SLR-SO-1

Seat No.	

Set P

M.Sc. (Semester-I) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Number Theory (MSC15108)

Day & Date: Wednesday, 19-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) If gcd $(a,m) = 1$ then a is primitive root of m if and only if _____ for every prime divisor p of $\varphi(m)$.
   a) $a^{\frac{\varphi(m)}{p}} \not\equiv 1(mod m)$
   b) $a^{\frac{\varphi(m)}{p}} \equiv 1(mod m)$
   c) $a^{\frac{\varphi(m)}{p}} \not\equiv 0(mod m)$
   d) $a^{\frac{\varphi(p)}{m}} \equiv 1(mod m)$

2) If p is prime and $p|a_1.a_2.a_3 \dots a_n$ then for $1 \le k \le n$ _____.
   a) $p|a_k$ for some k
   b) $p|a_k$ for all k
   c) $p \nmid a_k$ for all k
   d) $a_k|p$ for some k

3) If $f(n) = n^2 + 2$ and $n = 6$ then _____.
   a) $\sum_{d|6} f(d) = \sum_{d|6} f\left(\frac{d}{6}\right)$
   b) $\sum_{d|6} f(d) = \sum_{6|d} f(d)$
   c) $\sum_{d|6} f(d) = \sum_{d|6} f\left(\frac{6}{d}\right)$
   d) $\sum_{d|6} f(d) = 0$

4) Consider the statements:
   If p is a prime number then $(p - 1)! = 1(mod\ p)$
   If $a^{m-1} \equiv 1\ (mod\ m)$ then m is a prime number.
   a) only I is true
   b) only II is true
   c) both I and II are true
   d) both I and II are false

5) The number of integers of $S = \{1,2,3, \dots\dots, n\}$ divisible by a positive integer a is _____.
   a) $\sigma(a)$
   b) $\left[\frac{n}{a}\right]$
   c) $\left[\frac{a}{n}\right]$
   d) $\mu(n)$

6) If $a > 1$ and m, n are positive integers then $\gcd(a^m - 1, a^n - 1) =$ ____
   a) $a^{\gcd(m,n)} - 1$
   b) $\gcd(m,n) - 1$
   c) $a^{\gcd(m,n)}$
   d) $\gcd(m,n)$

7) If $a \equiv b(mod\ n_1)$ and $a \equiv c(mod\ n_2)$ where the integer $n = \gcd(n_1, n_2)$ then _____
   a) $b \equiv c(mod\ n_1)$
   b) $b \equiv c(mod\ n)$
   c) $a \equiv b(mod\ n_2)$
   d) $c \equiv b(mod\ n_2)$

8) Which of the followings are primitive root of 10?
   a) 3 and 7
   b) 3 and 9
   c) 5 and 7
   d) 7 and 9

9) If x and y be real numbers and [.] is the greatest integer function then which of the following is not true?
   a) $[x+y] \leq [x]+[y]+1$
   b) $[x+n] \leq [x]+n, n$ is any integer
   c) $\left[\frac{[x]}{n}\right] = \left[\frac{x}{n}\right], n$ is positive integer
   d) $[x-y] = [x]-[y]+1$

10) If p is a prime and $k > 0$ then which of the followings are true?
   a) $\varphi(p^k) = p^k - p^{k-1}$ b) $\varphi(p^k) = p^k + p^{k-1}$
   c) $\varphi(p^{k+1}) = p\varphi(p^k)$ d) Both a and c

B) Fill in the blanks. **06**

1) The number of integers less than 1896 and relatively prime to 1896 are _____.

2) If the orders of a_1 and a_2 modulo n be k_1 and k_2 respectively and $\gcd(k_1, k_2) = 1$ then the order of $a_1 a_2 (mod\ n)$ is _____.

3) The remainder when the sum S= 1! + 2! + 3! + …… + 999! + 1000! divisible by 8 is _____.

4) The congruence $x \equiv a\ (mod\ n)$ and $x \equiv\ b(mod\ m)$ admits a simultaneous solution iff

5) If a is a primitive root modulo n and b, c are any integers, then $ind.(bc)\ =$ _____.

6) An $gcd\ (ka, kb) =\ k.gcd\ (a, b)$ if _____.

Q.2 Answer the following **16**

a) If $c = ax + by$ and $d|a$ but $d \nmid c$ then show that $d \nmid b$

b) Factorize 340663 by Fermat's Factorization Method.

c) Find $\tau(n)$ and $\sigma(n)$ for n = 7056.

d) Solve the congruence $x^3\ \equiv\ 5(mod\ 13)$.

Q.3 Answer the following.

a) Show that every positive integer $n > 1$ can be expressed as the product of prime uniquely. **08**

b) Find the positive solution of the equation $11x + 5y = 17$ **08**

Q.4 Answer the following.

a) If a and b are two positive integers then show that $gcd(a, b)\ .lcm(a, b)\ =\ a.b$ **08**

b) If $n > 1$ be an integer then show that $\sigma(n)$ is odd if and only if n is a perfect square or twice of a perfect square. **08**

Q.5 Answer the following.

a) State and prove Chinese Remainder theorem. **10**

b) If $m, n > 2$ and $gcd(m, n) = 1$ then show that there exists no primitive root $(mod\ mn)$ **06**

Q.6 Answer the following.

a) State and prove Wilsons theorem and show that the converse of Wilsons theorem is also true. **10**

b) Find the last two digits of the number 9^{9^9} **06**

Q.7 Answer the following.

a) If p is a prime and $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_{0,}\ a_n \not\equiv 0(mod\ p)$ is a polynomial of degree $n \geq 1$ with integral coefficients then show that $f(x) = \ 0(mod\ p)$ has at least n incongruent solutions $mod\ p$. **08**

b) Find the highest power of 18 contained in 500! **08**

SLR-SO-2

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Object Oriented Programming using C++ (MSC15109)

Day & Date: Wednesday, 19-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If we want to free a dynamically allocated array, we must use the _____.
   a) cut b) remove
   c) delete d) erase

2) The _____ operator allows access to the global version of a variable.
   a) arithmetic b) scope resolution
   c) memory allocation d) static

3) Constructor is executed when _____.
   a) An object goes out of scope
   b) A class is declared
   c) An object is used
   d) An object is created

4) Which operator can not be _____ overloaded?
   a) + b) ::
   c) - d) *

5) _____ means the ability to take more than one form.
   a) Polymorphism b) Abstraction
   c) Inheritance d) None of these

6) A _____ is a collection of objects of similar type.
   a) Object b) Polymorphism
   c) Class d) Inheritance

7) _____ is used to declare integer data type.
   a) INT b) integer
   c) Integer d) int

8) _____ is the process by which objects of one class acquire the properties of another class.
   a) Encapsulation b) Inheritance
   c) Polymorphism d) Abstraction

9) _____is a ‘special’ member function whose task is to initialize the objects of its class.
   a) Constructor b) Pointer
   c) Constant d) inline

10) A _____ pointer refers to an object that currently invokes a member function.
   a) object   b) recursive
   c) this   d) multiple

B) State whether True or False. **06**

1) Constructors are invoked automatically when the objects are created.
2) Keywords refer to the names of variables, functions, arrays, classes, etc.
3) Function prototype describes the function interface to the compiler by giving details such as the number and type of arguments and type of return values.
4) A static function can have access to only other static member declared in the same class.
5) A non-member function can have an access to the private data of a class.
6) A class is a collection of objects of similar type.

Q.2 Answer the following. **16**
a) What is Flowchart? Explain the use of different symbols used in flowchart.
b) Explain the use of static member function with example.
c) What is call by reference? Explain with suitable example.
d) What is Constructor? Explain with example.

Q.3 Answer the following.
a) Explain different data types used in C++. **08**
b) What is friend function? Explain importance of friend function with example. **08**

Q.4 Answer the following.
a) What is Operator overloading? Explain with syntax and example. **08**
b) What is inheritance? Explain different types of inheritances. **08**

Q.5 Answer the following.
a) What is Virtual function? Explain the rules for virtual functions. **08**
b) Write a C++ program to implement operator overloading (use - (minus) operator) **08**

Q.6 Answer the following.
a) What is File? Explain different modes of file for opening. **08**
b) Write a C++ program to implement Class and object. (assume your own data) **08**

Q.7 Answer the following.
a) Explain the use of following statements with syntax and example. **08**
   i) width()   ii) precision()
   iii) fill()   iv) setf()
b) Write a C++ program to implement Single Inheritance. (assume your own data) **08**

SLR-SO-3

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Algebra - I (MSC15101)

Day & Date: Thursday, 20-07-2023 — Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Consider the following statements.
P: Every normal series is subnormal
Q: Every composition series is normal series
Then,
a) P is true but Q is false b) P is false but Q is true
c) Both P and Q is true d) Both P and Q is false

2) Which of the following is true in an integral domain D.
a) D is commutative ring
b) D has without zero divisor
c) D has unity
d) All of these

3) Which of the following is an integral domain?
a) Z b) 2Z
c) 3Z d) 5Z

4) If G is a cyclic group then which of the following is always true?
a) $G' = G$ b) $G' \neq \{e\}$
c) $G' = \{\,\}$ d) $G' = \{e\}$

5) If a group G is finite cyclic group of order p where p is prime, then number of generators of G is/are _____.
a) p b) p-1
c) p+1 d) 1

6) In $\mathrm{Z}[x]$, content of $x^2 + 2x - 3$ is _____.
a) 1 b) -1
c) -2 d) 2

7) Class equation of S_3 is _____.
a) $2 + 2 + 2$ b) $1 + 1 + 4$
c) $1 + 2 + 3$ d) $1 + 1 + 1 + 1 + 1 + 1$

8) Any group of order p^2, where p is prime then G is _____.
a) Abelian b) Non abelian
c) Cyclic d) None of these

9) If $\mathbb{F}$ is a field, then _____.
a) F is Integral domain b) F is Principal ideal domain
c) F is Euclidean domain d) All of these

10) If D is Euclidean domain, then D is _____.
   a) Principal ideal domain b) Unique factorization domain
   c) Integral domain d) All of these

B) Fill in the blanks. **06**

1) Class equation of $Q_8 = \{1, -1, i, -i, j, -j, k, -k\}$ is _____.
2) If G is abelian group then G' = _____.
3) Two subnormal series of a group G are have _____ refinement.
4) For every field F there exist at most _____ ideals.
5) Units in ring of Gaussian integer i.e. $\{a + ib / a, b \in Z\}$ is/are _____.
6) In principal ideal domain 'R' every ideal S of R is a _____.

Q.2 Answer the following **16**

a) State and prove Jordan - Holder theorem.
b) If $\{e\} = H_0 \lhd H_1 \lhd H_2 \lhd \cdots \lhd H_n = G$ is a subnormal series of a group G. $0\left(\frac{H_{i+1}}{H_i}\right) = S_{i+1}$ (say) for all $i = 0,1, \ldots, n-1$ then show that G is of finite order $S_1, S_2, \ldots, S_n$.
c) If $f(x) = x^4 - 3x^3 + 2x^2 + 4x - 1$ and $g(x) = x^2 - 2x + 3$ in $Z_5[x]$
Find $q(x)$ and $r(x)$ such that,
If $f(x) = g(x).q(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree $g(x)$.
d) Show that the cyclotomic polynomial
$p(x) = \frac{x^p - 1}{x - 1} = x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over Q for any prime.

Q.3 Answer the following.

a) If G be a finite group and X is finite G set if 'r' is no. of orbits of X in G then Prove that, $r.|G| = \sum_{g \in G} |X_g|$ where $|X_g| = \{x \in X / x.g = x, g \in G\}$ **08**

b) If G be a finite group and 'p' be a prime number such that $p \mid O\,(G)$ then prove that there exists an element $a \in G$ such that $a^p = e$ where e is an identity of G. **08**

Q.4 Answer the following.

a) Show that Subgroup of nilpotent group is nilpotent. **08**
b) If G be a finite p group and H be a p-subgroup of G then prove that, $(N(H){:}\,H) = (G{:}\,H)(mod\ p)$ **08**

Q.5 Answer the following.

a) Show that : No group of order 30 is simple. **08**
b) Define zero of the Polynomial, and find all the zeros of $f(x) = x^5 + 3x^3 + x^2 + 2x$ in $Z_5[x]$ **08**

Q.6 Answer the following.

a) If D be a principal ideal domain then prove that Every element neither 0 nor unit in D is product of irreducibles. **08**
b) If F is a field then prove that the ideal generated by $p(x) \neq 0$ of $F[x]$ is maximal iff $p(x)$ is irreducible over F **08**

Q.7 Answer the following.

a) If D is a unique factorization domain then prove that Product of two primitive polynomials in $D[x]$ is again primitive. **08**
b) State and prove 'Eisenstein's criteria of irreducibility over Q'. **08**

SLR-SO-4

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Real Analysis – I (MSC15102)

Day & Date: Friday, 21-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) **Choose correct alternative.** **10**

1) The lower integral of a function f on $[a, b]$ is_____
   a) infimum of set of upper sums
   b) infimum of set of lower sums
   c) supremum of set of upper sums
   d) supremum of set of lower sums

2) Consider the following statements:
   I) Every monotonic increasing function on $[a, b]$ is bounded
   II) Every monotonic increasing function on $[a, b]$ is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

3) By first mean value theorem, if a function f is continuous on $[a, b]$ then there exist a number ξ in $[a, b]$ such that $\int_a^b f(x)dx$ =_____
   a) $f(\xi)(a-b)$ b) $f(\xi)(b-a)$
   c) $f(\xi)(a+b)$ d) $f'(\xi)(a-b)$

4) If S is convex set then _____ for all $x, y \in S$
   a) $L(x,y) \subseteq S$ b) $L(x,y) \supseteq S$
   c) $L(x,y) = S$ d) None of these

5) Consider the following statements:
   I) Function having only one point discontinuity is integrable.
   II) Function having finite no. of points of discontinuity is integrable.
   a) only I is true b) only II is true
   c) both are true d) both are false

6) A function can have finite directional derivate $f'(C: u)$ but may fail to _____ at C.
   a) derivable b) finite
   c) integrable d) continuous

7) The directional derivative of $f(x,y) = xy$ at point $(1,1)$ in the direction $(1,0)$ is _____.
   a) 1 b) $(1,1)$
   c) y d) x

8) For a monotonic increasing function f on $[a, b]$, all values of function lies between _____ and _____.
   a) $0, f(a)$ b) $0, f(b)$
   c) $f(a), f(b)$ d) all of the above

9) If f and g are integrable functions then _____ is also integrable.
   a) $f+g$ b) $f-g$
   c) $f.g$ d) all of the above

10) The statement $\int_a^b f(x)dx$ exists implies that the function f is _____ and _____.
   a) continuous, integrable b) bounded, integrable
   c) bounded, continuous d) finite, continuous

B) Write True or False. **06**

1) Riemann sum for a function f on $[a,b]$ is defined as $S(P,f) =$ ____

2) The partial derivatives of a function describe the rate of change of a function in the direction of _____.

3) If f is non-negative on $[a,b]$ such that $\int_0^1 f(x)\,dx = 0$ then $f(x) =$_____ for all $x \in [0,1]$

4) For $\int_1^2 f(x)dx$, what is the value of Δx_i (length of n equal sub intervals)?

5) The condition of _____ is necessary for a function to assume its mean value ξ in given interval by first mean value theorem.

6) If $P1$ and $P2$ are two partitions of $[a,b]$ then their common refinement is given by $P^* =$

Q.2 Fill in the blanks. **16**

a) If $f(x,y) = (xy, x^2+y, x+y^2)$ then find $Df(x,y)$

b) If P_1, P_2 are any two partitions then with usual notations prove that $L(P,f,\alpha) \le U(P,f,\alpha)$

c) If f is bounded and integrable on $[a,b]$ and $K>0$ is a number such that $|f(x)| \le K$ for all $x \in [a,b]$ then prove that $|\int_a^b f(x)dx| \le K|b-a|$

d) If a function f is continuous on $[a,b]$ then prove that there exists a number ξ in $[a,b]$ such that $\int_a^b f(x)dx = f(\xi)(b-a)$

Q.3 Answer the following.

a) If f is bounded function on $[a,b]$ then prove that for every $\epsilon>0$ there corresponds $\delta>0$ such that **08**

1)
$$U(P,f) < \int_a^{\bar{b}} f(x)dx + \epsilon$$

2)
$$L(P,f) > \int_{\underline{a}}^{b} f(x)dx - \epsilon$$

for every partition P of $[a,b]$ with norm $\mu(P) < \delta$

b) Solve $\int_1^2 (x^2+3)dx$ by Riemann sum method. **08**

Q.4 Answer the following.

a) If a function f is bounded and integrable on$[a,b]$ then prove that the function F defined as, $F(x) = \int_a^x f(t)dt; \le a \le x \le b$ is continuous on $[a,b]$. **08** Furthermore if f is continuous at a point c of $[a,b]$ then prove that F is derivable at c and $F'(c) = f(c)$

b) If f_1 and f_2 are bounded and integrable functions on $[a,b]$ then prove that f_1+f_2 is also $\int_a^b (f_1+f_2)dx = \int_a^b f_1 dx + \int_a^b f_2 dx$ **08**

Q.5 Answer the following.

a) If f and all its partial derivatives of order less than m are differentiable at each point of an open set S in R^n and a, b are two points of S such that $L(a,b) \subseteq S$ then prove that there is a point z on the line segment $L(a,b)$ such that **08**

$$f(b) - f(a) = \sum_{k=1}^{m-1} \frac{1}{k!} f^{(k)}(a; b-a) + \frac{1}{m!} f^{m}(z, b-a)$$

b) Solve $\int_0^2 (x+9)d(x^2)$ **08**

Q.6 Answer the following.

a) If a function $f = (f_1, f_2 \ldots\ldots f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n and the Jacobian determinant $J_f(a) \neq 0$ for some point a in S then prove that there is an n-ball $B(a)$ on which f is one to one. **08**

b) If f is differentiable at c then prove that f is continuous at c. **08**

Q.7 Answer the following.

a) If P^* is a refinement of a partition P then for a bouded function f prove that **08**

1) $L(P^*, f, \alpha) \geq L(P, f, \alpha)$
2) $U(P^*, f, \alpha) \leq U(P, f, \alpha)$

b) Find extrema of a function $f(x) = 3x^4 + 4x^3 - 84x^2 - 288x$ **08**

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Differential Equations (MSC15103)

Day & Date: Saturday, 22-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The existence of partial derivative of a function _____ is the necessary for 'f'satisfies Lipschitz condition.
   a) $\left|\frac{\partial f}{\partial y}(x,y)\right| \leq K$ b) $\left|\frac{\partial f}{\partial y}(x,y)\right| = K$
   c) $\left|\frac{\partial f}{\partial y}(x,y)\right| \geq K$ d) $\left|\frac{\partial f}{\partial y}(x,y)\right| \neq K$

2) For A liner differential equation $a_0(x)y^n + a_1(x)y^{n-1} + \cdots + a_n(x)y = 0$ A singular point is any point $x = x_0$ for which $a_0\,(x_0) =$_____.
   a) One b) Two
   c) Zero d) Three

3) The regular singular point of Bessel's differential equation is _____.
   a) 0 b) -1
   c) 1 d) 2

4) Two solution $y_1(x)$ and $y_2(x)$ of the equation $a_0(x)y'' + a_1(x)y' + a_2(x)y = 0, a_0(x) \neq 0$ in $\mathrm{I} = (\mathrm{a}, \mathrm{b})$ are linearly independent iff their Wronskian is not zero at _____.
   a) Some x_0 in(a, b) b) at two points in (a, b)
   c) All x in (a, b) d) None of the above

5) If r_1 and r_2 are distinct roots of indicial polynomial $q(r) = 0$ of Euler's equation then solution of Euler's equation is, _____.
   a) $\varphi_1(x) = |x|^{r_1}, \varphi_2(x) = |x|^{r_1}$
   b) $\varphi_1(x) = |x|^{r_1}, \varphi_2(x) = x|x|^{r_1}$
   c) $\varphi_1(x) = |x|^{r_1}, \varphi_2(x) = |x|^{r_1}\log(x)$
   d) $\varphi_1(x) = |x|^{r_1}, \varphi_2(x) = 1$

6) The Wronskian of $\varphi_1\,(x) = x^2$ and $\varphi_2(x) = x|x|$ on$-\infty < x < \infty$ is _____.
   a) x^3 b) x^2
   c) 0 d) 2

7) The characteristic polynomial of a differential equation $y'' + a_1y' + a_2y = 0$
   a) $p(r) = r^2 + a_1r + a_2$
   b) $p(r) = a_1r + a_2$
   c) $p(r) = r^2 + a_2$
   d) $p(r) = r^2 + a_1r + a_2$

8) The differential equation $x^2y" + axy' + by = 0$ is _____.
   a) Euler equation b) Legendre equation
   c) Bessel equation d) Wave equation

9) Consider $L(y) = y" + a_1y' + a_2\,y = 0$ where $a_1,\ a_2$ are real constant then every solution of $\mathrm{L}(y) = 0$ tends to zero as $x \to \infty$ if _____.
   a) $a1 > 0$ b) $a1 < 0$
   c) $a1 = 0$ d) $a2 > 0, a1\ G\ 0$

10) Let φ is the solution of $L(y) = y'' + a_1y' + a_2y = 0$ on an interval I containing the point x_0 than for all $x\ \in \mathrm{I}$.
   a) $\|\varphi\ (x_0)\|\ e^{k|x-x_0|} \le \|\varphi(x)\|$
   b) $\|\varphi\ (x)\|\ \le e^{-k|x-x_0|}\|\varphi(x_0)\|$
   c) $\|\varphi\ (x)\|\ \le\ e^{k|x-x_0|}\ \|\varphi(x_0)\|$
   d) None of these

B) Fill in the blanks. **06**

1) The generating function of $J_n(x)$ is _____.
2) The solutions of $y'' - 16y = 0$ are _____.
3) Basis set of solutions of $y" - y = 0$ is _____.
4) The singular point of $(1 + x^2)y" + 2xy' + 2y = 0$ is _____.
5) A linear differential equation $L(y) = b(x)$ is said to be homogeneous if $b(x)$ _____.
6) If P is a polynomial such that deg $(P) \le 1$ then P has _____ root.

Q.2 Answer the following. **16**

a) Define Wronskian and find $\mathrm{W}(\varphi_1, \varphi_2)(x)$ if $\varphi_1 = \sin x\,, \varphi_2 = \cos x$.
b) Show that $f(x, y) = x^2\cos^2 x + y\sin^2 x$ on $S = \{(x, y)/|x| \le 1, |y| \le \infty\}$ satisfies Lipschitz condition.
c) Show that $\frac{d}{dx}x^{-n}.J_n(x) = -x^{-n}J_{n+1}(x)$, where $J_n(x)$ is Bessel's function of first kind of order n.
d) Find two linearly independent solution of equation $(3x - 1)^2\ y" + (9x - 1)y' - 9y = 0$

Q.3 Answer the following.

a) If $a_1$and a_2 be the constants and consider the equation $L(y) = y"a_1y' + a_2y = 0$ then show that, **10**
   i) If r_1, r_2 are distinct roots of characteristic polynomial $P(r) = r^2 + a_1r + a_2$ then the function $\varphi_1(x) = e^{r1(x)}\ \ \varphi_2(x) = e^{r_2(x)}$ is solution of $L(y) = 0$. **06**
   ii) If r_1 is repeated roots of characteristic polynomial $P(r) = r^2 + a_1r + a_2$ then the function $\varphi_1(x) = e^{r1(x)}\ \ \varphi_2(x) = xe^{r_1(x)}$ is solution of $L(y) = 0$.
b) Find the solution of $y'' + y =\ 0$ and,
   i) Compute the solution φ satisfies φ (0) = 1, φ (π/2) = 2
   ii) Compute the solution φ satisfies φ (0) = 0, φ (π) = 0

Q.4 Answer the following.

a) Find the general solution of the differential equation $4y" - y = e^x$ **08**
b) If x_0 be a point in I and $\alpha_1, \alpha_2 \ldots.\alpha_n$ be any n constants then show that there is at most one solution $\varphi(x)$ of $L(y) = y^n + a_1(x)y^{(n-1)} + a_2(x)y^{(n-2)} + \cdots + a_n(x)y = 0$ on I satisfying $\varphi(x_0) = \alpha_1$, $\varphi'(x_0) = \alpha_2 \ldots..\varphi^{(n-1)}(x_0) = \alpha_n$ **08**

Q.5 Answer the following.

a) If $\varphi_1, \varphi_2 \cdots\cdots \varphi_n$ be the 'n' solution of **08**

$$L(y) = y^n + a_1(x)y^{(n-1)} + a_2(x)y^{(n-2)} + \cdots + a_n(x)y = 0$$

on an interval I and X_0 be a point in I then prove that,

$$W(\varphi 1, \varphi 2, \dots \dots .., \varphi_n)(x) = \exp\left[-\int_{x_0}^{x} a_1\ (t)\right] W(\varphi_1, \varphi_2\ \cdots\cdots \varphi_n)(x_0)$$

b) If one solution of $x^2y" - 2y = 0$ is $\varphi_1\ (x) = x^2$ then find all the solutions of $x^2y" - 2y = 2x - 1, \quad 0 < x < \infty$ **08**

Q.6 Answer the following.

a) Define 'Legendre's differential equation' and discuss the nature of solution of Legendre's differential equation. **10**

b) Show that $\int_{-1}^{1} p_n(x).p_m(x)dx = 0\ ,\ \ m \neq n$ **06**

where $p_n(x)$ and $p_m(x)$ be the solution of Legendre's equation

Q.7 Answer the following. **08**

a) Find the solution of the form $\varphi_1(x) = x^r \sum_{k=0}^{\infty} c_k\ x^k$ of the following differential equation $(x > 0)$,

$$2x^2y" + (x^2 - x)y' + y = 0$$

b) Show that **08**

i) $x.J'_n\ (x) = n.J_n\ (x) - x.J_{(n+1)}(x)$

ii) $x.J'_n\ (x) = x.J_{(n-1)}\ (x) - n.J_n(x)$

Where $J_n(x)$ is Bessel's function of first kind of order n

SLR-SO-6

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Classical Mechanics (MSC15104)

Day & Date: Sunday, 23-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Newton's equation of motion can be derived from Lagrange's equation.
   a) True b) False
   c) Can't say d) May be

2) If constraints are scleronomic Kinetic energy is given by _____.
   a) $T = \sum qj' \frac{\partial T}{\partial qj'}$ b) $2T = \sum qj \frac{\partial T}{\partial qj'}$
   c) $2T = \sum qj' \frac{\partial T}{\partial qj'}$ d) $2T = \sum qj' \frac{\partial T}{\partial qj}$

3) Routhian is a function which usually replaces _____.
   a) Lagrangian b) Hamiltonian
   c) Both a and b d) None of a and b

4) A string of length 'l' moving in the plane then its degrees of freedom are _____.
   a) 3 b) 2
   c) 4 d) 1

5) Brachistochrone problem deals with _____.
   a) a curve with extremum length
   b) a curve with extremum area
   c) a curve with extremum volume
   d) a curve with extremum time

6) Geodesic on the surface of sphere is _____.
   a) parabola b) arc of great circle
   c) cycloid d) hyperbola

7) Determinant value of an orthogonal matrix is _____.
   a) 1 b) -1
   c) Either 1 or -1 d) Neither 1 nor -1

8) Which of the following does not represents a rotation?
   a) Orthogonal matrix with determinant -1
   b) Orthogonal matrix with determinant +1
   c) Eulerian angles
   d) Both (b) and (c)

9) The rotation matrix in 3-dimensions has _____ degrees of freedom.
   a) 9 b) 6
   c) 3 d) 1

10) Conservative force is only depend on _____.
   a) Time
   b) Velocity
   c) Co-ordinates
   d) Both (a) and (b)

B) Fill in the blanks. **06**

1) Number of generalized coordinates of Atwood machine is/are _____.
2) The extremum of the functional $J[y(x)]$ is called local maximum if ΔJ _____.
3) $\int_{t_1}^{t_2}(L+H)dt$ represents _____.
4) If two particles in the 3D-space are constrained to maintain a fixed distance from each other then degrees of freedom are _____.
5) Conservation theorem for energy states that _____.
6) The curve is _____ for which area of surface of revolution is minimum when revolved about y-axis.

Q.2 Answer the following. (Each of 04 marks). **16**

a) Show that frictional force is not conservative.
b) Define Degrees of freedom and Generalised co-ordinates and give one example each.
c) Show that: The generalised momentum corresponding to cyclic co-ordinates is conserved.
d) State modified Hamilton's principle.

Q.3 Answer the following.

a) Derive Newton's equation of motion from Lagrange's equation of motion. **08**
b) A particle of mass m moving in a plane under the action of an inverse square law of attractive force. Derive the Lagrangian L and hence equation of its motion. **08**

Q.4 Answer the following.

a) Find Euler-Lagrange's differential equation satisfied by $y(x)$ for which the integral $I=\int_{x1}^{x2} f(y,y',x)dx$ has extremum value, where $y(x)$ is twice differentiable function satisfying $y(x_1)=y_1$ and $y(x_2)=y_2$ **08**
b) Find the extremal of the function $I\big(y(x)\big)=\int_{x_0}^{x_1}(16y^2-(y'')^2+x^2)dx$ **08**

Q.5 Answer the following.

a) Derive the Hamilton's canonical equation of motion from variational principle. **08**
b) Establish the relation between δ - variation and Δ - variation. **08**

Q.6 Answer the following.

a) Prove that: In case of orthogonal transformation the inverse matrix is identified by its transpose. i.e. $A^{-1}=A^T$ **08**
b) Derive the equation of motion of Atwood's machine. **08**

Q.7 Answer the following.

a) Show that: The shortest distance between two points in a plane is a straight line. **08**
b) State and prove Hamilton's principle by using Lagranges's equation. **08**

SLR-SO-7

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Algebra – II (MSC15201)

Day & Date: Wednesday, 19-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative **10**

1) If C is field of complex number then the dimension of $\mathrm{C}(\mathrm{C})$ is _____.
   a) 1 b) 2
   c) 3 d) 0

2) The degree of extension of $Q(\sqrt{3},\sqrt{5})$ over Q is _____.
   a) 2 b) 4
   c) 5 d) 6

3) Which of the following is algebraic over Q?
   a) $\sqrt{2}$ b) π
   c) e d) None of these

4) The Splitting field of $x^2 - 2 \in R[x]$ over R is _____.
   a) Q b) R
   c) C d) None of these

5) The number of automorphism of field of complex number is / are ____.
   a) 1 b) 2
   c) 3 d) 0

6) The dimension of R over Q is ______.
   a) 1 b) 2
   c) 3 d) Infinite

7) If $[Q(\sqrt{3}):Q] = 2$ then each element in $Q(\sqrt{3})$ is algebraic over Q of degree.
   a) Equal to 2 b) less than 2
   c) greater than 2 d) at most 2

8) If K is finite extension of a field F and $\mathrm{G}(\mathrm{K},\mathrm{F})$ is finite group then which of the following is true,
   a) $\mathrm{O}(\mathrm{G}(\mathrm{K},\mathrm{F})) = [\mathrm{K},\mathrm{F}]$ b) $\mathrm{O}(\mathrm{G}(\mathrm{K},\mathrm{F})) < [\mathrm{K},\mathrm{F}]$
   c) $\mathrm{O}(\mathrm{G}(\mathrm{K},\mathrm{F})) > [\mathrm{K},\mathrm{F}]$ d) $\mathrm{O}(\mathrm{G}(\mathrm{K},\mathrm{F})) \leq [\mathrm{K},\mathrm{F}]$

9) If a and b are constructible then which of the following is constructible.
   a) $a+b$ b) $a-b$
   c) $a.b$ d) all of these

10) Which of the following is transcendental element over Q?
   a) $\sqrt{2}$ b) π
   c) $\sqrt{3}$ d) None of these

B) State true or false. **06**

1) If $'a'$ is constructible then $\sqrt{a}$ is not constructible.
2) For every prime number p and every positive integer m there exists a field having p^m elements
3) Any two field having same number of element are not isomorphic.
4) The field C of complex number is not a finite extension of the field of real number R.
5) The irrational number 'e' is algebraic over R.
6) If F is field then it is not an integral domain.

Q.2 Answer the following **16**

a) Prove that: Let K be an extension of field F and let $a_1, a_2, \dots . a_n$ be n elements in K are algebraic over F then $F(a_1, a_2, \dots . a_n)$ is finite extension of F and consequently an algebraic extension of F

b) Show that $\sqrt{2}$ *and* $\sqrt{3}$ are algebraic over Q. Exhibit the polynomial over Q of degree 4 satisfied by $\sqrt{2} + \sqrt{3}$.

c) If $'a'$ is constructible then show that $\sqrt{a}$ is constructible.

d) If F be a finite field then prove that F has p^m elements where the prime number p is characteristic of F.

Q.3 Answer the following.

a) If 'a' $\in K$ be algebraic over F then prove that any two minimal monic polynomial for 'a' over F are equal, where K is extension of F **08**

b) If a, b in K are algebraic over F of degree m and n respectively and 'm' & 'n' are relatively prime then prove that $F(a, b)$ is of degree 'mn' over F, where K is extension of F **08**

Q.4 Answer the following.

a) If F be a field and $g(x)$ be a polynomial of degree n in $F[x]$ and $V = < g(x) >$ be the ideal generated by $g(x)$ in $F[x]$ then Prove that $\frac{F[x]}{V}$ is an n dimensional vector space over F. **10**

b) If $a, b \in K$ are algebraic over F of degree m and n respectively then prove that $a \pm b,\ ab, \frac{a}{b}\ (b \neq 0)$ are algebraic over F of degrees at most mn, where K is extension of F. **06**

Q.5 Answer the following.

a) If $f(x) \in F(x)$ be the degree $n \geq 1$ then prove that there is a finite extension E of F of degree at most $n!$ In which $f(x)$ has n roots. **08**

b) If F be the field of rational number and $f(x) = x^4 + x^2 + 1 \in F[x]$ show that $f(w)$ where $w = (-1 + i\sqrt{3})/2$ is a splitting field of $f(x)$. Also determine the degree field of $f(x)$ over F **08**

Q.6 Answer the following.

a) If $A(K)$ be the collection of all automorphism of a field K then prove that $A(K)$ is a group w.r.t. composition of two functions. **08**

b) If K is finite extension of a field F of characteristic O and H is subgroup of $G(K, F)$ and K_H be the fixed field of H then prove that. **08**

$$[K{:}K_H] = O(H)$$

Q.7 Answer the following.

a) If F be a field of rational numbers, & $K = F(2^{1/3})$ where $2^{1/3}$ is a real cube root of 2 then show that the only automorphism of K is identity automorphism. Is K is a normal extension of F? **08**

b) Find the Galois group of $x^2 - 2$ over field of rational numbers. **08**

SLR-SO-8

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Real Analysis–II (MSC15202)

Day & Date: Sunday, 23-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) Every Borel set is _____.
   a) Measurable set b) Non measurable set
   c) Empty set d) Dense set

2) If $m^*(A) = 0$ then $m^*\ (A \cup B)$= _____.
   a) $m^*(A)$ b) $m^*(A + B)$
   c) $m^*(B)$ d) $m^*(A.B)$

3) A property is said to be hold almost everywhere if there exists a set of points where it fails to hold is of measure _____
   a) > 0 b) < 0
   c) $= 0$ d) All of these

4) If E be measurable subset of set of all real numbers then ______.
   a) E^c may not be measurable
   b) E^c is closed
   c) E^c is measurable
   d) none of the above

5) If C be cantor set then _____.
   a) $m^*(C) = \infty$ b) $m^*(C) = 1$
   c) $m^*(C) = -1$ d) $m^*(C) = 0$

6) Let ϕ be an empty set and R be the set of real numbers then _____
   a) both ϕ and R are not measurable
   b) both ϕ and R are measurable
   c) ϕ is measurable but R are not measurable
   d) R is measurable but ϕ is not measurable

7) Countable Union of collection of measurable sets is _____.
   a) Need not be measurable b) Uncountable
   c) Measurable d) Finite

8) Let $A + X = [y + x: y \in A\}$ then $m^*(A + X) =$ ______.
   a) $m^*(A)$ b) 0
   c) $m^*(X)$ d) Infinite

9) Let Z be the set of integers and Q be the set of rationals $Z \subset Q.$ If m is Lebesgue measure, then ______.
   a) $m(Z) < m(Q)$ b) $m(Z) = m(Q)$
   c) $m(Z) > m(Q)$ d) $0 < m(Z) < m(Q) < \infty$

10) A function f is measurable if for each α the set _____ is measurable.
   a) $\{x/f(x) > \alpha$ b) $\{x/f(x) < \alpha$
   c) $\{x|f(x) \leq \alpha$ d) All of the above

B) Fill in the blanks. **06**
1) If A is singletone set then $m^*\ (A)$ = _____.
2) The outer measure of an interval is its _____
3) The smallest σ-algebra containing all open sets is called family of _____.
4) A continuous function defined on measurable set is _____.
5) A set E is said to be measurable if for any set $A \subseteq R, m^*(A) =$ _____.
6) With usual notations, F_σ set is defined as _____.

Q.2 Answer the following. **16**
a) If A is countable set then prove that $m^*(A) = 0$
b) If E is measurable rlien prove that $\tilde{E}$ is also measurable.
c) If $\{f_n\}$ is a sequence of measurable functions with same domain then prove that *sup* $\{f_1, f_2, \ldots \ldots f_n\}$ is measurable.
d) If $\phi = 3.\chi_{A_1} + 2.\chi_{A_2}$when $A_1 = [1,2]$ and $A_2 = [4,7]$ then find $\int \phi dx$

Q.3 Answer the following.
a) Prove that outer measure m^* is translation invariant. **08**
b) If A is any set and $\{E_i\}$ is the sequence of disjoint measurable sets and then prove that, **08**

$$m^*\left(A \cap \bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$$

Q.4 Answer the following.
a) If f and g are the two measurable functions on the same domain then prove that functions $f + c, cf, f + g, f - g$ and $f.g$ are also measurable where c is constant. **08**
b) If f and g are two non negative measurable function then prove that, **08**
a) $$\int_E cf = c\int_E f, c > 0$$
b) $$\int_E f + g = \int_E f + \int_E g$$
c) $f \leq g$ a.e. then $$\int_E f \leq \int_E g$$

Q.5 Answer the following.
a) State and prove Fatou's Lemma. **08**
b) If f is function of bounded variations on $[a, b]$ then prove that
1) $P_a^b - N_a^b = f(b) - f(a)$
2) $T_a^b = P_a^b + N_a^b$

Q.6 Answer the following.
a) If f is absolutely continuous on $[a, b]$ then prove that f is a function of bounded variations on $[a, b]$ and hence f is differentiable a.e. on $[a, b]$ **08**
b) Prove that collection M of all measurable sets is σ-algebra. **08**

Q.7 Answer the following.

a) If $\{E_n\}_{n=1}^{\infty}$ be an infinite decreasing sequence of measurable sets and $m(E_1) < \infty$ then prove that **08**

$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

08

b) If $\phi = \sum_{i=1}^{n} a_i \chi_{E_i}$ Where $E_i \cap E_j = \phi$ for $i \neq j$ and each E_i is measurable set with measure finite measure then prove that $\int \phi = \sum_{i=1}^{n} a_i\, m\,(E_i)$

SLR-SO-9

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
General Topology (MSC15203)

Day & Date: Tuesday, 25-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative (MCQ). **10**

1) Collection of all topologies of set X is closed under _____ operation.
   a) Union b) Intersection
   c) Addition d) Subtraction

2) Which of the following is true?
   a) Every metric d on a set X induces a topology for X.
   b) If $< X, \tau >$ is a given topological space then there exists a metric d on X which induces topology τ
   c) Both a and b are correct.
   d) Both a and b are not correct.

3) If τ is cofinite topology on set of all positive integers N and $\{7,8\} \subseteq N$ then which of the following is an open set in τ.
   a) $\{5,6,7\}$ b) $\{8,9,10. \dots\}$
   c) $\{1,2,3. \dots\}$ d) $\{1,2,3,4,5,6,9 \dots\}$

4) A topological space is said to be regular if F is a closed set in X and if p is point of X not in F, then there exist two disjoint _____ sets G and H such that $p \in G$ and $F \in H$.
   a) closed b) compact
   c) connected d) open

5) Which of the following is not true?
   a) The set $\{n: n \geq 100\}$ is not open in indiscrete topology on set of natural numbers.
   b) set of even natural number is open in discrete topology on set of natural numbers.
   c) Both a and b are correct
   d) Both a and b are not correct

6) Which of the following is true?
   a) If$< X, \tau >$ is discrete topological space and $x \in X$ then every subset of X containing x is neighbourhood of x.
   b) If $< X, \tau >$ is indiscrete topological space and $x \in X$ then every subset of X containing x is neighbourhood of x.
   c) Both a and b are correct
   d) Both a and b are not correct

7) If $X = \{a, b\}, P(X) = \{\emptyset, X, \{a\}, \{b\}\}$ then which of the following is a topology?
   a) $\tau = \{\emptyset, \{a\}, \{b\}\}$ b) $\tau = \{\emptyset, X, \{a\}, \{b\}\}$
   c) $\tau = \{\emptyset, \{a\}\}$ d) $\tau = \{\emptyset, X\}$

8) Which of the following is not true?
   a) A second countable space is always first countable.
   b) A first countable space is always second countable.
   c) Every subspace of second countable space is second countable.
   d) A second countable space is always separable.

9) Which of the following is not true?
   a) R is a connected
   b) R^2 is a connected.
   c) If X is an infinite then finite topological space X is connected.
   d) None of these

10) Which of the following is not true?
   a) A closed subset of compact space is compact.
   b) An infinite discrete space is compact.
   c) Every cofinite topological space is infinite.
   d) closed and bounded subset of R is compact.

B) Fill in the blanks. **06**

1) Every T_3 -space is T_1 and _____ Space.
2) If A and B are connected subsets of topological space X then _____ is also connected.
3) Let $f: X \rightarrow Y$ be bijective continuous function. If X is compact and Y is T_2space, then f is _____.
4) In _____ space, any converging sequence converges to unique point.
5) If X is topological space, then _____ is called open covering.
6) In topological space $A \subseteq X$ _____ if and if A is open set.

Q.2 Answer the following. **16**

a) Show that derived set in any T-space is closed set.
b) Give example of separable space which is not second countable.
c) Prove or disprove Forts space is compact.
d) let $< X, \tau >$ be T-space $F \subseteq X$ such that F^c (complement of F) is open then show that F must be closed set in $< X, \tau >$.

Q.3 Answer the following.

a) Let $X = A|B$ and let C be a connected subset of X then show either $C \subseteq A$ or $C \subseteq B$. **08**
b) Show that $E \cup d(E)$ is a smallest set containing E. **08**

Q.4 Answer the following.

a) Define Compact space and regular space. Prove that every compact Housdroff space is a regular space. **10**

b) Prove that every $T_{3\frac{1}{2}}$ space is a T_3 space. **06**

Q.5 Answer the following.

a) Prove that being T_2 space is Hereditary property. **10**
b) Prove that every completely regular space is regular space. **08**

Q.6 Answer the following.

a) If A and B are separated sets in $< X, \tau >$, then prove that A and B are both open and closed in $A \cup B$ and conversly. **08**

b) Prove that T – space $< X, \tau >$ is Housdroff space if and only if two disjoint compact subsets of X be disjoint open sets. **08**

Q.7 Answer the following.

a) If $< X, \tau >$ and $< X^*, \tau^* >$ be two T-spaces then $f: X \to X^*$ is continuous mapping on X if and only if $f^{-1}[i^*(E^*)] \subseteq i[f^{-1}(E^*)]$ for any $E \subseteq X$. **08**
where $i^*(E^*)$ is interior of E^* in $< X^*, \tau^* >$
$i[f^{-1}(E^*)]$ is interior of $f^{-1}(E^*)$ in $< X, \tau >$.

b) If $< X, \tau >$ be any T-space. **08**
$X^* = \{a, b, c\}$ $\qquad$ $\tau^* = \{\emptyset, X^*, \{b\}, \{b, c\}\}$
be defined by $f(x) = a \ \forall x \in X$ discuss the continuity of f on X.

SLR-SO-10

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Complex Analysis (MSC15206)

Day & Date: Thursday, 27-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Critical points of $W = \frac{\alpha z+\beta}{\gamma z+\delta}, \alpha\delta - \beta\gamma \neq 0$ are _____.
   a) $-\frac{\delta}{\gamma}$ b) $-\frac{\delta}{\gamma}$ and 0
   c) $-\frac{\delta}{\gamma}$ and ∞ d) ∞ and 0

2) The bilinear transformation which maps the points $z = 1, z = 0, z = -1$ of z-plane into $w = i, w = 0, w = -i$ of w-plane respectively is _____.
   a) $w = z$ b) $w = iz$
   c) $w = i\,(z + 1)$ d) $w = z + 2$

3) If f is an entire function then _____.
   a) f has power series expansion
   b) f has not a power series expansion
   c) f is constant
   d) f is polynomial

4) $\int_c \frac{f(z)}{z-a}\, dz$ is equal to _____.
   a) $2\pi i\, f(a)$ b) $2\pi i\, Im\, f(a)$
   c) $2\pi i\, res\, f(a)$ d) $-2\pi i\, res\, f(a)$

5) The radius of convergence of the power series $\sum_{n=0}^{\infty}(n+2i)^n z^n$ is _____.
   a) 0 b) 0
   c) ∞ d) $n^2 + 4$

6) The function $f(z) = z^m$ at $z = \infty$ has _____.
   a) non-isolated essential singularity
   b) pole of order m
   c) pole of order $m + 1$
   d) removable singularity

7) Laurent series expansion of the function $\frac{1}{z^3-3z+2}$ for $|z| > 2$ is _____.
   a) $\sum_{n=0}^{\infty} \frac{2^n-1}{z^{n+1}}$ b) $\sum_{n=0}^{\infty} \frac{2^n}{z^{n+1}}$
   c) $\sum_{n=0}^{\infty} \frac{2^n+1}{z^{n+1}}$ d) $\sum_{n=0}^{\infty} \frac{2^n}{z^n}$

8) The function $f: C \to C$ defined by $f(z) = e^z + e^{-z}$ has _____.
   a) finitely many zeros b) no zeros
   c) only real zeros d) Infinitely many zeros

9) Which of the following functions does represent the series $\sum_{n=0}^{\infty} \frac{z^n}{n!}$ for $|z| < \infty$

a) $\sin z$ b) $\cos z$
c) e^z d) $\frac{e^z}{n!}$

10) A polygon with three sides is called _____.

a) Circle b) Simple curve
c) Triangular path d) Open set

B) Fill in the blanks. **06**

1) If $T_1(z) = \frac{z+2}{z+3}$ and $T_2(Z) = \frac{z}{z+1}$, then $T_2T_1(z)$ is _____.

2) A function which has poles as its only singularities in the finite part of the plane is said to be _____.

3) If $f(z)$ is analytic in a simply connected domain D, then for every cloth path C in D, $\oint_c f(z)dz =$ _____.

4) The magnification factor of the mapping $w = \sqrt{2}e^{\frac{\pi i}{4}}z + (1 - 2i)$ is _____.

5) The fixed points of the mapping $w = \frac{5z+4}{z+5}$ are _____.

6) A polynomial with no zeros in C is a _____ polynomial.

Q.2 Answer the following **16**

a) Calculate residue of $\frac{z^2-2z}{(z^2+4)(z+1)^2}$

b) Show that $\int_0^{\pi} \frac{1}{a+\cos\theta} d\theta = \frac{\pi}{\sqrt{a^2-1}} (a > 1)$

c) A Mobius map is uniquely determined by its action on any three distinct points in C_∞.

d) If f is analytic in $B(a, R)$ and suppose that $|f(z)| \leq M$ for all z in $B(a, R)$ than prove that $|f^n(a)| \leq \frac{n!M}{R^n}$

Q.3 Answer the following.

a) If G be a connected open set and $f: G \to C$ be analytic function, Then the following are equivalent statements: **10**

i) $f \equiv 0$ on G ; **06**

ii) $\{z \in G: f(z) = 0\}$ has a limit point in G ;

iii) There is a point a in G such that $f^n(a) = 0$ for each $n \geq 0$.

b) State and prove Argument Principle.

Q.4 Answer the following.

a) If z_1, z_2, z_3, z_4 be the four distinct points in C_∞, then prove that the cross ratio (z_1, z_2, z_3, z_4) is real iff all four points lie on a circle or straight line. **08**

b) If G be an open subset of the complex plane C and $f: G \to C$ be an analytic function. if γ is a closed rectifiable curve in G such that, $\eta(\gamma; w) = 0$; $\forall\, w \in C - G$ then for a $a \in G - \{\gamma\}$ prove that, **08**

$$f(a).\eta(\gamma; a) = \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w - a} dw$$

Q.5 Answer the following.

a) State and prove Morera's Theorem. **10**

b) Evaluate $\int_0^\infty \frac{1}{1+x^2} dx$. **06**

Q.6 Answer the following.

a) If G be a region and $f: G \to C$ be an analytic function such that there is a point 'a' in G with $|f(z)| \leq |f(a)| \; \forall z \in G$ then show that f is a constant. **06**

b) If f has an isolated singularity at $z = a$ then prove that the point $z = a$ is removable singularity iff $\lim_{z \to a} (z-a) f(z) = 0$. **10**

Q.7 Answer the following.

a) Explain Laurent series development. **10**

b) Prove that all the roots of equation $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$ **06**

SLR-SO-12

Seat No.	

Set P

M.Sc. (Semester-III) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Functional Analysis (MSC15301)

Day & Date: Monday, 10-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) In a quotient space $\mathrm{N/M}$, the addition is defined as $(\mathrm{x}+\mathrm{M})+(\mathrm{y}+\mathrm{M})=$ ______.
   a) $x+y+M$ b) $x+y+2M$
   c) M d) none of these

2) The set of all continuous linear transformations on a normed linear space N into normed linear space N’ is denoted by _____.
   a) B(N) b) B(N’)
   c) B(N, R) d) B(N, N’)

3) If $S'\ (x;r)$ is an open sphere centered at x and radius r then $S(x;r)=$ _____.
   a) $x+S(0;r)$ b) $x.S(0;r)$
   c) $x+S(0;1)$ d) $r+S(x;1)$

4) An idempotent linear transformation on a linear space N is called _____.
   a) operator b) norm
   c) projection d) metric

5) In Hilbert space X, with usual notations, $\langle x,y+z\rangle$ = _____.
   a) $\langle x,y\rangle+\langle y,z\rangle$ b) $\langle x,y\rangle+\langle x,z\rangle$
   c) $\langle x,z\rangle+\langle y,z\rangle$ d) All of the above

6) If N and N' are normed linear spaces and $T{:}\,N\rightarrow N'$ then graph of T is gives as $T_G=$ ____.
   a) $\{(x,T(x))/x\in N\}$ b) $\{(x,T(x))/x\in N'\}$
   c) $\{(x,T(x))/x\in\}$ d) $\emptyset$

7) A Banach space means _____.
   a) complete normed linear space
   b) complete inner product space
   c) normed linear space
   d) inner product space

8) A projection E on a linear space L determines two linear subspaces M and N such that $L=$ _____.
   a) $M+N$ b) $M\cup N$
   c) $M\oplus N$ d) $M\cap N$

9) In a normed linear space, the triangular inequality property is given as, _____.

a) $\|x+y\| \leq \|x\|+\|y\|$ b) $\|x+y\| \geq \|x\|+\|\mathcal{Y}\|$
c) $\|x+y\| = \|x\|+\|y\|$ d) $\|x-y\| \leq \|x\|-\|y\|$

10) In a Hilbert space, for any $x, y \in H$ the vectors x, y are said to be orthogonal if _____.

a) $\langle x, y\rangle \neq 0$ b) $\langle x, y\rangle = 0$
c) $\langle x, y\rangle \leq 0$ d) $\langle x, y\rangle \geq 0$

B) Fill in the blanks. **06**

1) A continuous linear transformation $\mathrm{T}: \mathrm{N} \rightarrow \mathrm{N}'$ is said to be open mapping if for every open set G in $\mathrm{N}, \mathrm{T}(\mathrm{G})$ is ____ in N'.

2) If $\mathrm{T}: \mathrm{X} \rightarrow \mathrm{Y}$ is a linear transformation and T is bounded then T maps bounded sets in X into _____ sets in Y.

3) In the set of all bounded linear transformations $\mathrm{B}(\mathrm{X}, \mathrm{Y})$ the scalar multiplication is defined as $(\alpha.T)(x)=$ _____.

4) A normed linear space, X is said to be complete if every Cauchy sequence is _____ in X.

5) In a normed linear space, a non-zero vector x can be converted to unit vector by _____.

6) The zero element of a quotient space N/M is _____.

Q.2 Answer the following. (Each of 04 marks) **16**

a) If x and y are two vectors in a Hilbert space then prove that $\|x+y\|^2+\|x-y\|^2 = 2(\|x\|^2+\|y\|^2)$

b) Prove that: Every complete subspace of normed linear space is closed.

c) If V be a normed linear space and defined $d(x, y) = \|x-y\|$, for all $x, y \in V$ Then prove that $< V, d >$ is metric space.

d) Define: Inner Product and Norm.

Q.3 Answer the following.

a) State and prove Hahn Banach theorem. **10**

b) Show that $|\ \| x \| - \| y \|\ | \leq \| x-y \|, \forall x, y \in V$ **06**

Q.4 Answer the following.

a) Prove that $\mathrm{B}(\mathrm{X}, \mathrm{Y})$ is normed linear space, where, $\| T \| = sup\{\| T(x) \| : x \in X, \| x \| \leq 1\}$ **08**

b) If X is a normed linear space over the field F, M is a closed subspace of X and define $\|.\|_1 : \frac{X}{M} \rightarrow \mathcal{R}$ by $\|x+m\|_1 = inf\{\|x+m\| / m \in M\}$ then prove that $\|.\|_1$ is a norm on $\frac{X}{M}$. **08**

Q.5 Answer the following.

a) If $\mathrm{T}: \mathrm{X} \rightarrow \mathrm{Y}$ be any linear transformation then prove that T is Continuous on X if and only if T is bounded X. **10**

b) Show that the linear space R^n and C^n of all n-tuples $x = (x_1, x_2, \ldots x_n)$ of real and complex numbers are Banach spaces under the norm $\|x\| = (\sum_{i=1}^{n} |x_i^2|)$ **06**

Q.6 Answer the following.

a) If H is Hilbert space then prove that H^* is also Hilbert space with the inner product defined by, $< f_x, f_y > = < y, x >$ **08**

b) Prove that: All norms on finite dimensional space are equivalent. **08**

Q.7 Answer the following.

a) If H Hilbert space then show that the adjoint operation $T \to T^*$ on $B(H)$ has the following properties. **08**

a) $(T_1 + T_2)^* = (T_1)^* + (T_2)^*$

b) $(T_1 . T_2)^* = (T_1)^* . (T_2)^*$

c) $(\propto . T)^* = \propto . (T)^*$

d) $T^{**} = T$

b) If M be a linear subspace of a Hilbert space H then prove that M is closed if and only if $M = M^{\perp\perp}$. **08**

SLR-SO-13

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
MATHEMATCIS
Advanced Discrete Mathematics (MSC15302)

Day & Date: Tuesday, 11-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The complete graph K_n is _____ regular.
   a) n b) n-1
   c) 2n d) $\frac{n(n-1)}{2}$

2) There are 5 different algebra books, 6 different complex analysis books and 8 different classical mechanics books. Then the number ways to pick an unordered pair of two books not both of the same course are _____.
   a) 118 b) 88
   c) 240 d) 19

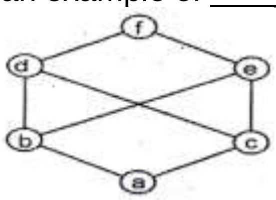
3) If u and v be vertices of a graph G then which of the following statement is true?
   I) Every u-v walk contains a u-v path.
   II) Every trail is a path.
   a) only I is true b) only II is true
   c) both I and II are true d) both I and II are false

4) The explicit formula for the sequence defined by the recurrence relation $a_n = a_{n-1} + 4;\ \forall\, n \geq 2$ with $a_1 = 2$ is _____.
   a) $a_n = 4n$ b) $a_n = 4n + 1$
   c) $a_n = 4n - 2$ d) $a_n = 4n + 2$

5) In any lattice L, Which of the following is true?
   a) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$
   b) $a \wedge (b \vee c) \leq (a \wedge b) \vee (a \wedge c)$
   c) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \vee c)$
   d) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

6) The generating function for the sequence $\left\{1, 1, \frac{1}{2!}, \frac{1}{3!}, \frac{1}{4!}, \ldots\ldots\right\}$ is _____.
   a) e^x b) e^{-x}
   c) $\log(1 + x)$ d) $(1 - x)^{-1}$

7) The graph given below is an example of _____.



   a) non-lattice poset b) complete lattice
   c) distributive lattice d) bounded lattice

8) A vertex v of a tree T is a cut vertex if and only if _____.
   a) $d(v) \leq 1$ b) $d(v) < 1$
   c) $d(v) \geq 1$ d) $d(v) > 1$

9) $^{12}C_r$ is greatest when r is equal to _____.
   a) 7 b) 6
   c) 12 d) 0

10) The number of different non-isomorphic spanning trees on the complete graph with 4 vertices are _____.
   a) 4 b) 16
   c) 2 d) 6

B) Fill in the blanks. **06**

1) If A and B are finite sets then $|A - B| =$ _____.
2) The number of three digits can be formed with the digits 2, 3, 4, 5, 6, 7 no digit being repeated are _____.
3) If L and M be any two lattices then the mapping $f: L \rightarrow M$ is called a meet homomorphism if _____.
4) If the edges of the walk W are distinct then W is called _____.
5) The coefficient of x^{10} in $((x^3 + x^4 + x^5 + - - -)^3$ is _____.
6) If G be a connected graph with vertex set V then for each $v \in V.$ the eccentricity of v i.e. $e(v)$ is given by _____.

Q.2 Answer the following. **16**
a) Draw all the spanning trees of K_4 graph.
b) In how many ways 4 boys and 4 girls be seated in a row so that boys and girls are alternate?
c) Define isomorphism of graph with two examples.
d) Show that every chain is a distributive lattice.

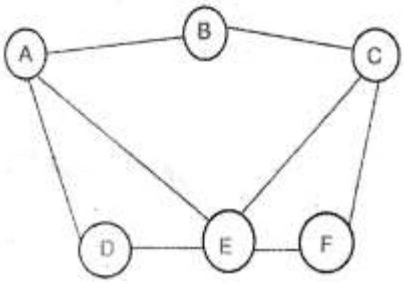
Q.3 Answer the following. **10**
a) If G be a graph with n vertices $v_1, v_2, v_3, \dots v_n$ & A denote the adjacency matrix of G with respect to this listing of vertices. Let $B = [b_{i,j}]$ be the matrix $B = A + A^2 + A^3 + \cdots + A^{n-1}$ .Then show that G is connected graph iff for every pair of distinct indices i, j we have $b_{i,j} \neq 0$.
b) Show that a graph G is connected if and only if it has a spanning tree, **06**

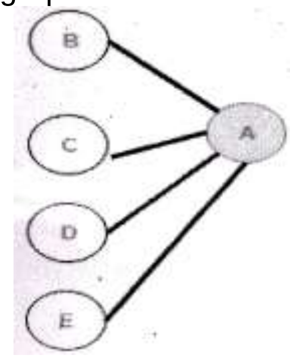
Q.4 Answer the following.
a) Find the primes less than 100 by using the principle of inclusion-exclusion? **08**
b) If $(\mathrm{L}, \lesssim)$ be a lattice then for any $\mathrm{a, b, c, d}$ in L prove that **08**
   i) $a \lesssim b \Longrightarrow a \vee c \lesssim b \vee c$
   ii) $a \lesssim b \Longrightarrow a \wedge c \lesssim b \wedge c$
   iii) $a \lesssim b \text{ and } c \lesssim d \Longrightarrow a \vee c \lesssim b \vee d$
   iv) $a \lesssim b \text{ and } c \lesssim d \Longrightarrow a \wedge c \lesssim b \wedge d$

Q.5 Answer the following.
a) State and prove bridge theorem. **10**
b) If $(L, \vee, \wedge)$ is a complemented distributive lattice, then prove that **06**
$(a \vee b)' = (a)' \wedge (b)'$ and $(a \wedge b)' = (a)' \vee (b)' \forall \quad a, b \in L$

Q.6 Answer the following.

a) Find the distance and diameter of the following graphs. **08**





b) Write a short note on the matrix representation of graph with two examples. **08**

Q.7 Answer the following.

a) Solve the recurrence relation by using the generating function. **10**

1) $a_n - 9a_{n-1} + 20a_{n-2} = 0\ ; a_0 = -3, a_1 = -10$

2) $a_{n+2} - 2a_{n+1} + a_n = 2^n\ ; a_0 = 2, a_1 = -1$

b) Given any two vertices u and v of a graph G, then prove that every u-v walk contains a u-v path. **06**

SLR-SO-14

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Linear Algebra (MSC15303)

Day & Date: Wednesday, 12-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) If $f: \mathbb{R}^3 \to \mathbb{R}$ is a linear functional defined by $f(x,y,z) = 2x + y + z, \ \forall (x,y,z) \in \mathbb{R}^3$ Then $nullity\ (f) =$
   a) 0 b) 1
   c) 2 d) 3

2) If V is a finite dimensional vector space and $\mathrm{V}^{\star\star}$ is its double dual space then _____.
   a) $\dim \mathrm{V} = \dim \mathrm{V}^{\star\star}$ b) $\dim \mathrm{V} < \dim \mathrm{V}^{\star\star}$
   c) $\dim \mathrm{V} > \dim \mathrm{V}^{\star\star}$ d) None of these

3) Which of the following set is orthogonal?
   a) $\{(1,2,0), (-2,1,1)\}$ b) $\{(1,2,0)\}$
   c) $\{(1,2,0), (-2,1,0)\}$ d) $\{(1,2,1), (-2,1,1)\}$

4) If A is any matrix of order n over the field $\mathbb{F}$ and let $f(x),\ m(x)$ denote its characteristic and minimal polynomial respectively, then which of the following is annihilating polynomial for A.
   a) $f(x)$ b) $m(x)$
   c) $g(x)$ such that $g(x) \mid m(x)$ d) all of the above

5) Which of the following types of matrix over $\mathbb{R}$ is always diagonalizable?
   a) Symmetric matrix
   b) Matrix with both eigenvalues equal
   c) Nilpotent matrix
   d) Sum of two nilpotent matrix of order 2

6) Characteristic values of $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$ are _____.
   a) $0, 0$ b) $1, 2$
   c) $1, -1$ d) $i, -i$

7) If V is a vector space and $\mathrm{W}_1, \mathrm{W}_2$ are two subspaces of V such that $\mathrm{V} = \mathrm{W}_1 \oplus \mathrm{W}_2$ Then _____.
   a) $\mathrm{W}_1 + \mathrm{W}_2 = \{0\}$ b) $\mathrm{W}_1 \cap \mathrm{W}_2 = \{0\}$
   c) $\mathrm{W}_1 = \mathrm{W}_2$ d) None of these

8) The characteristic values of a nilpotent matrix A of order n over the field $\mathbb{R}$ is ____.
   a) 1 and 0 b) 0
   c) $1, -1$ d) $0, -1$

9) Which of the following subspaces of U under linear transformation $T: V \to V$ satisfy $T(W) \subset W$?
   a) $W = N(T)$ b) $W = \{0\}$
   c) $W = R(T)$ d) All of the above

10) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is a projection on Y – axis. Then R(T) =
   a) $\{0\}$ b) $\mathbb{R}^2$
   c) X – axis d) Y – axis

B) Fill in the blanks. **06**

1) If $T : V \to V$ is a linear transformation and W is a subspace of V then W is called an invariant subspace of V if _____.
2) Let V be an inner product space over the field of complex numbers, then $< \alpha|c\beta + \gamma >=$ ______.
3) A finite dimensional complex inner product space is called _____.
4) If V is an inner product space, then $|< \alpha/\beta >| \leq$ _____.
5) Let V be an inner product space and S any set of vectors in $V.$ The orthogonal complement of S the set $S^{\perp}$ of _____.
6) If E is a projection defined on V, then E^2 _____.

Q.2 Answer the following. **16**

a) Define annihilator of a set in vector space. Prove that if W_1, W_2 are subspaces of a finite dimensional vector space, then prove that $W_1 = W_2 \; iff \; W_1^o = W_2^o$.

b) Let V be a finite-dimensional vector space over the field $\mathbb{F}$. For each vector $\alpha \in V$, define $L_\alpha(f) = f(\alpha), \forall f \in V^{**}.$ Then prove that mapping $\alpha \to L_\alpha$ is an isomorphism of V onto V^{**}

c) Find characteristic values for the matrix $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

d) Let V be a vector space over the field $\mathbb{F}$ and $T: V \to V$ be a linear transformation and f be any polynomial with $T(\alpha) = c\alpha$ for vector $\alpha \in V, c \in \mathbb{F}$. Then prove that $f(T)(\alpha) = f(c)\alpha$.

Q.3 Answer the following.

a) Find the rational canonical form for $A = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ -3 & -3 & -5 \end{bmatrix}$ **08**

b) Let W be a subspace of an inner product space V and let. β be a vector in $V.$ Then prove that **08**

1) the vector $\alpha \in W$ is the best approximation to β by vectors in W iff $\beta - \alpha$ is orthogonal to every vector in $W.$
2) if a best approximation to β by vectors in W exists, it is unique.
3) if W is a finite-dimensional and $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is any orthonormal basis for W, then vector $\alpha = \sum_{k=1}^{n} \frac{< \beta|\alpha_k >}{\|\alpha_k\|^2} \alpha_k$ is the best approximation to β by vectors in W.

Q.4 Answer the following.

a) Let V be a finite dimensional vector space over the field $\mathbb{F}$, and $\mathcal{B} = \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ be a basis of V. Then prove that there is a unique dual basis $\mathcal{B}^* = \{f_1, f_2, \ldots\ldots\ldots, f_n\}$ for V^* such that $f_i(\alpha_j) = \delta_{ij}$. Also prove that for each linear functional $f \in V^*, f$ can be given by **08**

$$f = \sum_{i=1}^{n} f(\alpha_i) f_i$$

and for any $\alpha \in V, \alpha$ can be written as

$$\alpha = \sum_{i=1}^{n} f_i(\alpha)\alpha_i.$$

b) Let T be a linear operator on an n-dimensional vector $V.$ Define minimal polynomial and characteristic polynomial for $T.$ Prove that the characteristic and minimal polynomials for T have the same roots, except for multiplicities. **08**

Q.5 Answer the following.

a) Let V be a finite-dimensional vector space over the field $\mathbb{F}$ and let T be a linear operator on V. Then T is diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)(x - c_2), \ldots, (x - c_k)$ where $c_1, c_2, \ldots, c_k$ are characteristic values of $T.$ **08**

b) Let V be a finite dimesnional inner product space. If T, U are linear operators on V and c is a scalar, then prove that **08**

1) $(T + U)^\star = T^\star + U^\star$
2) $(cT)^\star = \bar{c}T^\star$
3) $(TU)^\star = U^\star + T^\star$
4) $(T^\star)^\star = T$

Q.6 Answer the following.

a) Define a Hermitian form and self-adjoint linear transformation. Prove that if V is a complex vector space and f is a form on V such that $f(\alpha, \alpha)$ is real for every α, then prove that f is Hermitian. **08**

b) Consider the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{bmatrix}$. Prove that A is diagnoalizable over $\mathbb{R}$ and find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix. **08**

Q.7 Answer the following.

a) Let $V = P_2\ (\mathbb{R})$ be a vector space of set of all polynomials of degree at most 2 with an inner point $< f, g \geq \int_0^1 f(x)g(x)dx$ Apply Gram-Schmidt process to the basis $\{1, x, x^2\}$ to obtain an orthonormal basis for V **08**

b) Find the Jordan canonical form for the matrix $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$ **08**

SLR-SO-15

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Differential Geometry (MSC15306)

Day & Date: Thursday, 13-07-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Select the correct alternative. **10**

1) If $\bar{v}, \bar{w}$ are linearly independent then $\|\bar{v} \times \bar{w}\|$ represents area of _____.
   a) of a parallelogram with sides $\bar{v}, \bar{w}$
   b) of a triangle with sides $\bar{v}, \bar{w}$
   c) of a square with sides $\bar{v}, \bar{w}$
   d) of a rectangle with diagonals $\bar{v}, \bar{w}$

2) Formula for $\bar{v}_p[f] =$ _____.
   a) $f(p+tv)|_{t=0}$
   b) $f'(p+tv)_{t=0}$
   c) $f'(p+tv)|_{p=0}$
   d) $f(p+tv)|_{p=0}$

3) A curve $\alpha: I \to \mathbb{R}^3$ is said to be a regular curve if _____.
   a) $\alpha(t) = 0, \forall t \in I$
   b) $\alpha'(t) = 0, \forall t \in I$
   c) $\alpha'(t) \neq 0, \forall t \in I$
   d) $\alpha"(t) \neq 0, \forall t \in I$

4) A curve $\alpha: I \to \mathbb{R}^3$ is called an unit speed curve if _____.
   a) $\| \alpha'(t) \| = 1$
   b) $\| \alpha''(t) \| = 1$
   c) $\| \alpha'(t) \| = 0$
   d) $\| \alpha"(t) \| = 0$

5) If a vector field $\bar{Y}$ has constant length, the _____.
   a) $\bar{Y}.\bar{Y} = 0$
   b) $\bar{Y}.\bar{Y}' = 0$
   c) $\bar{Y}.\bar{Y}' = 1$
   d) None of these

6) Which of the following curves has zero acceleration?
   a) $\alpha(t) = (t, t+2, 3), \forall t \in I$
   b) $\alpha(t) = (t, t+2, 3t^2), \forall t \in I$
   c) $\alpha(t) = (t, t+2, \cos t), \forall t \in I$
   d) $\alpha(t) = (t^3, t+2, 3t), \forall t \in I$

7) If $\alpha: I \to \mathbb{R}^3$ is a curve, then its curvature B' _____.
   a) $-\tau N$
   b) τN
   c) $\tau N'$
   d) $-\tau N'$

8) In the Frenet apparatus for a curve α, plane spanned by B and N is known as _____.
   a) normal plane
   b) osculating plane
   c) principal plane
   d) rectifying plane

9) Which of the following is not a surface?
   a) Cone
   b) Closed disc
   c) Folded planes
   d) All of the above

10) Shape operator for a plane surface is _____.
   a) a unit vector
   b) vector of magnitude 2
   c) zero vector
   d) none of these

B) Fill in the blanks. **06**

1) Let $X: D \to E^3$ be a coordinate patch. Then for each (u_0, v_0) in D, the velocity vector at u_0 of u- parameter curve $v = v_0$ is denoted by _____.
2) For a patch $\mathrm{X}: \mathrm{D} \to E^3$ if $\mathrm{E} = \mathrm{X_u} \cdot X_u, F = X_u \cdot X_v, G = X_v \cdot X_v$ and $EG - F^2 \neq 0$ then X is _____.
3) For a non-unit speed curve in E^3, $B =$ _____.
4) Cylinders are surfaces obtained by translating a line along _____.
5) Let M be a surface and P be a point on M. Then the maximum and minimum values of the normal curvature $k(\bar{u})$ at P are called _____ curvature of M at point P.
6) A point P of M is called _____ if the normal curvature $k(\bar{u})$ is constant on all unit tangent vectors $\bar{u}$ at P.

Q.2 Answer the following **16**

a) Show that rotation is an orthogonal transformation.

b) Let f and g be real valued functions on E^3. If $\bar{v}_p$ and $\overline{w}_p$ are tangent vectors on E^3 and a, b are real numbers, show that

i) $\left(a\overline{v_p} + b\overline{w}_p\right)[f] = a\bar{v}_p[f] + b\overline{w}_p[f]$

ii) $\overline{v_p}\,[af + bg] = a\overline{v_p}[f] + b\overline{v_p}[g]$

c) For a patch $X: D \to E^3$, if $E = X_u \cdot X_u, F = X_u \cdot X_v, G = X_u \cdot X_v$, then prove that X is regular iff $EG - F^2 \neq 0$.

d) Show that the curves $\alpha(t) = (t, t^2 + 1, t), \beta(t) = (\sin t, \cos t, t)$ have the same initial speed.

Q.3 Answer the following.

a) Let v and w be tangent vectors at the same point p. Then prove that $v \times w$ is orthogonal to v and w and has length $\|v \times w\|^2 = (v \cdot v)(w \cdot w) - (v \cdot w)^2$. **08**

b) Consider the surface $M : z = f(x, y)$, where $f(0,0) = f_x(0,0) = f_y(0,0) = 0$. Then show that **08**

i) The vectors $u_1 = U_1(0), u_2 = U_2(0)$ are tangent vectors to M at the origin 0 and $U = \frac{-f_x U_1 - f_y U_2 + U_3}{\sqrt{f_x^2 + f_y^2 + 1}}$ is unit normal vector field on M.

ii) Show that
$$S(u_1) = f_{xx}(0,0)u_1 + f_{xy}(0,0)u_2\,, S(u_2) = f_{yx}(0,0)u_1 + f_{yy}(0,0)u_2.$$

Q.4 Answer the following.

a) Let $M: Z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ and let $X: E^2 \to E^3$ defined by $X(u, v) = (a(u + v), b(u - v), 4uv)$. Then show that X is a proper patch covering all of M. **08**

b) Show that $M: Z = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is a surface and $X(u, v) = (a\cos u\,, \cos v\,, b\cos u\ \sin u\,, c\sin u)$ defined on D is a parametrization where D: $-\frac{\pi}{2} < u, v < \frac{\pi}{2}$. **08**

Q.5 Answer the following.

a) Define 1-form. If φ is a 1 – form E^3, then prove that $\varphi \sum_{i=1}^{3} f_i \, dx_i$ where $f_i = \varphi(U_i)$. **08**

b) Determine the curvature for the ellipse $a(t) = (a\cos t, b\sin t, 0)$ and deduce from it that the curvature of a circle of a radius r. Also prove that the curve is a plane curve. **08**

Q.6 Answer the following.

a) Define Mean and Gaussian curvature. Prove that if k_1 and k_2 are principal curvatures at a point $P \in M$ then show that the Guassian curvature and mean curvature are respectively given by **08**

$K(P) = k_1, k_2$ and $H(P) = \frac{k_1+k_2}{2}$ $\forall P \in M$

b) i) If $\bar{V} = x^2\bar{U}_1 + yz\bar{U}_3,\quad \bar{v} = (-1,0,2),\ p = (2,1,0)$ find $\nabla_{\bar{v}}\bar{V}(p)$. **08**

ii) If $\bar{W} = x\bar{U}_1 + x^2\bar{U}_2, -z^2\,\bar{U}_3,\ \ \bar{v} = (1,-1,2),\ p = (1,3,-1)$ find $\nabla_{\bar{v}}\bar{W}(p)$

Q.7 Answer the following.

a) Prove that every isometry of E^3 can be uniquely described as orthogonal transformation followed by translation. **08**

b) Prove that **08**

i) If $\bar{S}$, $\bar{T}$ are translations then $\bar{S}\bar{T} = \bar{S}\bar{T}$ is also a translation.

ii) Prove that if $\bar{T}$ is a translation by $\bar{a}$, then $\bar{T}^{-1}$ is a translation by $-\bar{a}$.

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Measure & Integration (MSC15401)

Day & Date: Monday, 10-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose correct alternative. **10**

1) Two measures γ_1 and γ_2 on $(\mathrm{X}, \mathcal{B})$ are said to be mutually singular if there are disjoint measurable sets A and B with $X = A \cup B$ such that
   a) $\gamma_1(A) = \gamma_2(B) \neq 0$ b) $\gamma_1(A) = \gamma_1(B) = 0$
   c) $\gamma_2(A) = \gamma_2(B) = 0$ d) $\gamma_1(A) = \gamma_2(B) = 0$

2) If $(X, \mathcal{B}, \mu)$ be a complete measure space. $E_1 \in \mathcal{B}$ and μ $(E_1 \Delta E_2)$ =0 then _____.
   a) $E_2 \in \mathcal{B}$ b) $\mu(E_1 - E_2) = 0$
   c) $\mu(E_2 - E_1) = 0$ d) All of these

3) For a measurable set A, define $\gamma(A) = \int_A e^x \, dm$, where m is a Lebesgue measure and $\gamma << m$ then $\left[\frac{dy}{dm}\right] =$
   a) e^{-x} b) e^x
   c) $2e^x$ d) 0

4) Any set in $R\sigma$ is a _____ of measurable rectangles.
   a) disjoint union b) intersection
   c) sum d) difference

5) If $(X, \mathcal{B}, \mu)$ be a measure space. If every locally measurable set in X is a measurable set then _____.
   a) μ is complete b) μ is finite
   c) μ is saturated d) μ is σ −finite

6) The linear combination of two measures μ *and* γ on a same measurable space is _____.
   a) always positive
   b) always non-negative
   c) never positive
   b) need not be always non-negative

7) If E be set with $\mu_*(E) < \infty$. Then there exists a set $H \in \mathcal{B}_{\delta\sigma}$ such that
   a) $H \subseteq E$ and $\mu^-(H) = \mu^*(E)$ b) $E \subseteq H$ and $\mu^-(H) < \mu^*(E)$
   c) $H \subseteq E$ and $\mu^-(H) = \mu_*(E)$ d) $H \subseteq E$ and $\mu^-(H) \geq \mu_*(E)$

8) If $\{f_n\}$ be sequence of non-negative measurable functions then $\int \sum_{n=1}^{\infty} f_n =$ _____.
   a) $\bigcup_{n=1}^{\infty} \int f_n$ b) $\sum_{n=1}^{\infty} \int f_n$
   c) $\int f_n$ d) $-\sum_{n=1}^{\infty} \int f_n$

9) If N be the set of natural numbers and $N \times Z$ be the product set of naturals with integers. If m is Lebesgue measure, then
   a) $m(N) = 0$ b) $m(N) = m(N \times Z)$
   c) $m(N \times Z) = 0$ d) All of these

10) Which of the following is true?
   a) If f is measurable then so is $| f |$
   b) If $| f |$ is measurable so is f
   c) both a and b are true
   d) both a and b are false

B) Fill in the blanks. **06**

1) If $A \subseteq R$ and A is countable. If $B \subseteq R$ is any set then $\mu^*(A \cup B) =$ ____
2) A subset E of X is said to be μ^*-measurable, if for any set A we have $\mu * (A) =$ _____.
3) The smallest σ-algebra containing all closed sets and also open intervals is _____.
4) If $(X, \mathcal{B}, \mu)$ be measure space and f is any function on X then $f^-(x) =$ _____.
5) If $\gamma << \mu << \omega << \lambda$ than $\left[\frac{d\gamma}{d\lambda}\right] =$ _____.
6) If A and B are two disjoint set's then the characteristic function $\chi_{A \cup B} =$ _____.

Q.2 Answer the following. **16**

a) Show that for any set E, $\mu_*(E) = \mu^*(E)$.

b) If $\gamma_1 \ll \mu$ and $\gamma_2 \ll \mu$ and μ is a $\sigma -$ finite measure then prove that

$$\left[\frac{d(\gamma_1 + \gamma_2)}{d\mu}\right] = \left[\frac{d\gamma_1}{d\mu}\right] + \left[\frac{d\gamma_2}{d\mu}\right]$$

c) Show that every σ-finite measure is saturated.

d) If f and g are integrable functions and E is measurable set then prove that

$$\int_E c_1 f + c_2 g = c_1 \int_E f + c_2 \int_E g$$

Q.3 Answer the following.

a) If E and F are disjoint sets, then prove that **08**

$\mu_*(E) + \mu_*(F) \le \mu_*(E \cup F) \le \mu_*(E) + \mu^*(F)$.

b) State and prove Lebesgue convergence theorem. **08**

Q.4 Answer the following.

a) If X be an uncountable set and let **08**
$\mathfrak{B} = \{A \subseteq X |\ A$ is countable or $A = E^c$, where E is countable$\}$
Define $\mu: \mathfrak{B} \rightarrow [0, \infty] \cup \{\infty\}$ by

$$\mu(A) = \begin{cases} 0; & if\ A \text{ is countable} \\ 1; & if\ A = E^c \text{ is countable} \end{cases}$$

Then show that (X,$\mathfrak{B}$,μ) is a measure space.

b) If γ be a signed measure on a measurable space (X, $\mathcal{B}$), then prove that there exist a positive set A and a negative set B such that $X = A \cup B$ and $A \cap B = \varphi$ **08**

Q.5 Answer the following.

a) State and prove Lebesgue decomposition theorem. **10**

b) Prove that the collection $\mathcal{R}$ of measurable rectangles is a semi-algebra. **06**

Q.6 Answer the following.

a) Define an inner measure of a set. Show that for any set E We have $\mu_*(E) \leq \mu^*(E)$ Further if $E \in \mathcal{A}, \mu_*(E) = \mu^*(E)$. **08**

b) If μ_1 and μ_2 be two measures on $(X,\mathfrak{B})$ such that atleast one of them is finite. Define $\gamma(E) = \mu_1(E) - \mu_2(E) \forall E \in \mathfrak{B}$. Then show that γ is a signed measure. **08**

Q.7 Answer the following.

a) If γ be a signed measure on a measurable space (X,$\mathcal{B}$). Let E be a measurable set such that $0 < \gamma(E) < \infty$ then prove that there is a positive set A contained in E with $\gamma(A) > 0$. **08**

b) If $\{A_i\}$ be a disjoint sequence of sets in $\mathcal{A}$ then prove that **08**

$$\mu_*(E \cap \cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$$

SLR-SO-18

Seat No.	

Set P

M.Sc. (Sem-IV) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Partial Differential Equations (MSC15402)

Day & Date: Wednesday, 12-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Number of arbitrary constant is less than number of independent variable then the elimination of arbitrary constant usually gives rise to _____.
   a) Less than two partial differential equation of order one
   b) More than one partial differential equation of order two
   c) More than one partial differential equation of order one
   d) One partial differential equation of order one

2) A first order partial differential equation is said to be linear equation if it is linear in
   a) p,q and z b) p,q and x
   c) q,z,x and y d) x,y,p,q, and z

3) General integral is envelope of _____ parameter subfamily of the family of solutions.
   a) 1 b) 2
   c) 3 d) none

4) Singular integral is envelope of _____ parameter subfamily of the family of solution.
   a) 1 b) 2
   c) 3 d) none

5) Necessary and sufficient condition for integrability of $\mathrm{dz} = \Phi(x,y,z)dx + \Psi(\mathrm{x,y,z})\ dy$ is _____.
   a) $[f,g] = 0$ b) $[f,g] \neq 0$
   c) $[f,g] = 1$ d) $[f,g] \neq 1$

6) Compatible system of first order partial differential equation has _____.
   a) Two parameter family of common solutions
   b) Envelope of one parameter family of common solutions
   c) One parameter family of common solutions
   d) no solutions

7) $f(p,q) = 0$ be the partial differential equation which does not involved x, y, and z explicitly then complete integral is given by _____.
   a) $z = ax + \Phi(a).y + b$ b) $z = ax + \Phi(a).y^2 + b$
   c) $z = ax^3 + \Phi(a).y + b$ d) $z = ax^2 + \Phi(a).y + b$

8) The given second order partial differential equation $e^{2x}u_{xx}2e^{x+y}u_{xy} + e^{2y}u_{yy}$= 0 is of the form _____.
   a) Hyperbolic b) Parabolic
   c) Elliptical d) None

9) If the discriminant $S^2 - 4RT > 0$ of the quadratic equation $R\lambda^2 + S\lambda + T = 0$ then roots are _____.
   a) Real and Repeated b) purely imaginary and Distinct
   c) Purely imaginary d) Real and Distinct

10) The solution of Dirichlet problem, _____.
   a) Not unique b) Always exist
   c) Not exist d) if it exists then it is unique

B) fill in the blanks. **06**

1) The complete integral of partial differential equation pq=c is given by, _____.

2) If u(x,y) be harmonic function in bounded closed region D and continuous in $D \cup B$, where B is boundary of region D, then 'u' attains its minimum on _____.

3) For n=1 the given equation $(n-1)^2u_{xx} - y^{2n}u_{yy} = n.y^{2n-1}u_y$ reduces to the hyperbolic canonical form.

4) If the discriminant $S^2 - 4RT = 0$ of the quadratic equation $R\lambda^2 + S\lambda + T = 0$ then roots are _____.

5) The canonical form of the differential equation $y^2 u_{xx} - 2xy\, u_{xy} + x2\, u_{yy} = 0$ is _____.

6) If g(y,q) - h(x,y) = 0 be the partial differential equation then complete integral is given by _____.

Q.2 Answer the following. **16**

a) Find the partial differential equation satisfied by all the surfaces of the forms $F(u, v) = 0$
where,$u = u(x, y, z) v = v(x, y, z)$ and F is arbitrary function of u and v.

b) Eliminate the arbitrary function and find the corresponding partial differential equation for the function $F(x - z, y - z) = 0$

c) Define:
   i) Interior Dirichlet problem
   ii) exterior Dirichlet problem

d) Show that the necessary condition for the existence of the solution of Neumann problem is that $\int f(s)\, ds.$ should vanish on boundary.

Q.3 Answer the following. **16**

a) Prove that: A necessary and sufficient condition that there exist a relation between two function $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$ or $u = H(v)$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.

b) Show that $(x - a)^2 + (y - b)^2 + z^2 = 1$ is complete integral of $z^2(1 + p^2 + q^2) = 1$ by taking $b = 2a$ and the envelope of the sub family is $(y - 2x)^2 + 5z^2 = 5,$ which is particular solution.

Q.4 Answer the following. **16**

a) If $\bar{X}$ is a vector such that $\bar{X}$= (P, Q, R) i.e. $\bar{X} = Pi + Qj + Rk$ where i, j, k are unit vector along respective co-ordinate axis and $\bar{X}.\ curl\bar{X} = 0$, μ is an arbitrary differentiable function of x, y, z then prove that $\mu\bar{X}.\ curl\mu\bar{X} = 0$.

b) Check the integrability and find the solution of following differential equation.

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

Q.5 Answer the following. **16**

a) Show that the following Pfaffian differential equation is integrable and find its integral.

$$y\,dx + x\,dy + 2z\,dz = 0$$

b) Describe Charpits method for solving a first order partial differential equation.

$$f(x, y, z, p, q) = 0$$

Q.6 Answer the following. **16**

a) Find the integral surface of the given partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the curve

$$x_0(s) = 1, y_0(s) = 0, z_0(s) = s$$

b) Obtain an equation observing small transverse vibration of an elastic string.

Q.7 Answer the following. **16**

a) Reduce the equation $u_{xx} - x^2 u_{yy} = 0$ to a canonical form.

b) Find the condition that a one parameter family of surfaces forms a family of equipotential surfaces.

SLR-SO-19

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Integral Equation (MSC15403)

Day & Date: Friday, 14-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) An integral equation $g(x)u(x) = f(x) + \int_a^b k(x,t)\, u(t)dt$ is said to be of the first kind if _____.
   a) $g(x) = 0$ b) $g(x) = 1$
   c) $f(x) = 0$ d) $f(x) = 1$

2) Which of the following is not a degenerate kernel?
   a) $K(x,t) = xt$ b) $K(x,t) = x - t$
   c) $K(x,t) = sin(x - t)$ d) $K(x,t) = e^{xt}$

3) Which of the following is not a symmetric kernel?
   a) $K(x,t) = x + t$ b) $K(x,t) = \sin(x - t)$
   c) $K(x,t) = e^{x^{2+t^2}}$ d) $K(x,t) = \log(x + t)$

4) Eigenvalues of symmetric kernel of a Fredholm integral equation are _____.
   a) always positive b) always negative
   c) always real d) purely imaginary

5) A Volterra integral equation can be solved using Laplace transform if the kernel is _____.
   a) Symmetric b) Separable
   c) convolution type *d)* positive

6) The second iterated kernel $K_2(x,t)\ for\ K(x,t) = 1$ of a volterra integral equation is _____.
   a) $(x - t)$ b) $\frac{(x-t)^2}{2}$
   c) $\frac{(x-t)^2}{3}$ d) $\frac{(x-t)^3}{3!}$

7) Solution of $y(x) = 1 + \int_0^x y(t)$ *dt* is _____.
   a) 1 b) e^x
   c) x d) None of these

8) $\int_0^x \int_0^x \int_0^x y(t)dt^3 =$ _____.

a) $\int_0^x y(t)dt$

b) $\int_0^x \frac{(x-t)^2}{2} y(t)dt$

c) $\int_0^x \frac{(x-t)^3}{3} y(t)dt$

d) $\int_0^x \frac{(t-x)^2}{2} y(t)dt$

9) Which of the following is a formula to find n-th iterated kernel of a Volterra integral equation?

a) $K_n(x,t) = \int_0^x K(x,z)K_{n-1}(z,t)dz$

b) $K_n(x,t) = \int_0^x K(x,z)K_{n-1}(x,z)K(z,t)dz$

c) $K_n(x,t) = \int_t^x K(x,z)K_{n-1}(z,t)dz$

d) All of the above

10) . Green's function for a BVP exists if _____.

a) the BVP has only non-trivial solution

b) the BVP has no solution

c) the BVP has no trivial solution

d) the BVP has only trivial solution

B) State whether True or False. **06**

1) A homogenous Fredholm integral equation can never have eigen values.

2) The integral equation $y(x) = \int_0^1 (x - t)y(t)$dt is homogeneous.

3) The kernel $K(x,t) = e^{x+t}$ is symmetric.

4) Green's function exists for every boundary value problem.

5) Every Volterra integral equation can be solved using Laplace transform.

6) Eigen values of every Fredholm integral equation are always real numbers.

Q.2 Answer the following. **16**

a) Show that the function $y(x) = \frac{1}{(1+x^2)\sqrt{1+x^2}}$ is a solution of $y(x) = \frac{1}{1+x^2} - \int_0^x \frac{t}{1+x^2} y(t)dt$.

b) Define Green's function.

c) Solve: $\int_0^\infty F(x) cospx\, dx = \begin{cases} 1-p, 0 \le p \le 1 \\ 0, p > 1 \end{cases}$

d) Solve: $y(x) = \lambda \int_0^{\frac{\pi}{4}} \sin^2 xy(t)dt$

Q.3 Answer the following.

a) Obtain the integral equation from $y'' + \lambda y = x, y(0) = 0, y(1) = 1$. Also recover the boundary value problem from the integral equation obtained. **08**

b) Prove that the eigen functions of a symmetric kernel, corresponding to distinct eigenvalues are orthogonal. **08**

Q.4 Answer the following.

a) Find the Green's function for the boundary value problem $y'' + \mu\text{^}2\, y = 0, y(0) = y(1) = 0.$ **10**

b) Solve: $Y'(t) = t + \int_0^t Y(t - x)\cos x\, dx, Y(0) = 4.$ **06**

Q.5 Answer the following.

a) Solve: $y(x) = \cos x + \lambda \int_0^{\pi} sin\,(x - t)y(t)dt.$ **08**

b) Solve by the method of successive approximation. **08**

$y(x) = 1 + x^2 + \int_0^x \frac{(1+x^2)}{(1+t^2)} y(t)dt.$

Q.6 Answer the following.

a) Solve by the method of successive approximations. **08**

$y(x) = 1 + \int_0^x (x - t)y(t)dt, y_0(x) = 1$

b) Solve by the iterative method. **08**

$y(x) = \sin x - \frac{x}{4} + \frac{1}{4}\int_0^{\frac{\pi}{2}} xty(t)dt$

Q.7 Answer the followings.

a) Find the eigenvalues and eigen functions of the following integral equation. **10**

$$y(x) = \lambda \int_0^{\pi} K\,(x,t)y(t)dt$$

Where $K(x,t) = \begin{cases} \cos x \sin t\,, 0 \le x \le t \\ \cos \mathrm{t} \sin x\,, t \le x \le \pi \end{cases}.$

b) Solve the symmetric integral equation **06**

$y(x) = (x+1)^2\ \int_{-1}^{1} (xt + x^2t^2)y(t)dt$ by using Hilbert-Schmidt theorem.

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Operations Research (MSC15404)

Day & Date: Sunday, 16-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) In the simplex method the slack, surplus and artificial variable restricted to be _____.
   a) Multiplied b) Negative
   c) Non negative d) None of these

2) According to simplex method the slack variable assigned zero coefficients because _____.
   a) No contribution in objective function
   b) High contribution in objective function
   c) divisors contribution in objective function
   d) base contribution in objective function

3) For a maximization problem, the objective function co-efficient for an artificial _____ variable is
   a) +M b) -M
   c) Zero d) None of these

4) A quadratic form Q(x) is said to be positive semi definite if _____.
   a) $Q(x) \geq 0$ for all $x \neq 0 \in \mathrm{R}^n$ b) $Q(x) > 0$ for all $x \neq 0 \in \mathrm{R}^n$
   c) $Q(x) < 0$ for all $x \neq 0 \in \mathrm{R}^n$ d) $Q(x) \leq 0$ for all $x \neq 0 \in \mathrm{R}^n$

5) Identify the wrong statement _____.
   a) A game without saddle point is probabilistic
   b) Game with saddle point will have pure strategies
   c) Game with saddle point cannot be solved by dominance rule
   d) Game without saddle point uses mixed strategies

6) In game theory, a situation in which one firm can gain only what another firm loses is called _____.
   a) nonzero-sum game, b) prisoners' dilemma
   c) zero-sum game d) Predation game

7) If at least one Δ_j is negative then the solution of linear programming problem is _____.
   a) Not optimal b) not feasible
   c) not bounded d) not basic

8) Consider the following statements:
   i) The closed ball in R^n is a convex set.
   ii) A hyperplane in R^n is a convex set.
   a) only I is true b) only II is true
   c) both are true d) both are false

9) For a set of m equations in n variables (n>m), a solution obtained by setting (n-m) variables equal to zero and solving for remaining m equations in m variables is called _____.
   a) Basic solution b) feasible solution
   c) basic feasible solution d) optimum solution

10) If the set of feasible solutions of the system $AX = B, X \geq 0$, is a convex polyhedron, then at least one of the extreme points gives a/an:
   a) Unbounded solution b) Bounded but not optimal
   c) Optimal solution d) Infeasible solution

B) Write True or False. **06**

1) Beal's method is used to solve _____ programming problem.
2) The intersection of finite number of closed half spaces in R^n is called _____.
3) The convex hull of X is the _____ convex set containing X.
4) In Gomory's method, the negative fraction in k^{th} row of optimum simplex table is expressed as sum of _____.
5) If dual has an unbounded solution, then primal has _____ solution.
6) The dual of dual of a given primal problem is _____.

Q.2 Answer the following **16**

a) Find the dual of

$$Max\ Z = 2x_1 + 3x_2 + 4x_3 \ subject\ to\ constraint$$
$$2x_1 + x_2 \leq 1,\ \ x_1 + 5x_2 - x_3 \leq 2, 7x_1 + x_2 + x_3 = 3, x_{1,}x_{2,}x_3 \geq 0$$

b) Show that: The intersection of two convex set is a convex set.
c) Write the general rules for converting any primal into its dual.
d) If X is an feasible solution of the primal problem and W is any feasible solution to the dual problem then prove that $CX \leq b^T W$.

Q.3 Answer the following. **16**

a) State and prove that fundamental theorem of linear programming problem.
b) Solve the following linear programming problem by Big-M method.

$$Max\ Z = 2x_1 + x_2 \ \ subject\ to\ condition$$
$$3x_1 + x_2 = 3,\ \ 4x_1 + 3x_2 \geq 6, \qquad x_1 + 2x_2 \leq 4 \quad \& \quad x_1, x_2 \geq 0$$

Q.4 Answer the following. **16**

a) If the k^{th} constraint of the primal is an equality then prove that the dual variable w_k is unrestricted in sign.
b) State and prove Complementary Slackness Theorem.

Q.5 Answer the following. **16**

a) Describe the algorithm of Gomory's cutting plane method to solve integer programming problem.
b) Solve the following integer programming problem.

$$Max\ Z = 7x_1 + 9x_2 \ \ subject\ to\ condition$$
$$-x_1 + 3x_2 \leq 6,\ \ 7x_1 + x_2 \geq 35, \qquad x_{1,}x_2 \geq 0\ and\ are\ integer$$

Q.6 Answer the following. **16**

a) Explain the construction of Kuhn-Tucker condition for solving the quadratic programming problem.

b) Solve the 3*3 game by simplex method of linear programming problem whose payoff matrix is given by,

$$\begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix}$$

Q.7 Answer the following. **16**

a) Solve the following quadratic programming problem by Wolfe method

$$Max\ Z_x = 2x_1 + x_2 - x_1^2 \ subject\ to\ condition$$
$$2x_1 + 3x_2 \leq 6,\ \ 2x_1 + x_2 \leq 4,\ \ \ x_1, x_2 \geq 0$$

b) If $\{v_{ij}\}$ be the payoff matrix for two person zero sum game and $\underline{v}$ denotes maximin value and $\bar{v}$ is the minimax value of the game then prove that $\bar{v} \geq \underline{v}$. .

SLR-SO-21

Seat No.	

Set P

M.Sc. (Semester-IV) (New) (CBCS) Examination: March/April-2023
MATHIMATICS
Numerical Analysis (MSC15408)

Day & Date: Tuesday, 18-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) LU decomposition method is an iterative method to solve system of liner equations.
a) True b) False
c) Can not say d) None of these

2) Gauss-seidal method is _____ as efficient as Jacobi method.
a) Fifth b) Fourth
c) Thrice d) Twice

3) Using bisection method, the real roots of $x^3 + x^2 + x + 7 = 0$ between $x = -2$ and $x = -3$ is near to _____.
a) 2.75 b) -2.105
c) 2.2 d) -3.1

4) 3.142 is _____ value of π and 1/2 is _____ number.
a) Approximate, exact b) Real, integer
c) Exact, integer d) Real, approximate

5) Which of the following is best approximate for 1/3 _____.
a) 0.30 b) 0.33
c) 0.34 d) None of these

6) Newton- Raphson method _____.
a) Converges fastly b) Diverges
c) Converges quadratically d) Converges slowly

7) _____ is used in Regula-falsi method.
a) Slope formula b) Derivative
c) Taylor series d) None of these

8) _____ is used for derivation of secant method.
a) Quadratic polynomial b) n^{th} degree polynomial
c) Linear equation d) None of these

9) The order of convergence of secant method is _____.
a) 1.618 b) 2
c) 2.1 d) 3

10) In the gauss elimination method for solving a system of linear algebraic equations, triangulation leads to _____.
a) Upper triangular matrix b) Lower triangular matrix
c) Zero matrix d) None of these

B) Fill in the blanks. **06**

1) If f(x) =0 has a root between a and b than f(a) and f(b) are of _____ signs.
2) _____ method is used for finding the dominant Eigen-value of a matrix.
3) Using Taylor series we can drive formula for _____ method for finding roots of equation.
4) Newton Raphson's formula for reciprocal of number is _____.
5) In gauss elimination method the coefficient matrix is reduced to _____.
6) Householder method consist of converting real symmetric matrix to _____ matrix.

Q.2 Answer the following **16**

a) Find a positive root of the equation $xe^x = 1$ using network Raphson method up to three iterations.

b) Preform three iteration of secant method to find root of in the interval (0,1).

c) Preform two iterations of the newton Raphson method to obtain the approximate value of 17^3 correct to three decimal places.

d) An approximate value of π is 3.1428571 and its true value is 3.1415926. find the absolute and relative errors.

Q.3 Answer the following.

a) Using the Householders transformation reduce the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ into tridiagonal matrix. **08**

b) Explain rate of convergence of secant method. **08**

Q.4 Answer the following.

a) Solve the system of equation $2x_1 - x_2 = 1, -x_1 + 2x_2 - x_3 = 1,\ x_2 + 2x_3 = 1$ Using gauss seidel method **08**

b) Find the largest eigen value in modulus and the corresponding eigenvector of the matrix $\begin{bmatrix} 15 & 4 & 3 \\ 10 & -12 & 6 \\ 20 & -4 & 2 \end{bmatrix}$ using the power method. **08**

Q.5 Answer the following.

a) Explain alternating direction implicit (ADI) method for numerical solution of partial differential equation. **08**

b) Determine the value of y using modified Euler method when $x = 0.1$ given that $y(0) = 1, h = 0.5\ and\ y' = x^2 + y$. **08**

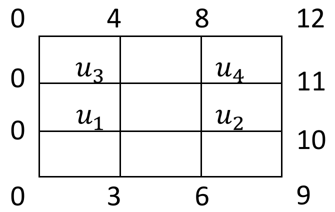
Q.6 Answer the following.

a) Find the root of equation $x^3 + x - 1 = 0$ by using bisection method. **08**

b) Solve the equation $x_1 + x_2 + x_3 = 1, 4x_1 + 3x_2 - x_3 = 6, 3x_1 + 5x_2 + 3x_3 = 4$ using LU decomposition method. **08**

Q.7 Answer the following.

a) If function $u(x,y)$ satisfies Laplace's equation at all points within the square given below and has boundary values as indicated. **08**



Compute solution correct upto two decimal places by finite difference method.

b) Explain Picard's method of successive approximations, solve $\mathcal{Y}' = x + \mathcal{Y}^2 \; y(0) = 1$ using Picard's method **08**

SLR-SO-23

Seat No.	

Set P

B.Sc. (Semester - IV) (New) (CBCS) Examination: March/April-2023
MATHEMATICS
Probability Theory (MSC15410)

Day & Date: Tuesday, 18-07-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) If $\{A_n\}$ is decreasing sequence of sets, then it converges to _____
   a) lim inf $\{A_n\}$ b) lim *sup* $\{A_n\}$
   c) both (a) and (b) d) None of the above

2) If for two independent events A and B, $P(A) = 0.3$, $P(B) = 0.1$, then $P(AUB) =$ _____.
   a) 0.68 b) 0.37
   c) 0.40 d) 0.68

3) Which of the following is the weakest mode of convergence?
   a) convergence in rth mean
   b) convergence in probability
   c) convergence in distribution
   d) convergence in almost sure

4) If events A and B are independent events, then which of the following is correct?
   a) $P(A \cap B) = P(A) + P(B)$
   b) $P(A \cup B) = P(A) + P(B) - P(A) * P(B)$
   c) $P(A \cup B) = P(A) * P(B)$
   d) $P(A \cap B) = P(A) - P(B)$

5) If F_1 and F_2 are two fields defined on subsets of Ω, then which of the following is/are always a field?
   a) $F_1 \cup F_2$ b) $F_1 \cap F_2$
   c) Both a) and b) d) Neither a) and b)

6) A class Ƒ is said to be closed under finite intersection, if $A, B \in$ Ƒ implies
   a) $A \cap B \in$ Ƒ, for all $A, B \in$ Ƒ
   b) $A^C \in$ Ƒ, $B^C \in$ Ƒ
   c) Both a) and b)
   d) None of these

7) Lebesgue measure of a singleton set $\{k\}$ is _____.
   a) 0 b) 1
   c) k d) None of these

8) The sequence of sets $\{An\}$, where $An = (0,2 + \frac{1}{n})$ converges to _____.
   a) (0,2) b) (0,2]
   c) [0,3) d) $[0,2]$

9) The σ- field generated by the intervals of the type (-∞,x), $x \in R$ is called _____.
   a) Standard σ- field b) Borel σ- field
   c) Closed σ- field d) None of these

10) Indicator function is a_____
   a) Simple function b) Elementary function
   c) Arbitrary function d) All of these

B) Fill in the blanks. **06**

1) A well-defined collection of sets is called as ...
2) If F(.) is a distribution function for some random variable, then $\lim_{n\rightarrow\infty} F(x)$ = _____.
3) If P is a probability measure defined on $(\Omega, \mathcal{A})$, then P(Ω)= _____.
4) If A ⊂ B, then $P(A)$ _____ $P(B)$.
5) The convergence in _____ is also called as a weak convergence.
6) Expectation of a random variable X exists, if and only if _____ exists.

Q.2 Answer the following. **16**

a) Prove that inverse mapping preserves all set relations.
b) Write a note on Lebesgue measure.
c) Prove or disprove: Arbitrary union of fields is a field.
d) Write a note on characteristic function of a random variable.

Q.3 Answer the following.

a) State and prove monotone convergence theorem. **08**
b) Prove that probability measure is a continuous measure. **08**

Q.4 Answer the following.

a) Prove that collection of sets whose inverse images belong to a σ-field, is a also a σ-field. **08**
b) Prove that an arbitrary random variable can be expressed as a limit of sequence of simple random variables. **08**

Q.5 Answer the following.

a) Define, explain and illustrate the concept of limit superior and limit inferior of a sequence of sets. **08**
b) Prove that inverse image of σ -field is also a σ -field. **08**

Q.6 Answer the following.

a) Prove or disprove: **08**
   i) Convergence in distribution implies convergence in probability
   ii) Convergence in probability implies convergence in distribution.
b) Define expectation of simple random variable. If X and Y are simple random variables, prove the following: **08**
   i) $E(X + Y) = E(X) + E(Y)$
   ii) $E(cX) = c\,E(X)$, where c is a real number.
   iii) $If\ X > 0$ a.s., then $E(X) > 0$.

Q.7 Answer the following.

a) Prove that expectation of a random variable X exists, if and only if $E|X|$ exists. **08**
b) State and prove Borel-Cantelli lemma. **08**