

1) The variation due to _____ causes cannot be identified and removed from the process.
2) For a variable sampling plan, the distribution of quality characteristic is assumed to be _____.
3) The number of inspected units between two consecutive nonconforming units, including the end nonconforming unit is known as _____.
4) In demerit system, the occurrence of defects in each class is modelled by _____ distribution.
5) The ASN of a double sampling plan reduces to that of a single sampling plan if probability of making a decision on the basis of first sample is _____.
6) An appropriate distribution of run length is _____.

Q.2 Answer the following **16**

a) Distinguish between process control and product control. Discuss the situations where they are used.
b) Write a short note on DMAIC Cycle.
c) Explain the use of Pareto chart with suitable example.
d) Explain the terms:
   i) Consumer's risk
   ii) Producer's risk

Q.3 Answer the following **16**

a) List seven SPC tools and explain in detail any two of off-line tools.
b) Discuss the various steps involved in the construction of $\bar{X}$ and R charts.

Q.4 Answer the following **16**

a) Define single sampling plan for attributes. Give an algorithm to design the single sampling plan. Obtain OC function of the same.
b) Explain variable sampling plan when lower specification is given and standard deviation is known.

Q.5 Answer the following **16**

a) Discuss the control chart for fraction nonconforming when the sample size is,
   i) Fixed
   ii) Variable
b) What is an EWMA control chart? In which situation it is preferred to $\bar{X}$ chart? Explain the procedure of obtaining control limits for the same.

Q.6 Answer the following **16**

a) Stating the underlying assumptions, define process capability indices C_p and C_{pk}. Derive the relationship between them.
b) What is CUSUM chart? Explain its construction and operation

Q.7 Answer the following **16**

a) Stating the assumptions, explain the construction and operations of the Hotelling's T^2 chart to monitor process mean vector.

b) Explain the basic concept of six-sigma methodology. Also explain the benefits of implementing the same.

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
STATISTICS
Reliability and Survival Analysis (MSC16403)

Day & Date: Friday, 14-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If $\phi(x)$ is a structure function then dual of $\phi(x)$ is_____.
 a) $1 - \phi(x)$ b) $1 - \phi(1-x)$
 c) $\phi(1-x)$ d) none of these

2) Suppose $R(t)$ is the reliability of parallel system of two components having reliabilities $R_1(t)$ and $R_2(t)$ respectively then______.
 a) $R(t) > Max\{R_1(t), R_2(t)\}$
 b) $R(t) = Min\{R_1(t), R_2(t)\}$
 c) $R(t) \geq Min\{R_1(t), R_2(t)\}$
 d) $R(t) < Min\{R_1(t), R_2(t)\}$

3) A vector $\underline{X}$ is called cut vector if_____.
 a) $0 \leq \phi(\underline{X}) \leq 1$ b) $\phi(\underline{X}) = 0.5$
 c) $\phi(\underline{X}) = 1$ d) $\phi(\underline{X}) = 0$

4) The i^{th} component of a system is irrelevant if_____.
 a) $\phi(1_i, \underline{x}) \leq \phi(0_i, \underline{x})$ b) $\phi(1_i, \underline{x}) \geq \phi(0_i, \underline{x})$
 c) $\phi(1_i, \underline{x}) = \phi(0_i, \underline{x})$ d) none of the above

5) If distribution F is IFR then_____ is Polya function of order 2.
 a) $h(t)$ b) $R(t)$
 c) $\log R(t)$ d) $Z(t)$

6) The minimal cut sets of structure ϕ are_____ for its dual.
 a) minimal cut vectors b) minimal cut sets
 c) minimal path sets d) none of the above

7) A distribution function $F(t)$ said to have new worse than used (NWU) if
 a) $\bar{F}(t+x) \geq \bar{F}(t)\bar{F}(x)$ b) $\bar{F}(t+x) \leq \bar{F}(t)\bar{F}(x)$
 c) $\bar{F}(t+x) = \bar{F}(t)\bar{F}(x)$ d) none of the above

8) In type I censoring_______.
 a) the number of failures is fixed
 b) duration of an experiment is fixed
 c) both time and number of failures is fixed
 d) none of these

9) Censoring technique is used for reducing_____.
 a) time of experiment b) cost of experiment
 c) number of failures d) none of the above

10) In survival analysis, the data set may contain______.
   a) only left censored observations
   b) only right censored observations
   c) both left and right censored observations
   d) none of the above

B) Fill in the blanks. **06**

1) The number of minimal paths in 2-out-of-3 system are_______.
2) DFRA property is preserved under_____.
3) As the number of components n increases, the reliability of series system______.
4) The survival function ranges between______.
5) In type I censoring, the number of uncensored observations has ___distribution.
6) To obtain confidence band for survival function_______ statistic is used

Q.2 Answer the following. **16**

a) Define reliability of component. Obtain the reliability of series system of n independent components.
b) Write a short note on Polya function of order 2.
c) Describe each of the following with one illustration:
1) Type-I censoring
2) Type-II censoring
d) Write a short note on estimation of survival function under uncensored data.

Q.3 Answer the following.

a) Define mean time to failure (MTTF) and mean residual life (MRL) function. Obtain the same for exponential distribution. **08**
b) Define star shaped function. Prove that F ∈ IFRA if and only if $-\log R(t)$ is star shaped. **08**

Q.4 Answer the following.

a) Define IFR and IFRA class of distributions. If F ∈ IFR then show that F ∈ IFRA . **08**
b) If failure time of item has Weibull distribution with distribution function **08**

$$F(t) = \begin{cases} 1 - e^{-(\lambda t)^{\alpha}}, & t > 0 \\ 0, otherwise \end{cases}$$

Examine whether it belongs to IFR or DFR.

Q.5 Answer the following.

a) Describe the need of censoring experiment. Describe situations where random censoring occurs naturally. **08**
b) Obtain maximum likelihood estimator of the mean of exponential distribution under type I censoring. **08**

Q.6 Answer the following.

a) Describe actuarial method of estimation of survival function, with suitable illustration. **08**
b) Describe Gehan's test for two sample testing problem in presence of censoring. **08**

Q.7 Answer the following.

a) For a coherent system with n components prove that: **08**

1) $\phi(0) = 0$ and $\phi(1) = 1$
2) $$\prod_{i=1}^{n} X_i \leq \phi(X) \leq \coprod_{i=1}^{n} X_i$$

b) Obtain the nonparametric estimator of survival function based on complete data. Also obtain confidence band for the same using Kolmogorov-Smirnov statistic. **08**

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
STATISTICS
Optimization Techniques (MSC16404)

Day & Date: Sunday, 16-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Select correct alternatives of the following questions. **10**

1) If ith constraint of LPP is deleted then the optimum solution is also changed then such constraint is called _____.
   a) Redundant constraint
   b) Binding constraint
   c) Unbinding constraint
   d) none of these

2) Which of the following is not correct with respect to standard form of LPP?
   a) All Constraints are "<=" type
   b) All Constraints are "=" type
   c) All right hand side coefficients ">=" type
   d) Objective function may be maximization or minimization

3) Linear programming problem means _____.
   a) Linear objective function but linear or non linear constraints
   b) Linear constraints but linear or non linear objective function
   c) All constraints and objective function is linear
   d) None of the above

4) At any iteration of the Big-M simplex method, if there exist at least one artificial variable in the basis at zero level and all $z_j - c_j \geq 0$, the current solution is _____.
   a) Infeasible
   b) Unbounded
   c) Non-degenerate optimum basic feasible solution
   d) Degenerate optimum basic feasible solution

5) To maintain feasibility of current optimum solution, a range of change in the constants $b_k(\Delta b_k)$, is _____.
   a) $\max\left\{-\frac{x_{B_i}}{\beta_{ik}}, \beta_{ik} > 0\right\} \leq \Delta b_k \leq \min\left\{-\frac{x_{B_i}}{\beta_{ik}}, \beta_{ik} < 0\right\}$
   b) $\min\left\{-\frac{x_{B_i}}{\beta_{ik}}, \beta_{ik} > 0\right\} \leq \Delta b_k \leq \max\left\{-\frac{x_{B_i}}{\beta_{ik}}, \beta_{ik} < 0\right\}$
   c) $-\infty \leq \Delta b_k \leq \min\left\{-\frac{x_{B_i}}{\beta_{ik}}, \beta_{ik} < 0\right\}$
   d) $\max\left\{-\frac{x_{B_i}}{\beta_{ik}}, \beta_{ik} > 0\right\} \leq \Delta b_k \leq \infty$

6) Which of the following is not correct?
   a) Number of dual constraints equal to number of primal variables
   b) Number of dual variables equal to number of primal constraints
   c) Primal objective is minimization type then dual objective is maximization type
   d) Dual variables are always unrestricted in sign

7) Addition of a constraint in linear programming problem.
   a) Always affects on feasible solution space
   b) It always changes the current optimum solution
   c) It may nor may not affect on solution space
   d) It enlarged solution space

8) In mixed integer programming problem.
   a) Different objective functions are mixed together
   b) All of the decision variables require integer solutions
   c) Only few of the decision variables requires integer solutions
   d) None of these

9) Consider two-person zero sum game with payoff matrix as _____

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

   Then saddle point is _____.
   a) 5 b) 6
   c) 7 d) 8

10) If the quadratic form X^TQX is positive definite, then it is _____.
   a) Strictly convex b) Strictly concave
   c) Convex d) Concave

B) Fill in the blanks **03**

1) Artificial variable added in constraints to solve LPP, if _____ does not exist.
2) If the players select the same strategy each time, then it is referred as _____.
3) The quadratic form X^TQX is negative definite, then it is _____.

C) State whether following statements are true or false **03**

1) Branch and bound method used to solve QPP.
2) Degenerate basic feasible solution means at least one basic variable having value zero.
3) Dual constraints corresponding to maximization primal problem are “>=” type _____.

Q.2 a) Explain graphical method to solve two persons zero sum problem. **04**

b) State and prove weak duality theorem. **04**

c) Show that set of all convex combinations of finite number of points of $S \subseteq R^n$ is convex set. **04**

d) Define the following terms. **04**

i) Convex polyhedral
ii) Supporting hyperplane
iii) Separating hyperplane

Q.3 **a)** Describe dual simplex algorithm to solve linear programming problem. **08**

b) What is linear programming problem? Explain advantages and its limitations. **08**

Q.4 **a)** Solve the following LPP using simplex method **08**

Max $Z = 2x_1 + 3x_2$,

Subject to, $x_1 + x_2 \leq 30$, $x_1 - x_2 \geq 0$, $0 \leq x_1 \leq 20$, $0 \leq x_2 \leq 12$

b) Describe Beale's method to solve quadratic programming problem. **08**

Q.5 **a)** Show that game problem can be modelled as LPP. **08**

b) Describe use of artificial variable to solve LPP and explain Big-M method in detail. **08**

Q.6 **a)** What is quadratic programming problem? Obtain necessary KKT conditions. **08**

b) State and prove basic duality theorem. **08**

Q.7 **a)** Describe Gomory's cutting plane method for mixed IPP. **08**

b) Use dual simplex method to solve following LPP. **08**

Max $Z = x_1 + x_2$

Subject to the constraints:

$2x_1 + x_2 \geq 16$,

$x_1 + 2x_2 \leq 6$,

$x_1, x_2 \geq 0$

SLR-SR-21

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
STATISTICS
Time Series Analysis (MSC16407)

Day & Date: Tuesday, 18-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Which of the following is additive model of time series?
   a) $X_t = m_t \times s_t \times Y_t$ b) $X_t = m_t + s_t + Y$
   c) $X_t = m_t \times s_t + Y_t$ d) $X_t = m_t \times Y_t$

2) The variance of white noise process is _____.
   a) always zero b) constant
   c) non-constant d) None of these

3) A weak stationary process is also known as _____.
   a) variance stationary b) second order stationary
   c) both A) and B) d) none of these

4) AR (2) model can be presented as
   a) $X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + Z_t \; where \; \{Z_t\} \sim WN(0, \sigma^2)$
   b) $X_t = \phi_1 X_{t-1} \times \phi_2 X_{t-2} + Z_t \; where \; \{Z_t\} \sim WN(0, \sigma^2)$
   c) $X_t = \phi_1 X_{t-1} \times \phi_2 X_{t-2} \times Z_t \; where \; \{Z_t\} \sim WN(0, \sigma^2)$
   d) $X_t = \mu \times \phi_1 X_{t-1} + \phi_2 X_{t-2} + Z_t \; where \; \{Z_t\} \sim WN(0, \sigma^2)$

5) The ACF of MA (1) process is zero after lag
   a) One b) Two
   c) Three d) Zero

6) The singe exponential smoothing equation is used when _____ present in the given time series
   a) there is trend component
   b) there is trend and seasonal component
   c) there is only level
   d) there is cyclic component

7) Turning point test is used for testing _____.
   a) trend in the given series
   b) seasonality in the given series
   c) average value of the given series
   d) none of these

8) Holt - Winter smoothing method is used when there is
   a) Trend component present only
   b) Seasonal component present only
   c) Trend and seasonal both present
   d) There is level component present only

9) The linear process $X_t = \sum_{j=\infty}^{\infty} \Psi_j Z_{t-j}$ is called Moving average process of infinity order if _____

a) $\Psi_j = 1; j > 0$ b) $\Psi_j = 0; j > 0$
c) $\Psi_j = 0; j < 0$ d) $\Psi_j = 1; j < 0$

10) The process $X_t = \phi_1 X_{t-1} + Z_t$ where $\{Z_t\} \sim WN(0, \sigma^2)$ is causal process if ______

a) $|\phi_1| < 1$
b) $|\phi_1| > 1$
c) $|\phi_1| = 1$
d) $|\phi_1| < 1.5$

B) Fill in the blanks. **06**

1) A linear filter when applied to stationary input series, produces _____ series.
2) The ACF of AR (1) process $X_t = \phi_1 X_{t-1} + Z_t$ is _____.
3) ACF of MA (q) process has insignificant autocorrelations after lag ___.
4) If *d* is non-negative integer, then $\{X_t\}$ is said to be ARIMA *(p, d, q)* process if $Y_t = (1 - B)^d X_t$ is _____ process.
5) Differencing method can be used to eliminate _____ and _____ component in the given time series if present.
6) The causal representation of ARMA(p, q) process $\{X_t\}$ is _____.

Q.2 Answer the following. **16**

a) Define weakly stationary and strong stationary process.
b) Describe any one method of estimation and elimination of trend component in time series.
c) Write a short note on double exponential smoothing.
d) Define AR (1) process. Obtain its autocorrelation function.

Q.3 Answer the following. **16**

a) Define causal process. Describe the method to obtain Ψ_j weights in a given causal process.
b) Define autocovariance function. State and prove its elementary properties.

Q.4 Answer the following. **16**

a) Discuss the identification of mixed models.
b) Verify whether the process $X_t + 0.6X_{t-1} = Z_t$ is causal or not. Obtain the autocovariance function of the same process.

Q.5 Answer the following **16**

a) Define ARMA $(1,1)$ process and hence obtain its autocorrelation function.
b) Discuss in detail ARCH and GARCH volatility models.

Q.6 Answer the following **16**

a) Determine which of the following processes are causal and/or invertible

1) $X_t + 0.6X_{t-1} = Z_t + 0.04Z_{t-1}$
2) $X_t + 1.6X_{t-1} = Z_t - 0.4Z_{t-1} + 0.04Z_{t-1}$

in both process $\{Z_t\} \sim)WN(0, \sigma^2)$

b) Describe analysis of Seasonal ARIMA $(p, d, q) \times (P, D, Q)$ process.

Q.7 Answer the following **16**

a) Define MA (q) process. Obtain PACF of MA (q) process.

b) Discuss the preliminary transformations in the time series analysis.