Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Real Analysis

Day & Date: Monday, 13-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) Subset of a countable set is always _____.
   a) Countable b) Uncountable
   c) partially countable d) none of these

2) The limit of sequence $S_n = \frac{1}{n}$, $n \epsilon N$ is _____
   a) 1 b) 0
   c) 10 d) 2

3) The set of all integers is _____.
   a) Countable b) Uncountable
   c) both (a) and (b) d) none of these

4) The number $\sqrt{9}$ is _____ number.
   a) Irrational b) Rational
   c) Integer d) none of these

5) If there exists one to one correspondence between given set and set of natural numbers, then the given set is _____
   a) perfect set b) Good set
   c) countable set d) Uncountable set

6) Every infinite bounded set has a _____.
   a) interior point b) limit point
   c) initial point d) None of these

7) Which of the following is not correct?
   a) Every continuous function is differentiable.
   b) Every differentiable function is continuous.
   c) Every continuous function is right continuous.
   d) Every continuous function is left continuous.

8) The $\alpha(.)$ function in R-S integral is ______.
   a) Always non negative
   b) Always monotonic non-increasing
   c) Always monotonic non-decreasing
   d) Always constant

9) The set s$\left\{\left(1+\frac{(-1)^n}{n}\right), n \in N\right\}$ has _____ limit points.
   a) One b) Two
   c) Zero d) Four

10) If set $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n =$

a) Zero b) 1

c) Infinity d) -1

B) Fill in the blanks. **06**

1) A set $\{1,3,7,20\}$ has total of _____ limit points.
2) The maximum value of the function $f(x) = -x2 + 2\% + 3$ is _____.
3) Greatest Lower Bound of a set is also called as _____.
4) If A and B are open sets, then $A \cap B$ is _____.
5) A superset of uncountable set _____.
6) A geometric series with common ratio r converges, if _____.

Q.2 Answer the following **16**

a) State:
i) Heine-Borel theorem
ii) Bolzano-Weistrauss theorem

b) Write a short note on mean value theorem.

c) State implicit function theorem. Also state its applications.

d) Define and illustrate:
i) Open set
ii) Closed set

Q.3 Answer the following **16**

a) Prove or disprove: Arbitrary intersection of closed sets is always closed.

b) Define a bounded sequence. Show that a convergent sequence is always bounded. Is every bounded sequence convergent? Justify.

Q.4 Answer the following **16**

a) Explain with illustration the concept of infimum and supremum of a set.

b) Define open set. Prove or disprove: Arbitrary union of open sets is open.

Q.5 Answer the following **16**

a) Discuss the convergence of following series:

i) $\sum_{n=2}^{\infty} \frac{n+1}{n^2-1}$

ii) $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$

b) Define radius of convergence. Illustrate using any power series.

Q.6 Answer the following **16**

a) Discuss in detail Riemann integration.

b) Explain Lagrange's method for obtaining constrained maxima or minima.

Q.7 Answer the following **16**

a) Examine the convergence of p-series for various values of p.

b) State and prove rule of integration by parts.

SLR-GR-2

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Linear Algebra & Liner Models

Day & Date: Tuesday, 14-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q.Nos. 1 and 2 are compulsory.
2) Attempt any Three questions from Q.3 to Q.7
3) Figures to the right indicate full marks.

Q.1 A) Choose the correct Alternative. **10**

1) If A and B are two matrices of order n x n with ranks r_1 and r_2, then -
a) $\mathrm{Rank(AB)} = r_1 + r_2$
b) $\mathrm{Rank(AB)} > r_1 + r_2$
c) $\mathrm{Rank(AB)} \geq r_1 + r_2 - n$
d) $\mathrm{Rank(AB)} \leq r_1 - r_2$

2) For non-homogenous system of equations Ax=b with k unknowns, unique solution exists if-
a) $\mathrm{Rank[A{:}\,b]} > \mathrm{rank(A)}$
b) $\mathrm{Rank[A{:}\,b]} = \mathrm{rank(A)} = k$
c) $\mathrm{Rank[A{:}\,b]} = \mathrm{rank(A)} < k$
d) All of the above

3) Matrix addition is
a) Commutative
b) Associative
c) Both(a) and (b)
d) Neither (a) nor (b)

4) If v_1, v_2, v_3 are three vectors such that $4v_1 + 2v_2 + v_3 = 0$, then
a) v_1, v_2, v_3 are linearly dependent vectors
b) v_1, v_2, v_3 are linearly independent vectors
c) Need to verify other linear combinations to check independence.
d) None of these

5) If G is a g-inverse of matrix A, then _____.
a) $\mathrm{rank(A)} \geq \mathrm{rank(G)}$
b) $\mathrm{rank(A)} = \mathrm{rank(G)}$
c) $\mathrm{rank(GAG)} < \mathrm{rank(A)}$
d) $\mathrm{rank(A)} \leq \mathrm{rank(G)}$

6) Linear combinations of estimable functions are _____.
a) Always non-estimable
b) Always estimable
c) May or may not be estimable
d) None of these

7) For a matrix N with 5 rows and 3 columns, $\rho(N)$ is rank of N then
a) $\rho(N) \leq 5$
b) $\rho(N) \geq 3$
c) $\rho(N) \geq 5$
d) $\rho(N) \leq 3$

8) For a Gauss-Markov model $Y = X\beta + \varepsilon$,
a) $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) = 0, if\ i \neq j$
b) $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) < 0, if\ i \neq j$
c) $\mathrm{Cov}(\varepsilon_i, \varepsilon_j) > 0, if\ i \neq j$
d) None of these

9) The quadratic form $2X_1^2 + X_2^2$ is -
a) positive definite
b) negative definite
c) positive semi definite
d) negative semi definite

10) For which value of x will the matrix given below will become singular?

$$\begin{bmatrix} 8 & x & 0 \\ 4 & 0 & 2 \\ 12 & 6 & 0 \end{bmatrix}$$

a) 4　　b) 6
c) 8　　d) 12

B) Fill in the blanks. **06**

1) Multiplication of a matrix with a scalar constant is called as _____.
2) The rank of identity matrix of order 7 is _____.
3) If transpose of the given matrix is equal to the matrix itself, then it is called _____.
4) The product of matrix A and its inverse A^{-1} is _____.
5) The dimension of the vector space R^2 over the field R is _____.
6) If number of columns is less than number of rows, then the matrix is called as _____.

Q.2 Answer the following. **16**

a) Write a note on matrix multiplication. Also illustrate with one example.

b) Define-
   i) Symmetric matrix
   ii) Skew-symmetric matrix

c) Define:
   i) Span of a set of vectors.
   ii) Spanning set

d) Define subspace. State the conditions needed to verify whether a subset of a vector space is a subspace.

Q.3 Answer the following.

a) Determine whether $S = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in R^3 / y = 0 \right\}$ is a vector space under regular addition and scalar multiplication. **08**

b) Prove: For any vector $\underline{u}$ in vector space $\mathrm{V}, 0.\underline{\mathrm{u}} = \underline{0}$ **08**

Q.4 Answer the following.

a) How the independence of vectors is examined? Also verify whether following set is a set of independent vectors. **08**

$S = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 4 \end{pmatrix} \right\}$

b) Define: **08**
   i) Symmetric matrix
   ii) Skew-symmetric matrix

Prove or disprove: Every square matrix can be written as sum of symmetric and skew symmetric matrix.

Q.5 Answer the following.

a) Define norm of a vector. Using Gram Schmidt orthogonalisation process, obtain orthonormal basis from the given set S. **08**

$$S = \left\{ \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\}$$

b) Describe row-reduced form of a matrix. Also give one example of such a matrix of order 5 x 5. **08**

Q.6 Answer the following.

a) Define inverse of a matrix. Show that it is unique. **08**

b) Show that rank of a matrix is unaltered by multiplication with a non-singular matrix **08**

Q.7 a) Show that the following system of equations is consistent. Also find solution for the same. **08**

$x + y + z = 6$

$x + 2y + 3z = 14$

$x + 4y + 7z = 30$

b) Define linear model. Discuss the estimation of parameters involved in the model. **08**

SLR-GR-3

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Distribution Theory

Day & Date: Wednesday, 15-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) Which of the following is not a scale family?
   a) $\mathrm{U}(0,1)$ b) $\mathrm{U}(0,\theta)$
   c) $\mathrm{N}(0,\sigma^2)$ d) $\mathrm{Exp}(\theta)$

2) A random variable X is said to be symmetric about point α if _____.
   a) $P(X \geq \alpha + x) = P(X \geq \alpha - X)$
   b) $P(X \geq \alpha + x) = P(X \leq \alpha - X)$
   c) $P(X \leq \alpha + x) = P(X \leq \alpha - X)$
   d) $P(X \leq \alpha + x) = P(X \geq \alpha - X)$

3) Let $F(x)$ denotes the distribution function of random variable X. Then which of the following is not true?
   a) $0 \leq F(x) \leq \infty$
   b) $F(x_1) \leq F(x_2)$ if $x_1 < x_2$
   c) $F(-\infty) = 0$
   d) $F(+\infty) = 1$

4) Let X be distributed as Exp$(\mathrm{Mean}\ \theta)$ .Then distribution of $Y = X|\theta$ is _____.
   a) $\mathrm{Exp}\ (\mathrm{Mean}\ \theta)$ b) $\mathrm{Exp}\ (\mathrm{Mean}\ 1)$
   c) $\mathrm{U}(0,1)$ d) $\mathrm{U}(0,\theta)$

5) Let X be a non-negative random variable with distribution function F. If $E(X)$ exists then $E(X)$= ______.
   a) $\int_0^\infty F(x)dx$ b) $\int_0^\infty [F(x) - 1]dx$
   c) $\int_0^\infty [1 + F(x)]dx$ d) $\int_0^\infty [1 - F(x)]dx$

6) For $X > 0$, which of the following is not true?
   a) $E[X^2] \geq [E(X)]^2$ b) $E[1/X] \geq 1/E(X)$
   c) $E[\sqrt{X}] \geq \sqrt{E(X)}$ d) $E[\log X] \leq \log[E(X)]$

7) For which of the following distribution, $E(X)$ does not exist?
   a) Cauchy b) Uniform
   c) Normal d) Exponential

8) If $M_X(t)$ denotes MGF of random variable X. If $Z = a\,X$ then $M_2(t)$ is _____.
   a) $aM_X(t)$ b) $aM_X(at)$
   c) $M_X(at)$ d) $aM_X(t/a)$

9) Let (X, Y)has bivariate normal $BVN(3,4,25,36,3/5)$ then the conditional mean of X given $Y = 10$ is _____.
   a) 5 b) 4
   c) 3 d) None of these

10) Let $X_1, X_2, \dots \dots, X_n$ be a random sample from pdf $f_x(x)$ and $Y_1 \le Y_2 \le \cdots \le Y_n$ be its order statistics. If pdf of Z is $n[F_x(z)]^{n-1} f_x(z)$ then Z is _____.
   a) sample median b) sample range
   c) smallest observation d) largest observation

B) Fill in the blanks. **06**

1) The mean of first order statistic in $U(0,1)$ distribution is ______.
2) Let X and T be two iid random variables with pdf $f(x) = 2e^{-2x}, x \ge 0$. The distribution of $Z = X - Y$ is _____.
3) Suppose $X_1, X_2, \dots \dots, X_k$ is a multinomial random variate then $Cov\left(X_i, X_j\right), i = j = 1,2, \dots \dots .k, i \ne j$ is _____.
4) If Z is standard normal variate then variance of Z^2 is _____.
5) Let X be distributed as $B(n, p)$. The distribution of $Y = n\ - X$ is ______.
6) Suppose X is $U(0,1)$ random variable then $Y = -\log X$ has _____ distribution.

Q.2 Answer the following **16**

a) Define scale family. Illustrate it with one example.
b) Let X has $N\ (0,1)$ distribution. Find the distribution of $Y\ = |X|$.
c) Define power series distribution. Obtain its MGF.
d) If X is symmetric about α then show that $E(X) = \alpha$.

Q.3 Answer the following. **16**

a) Define distribution function of a random variable X. State and prove its important properties.
b) Define truncated normal distribution truncated below a. Obtain its mean.

Q.4 Answer the following. **16**

a) State and prove Markov's inequality.
b) Let X is a non-negative continuous random with distribution function $F(x)$. If $E(X)$ exist then show that $E(X) = \int_0^\infty [1 - F(u)]\, du$

Q.5 Answer the following. **16**

a) Define moment generating function (MGF) of a random variable X. Explain how it is used to obtain moments of a random variable X.
b) Let X has Poisson (λ) distribution. Obtain the MGF of X. Hence obtain its mean and variance.

Q.6 Answer the following. **16**

a) Define multinomial distribution. Obtain its moment generating function. Hence obtain the pmf of trinomial distribution.
b) Derive the pdf of largest order statistic based on a random sample of size n from a continuous distribution with $pdf\ f(x)$ and $cdf\ F(x)$.

Q.7 Answer the following. **16**

a) Let (X, Y) has $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Obtain the conditional distribution of X given $Y = y$.

b) If X and Y are jointly distributed with probability density function (p.d.f.).
$f(x, y) = 24xy, x \geq 0, y \geq 0$ and $x + y \leq 1$
Find.
1) Marginal distributions of X and Y.
2) Conditional distribution of Y given $X = x$.
3) $E(Y/X = x)$

SLR-GR-4

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Estimation Theory

Day & Date: Thursday, 16-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) Which of the following statements is correct _____.
 a) MLE always exists b) MLE is always unique
 c) MLE is always unbiased d) None of the above is true

2) Let X_1, X_2 be iid Poisson (θ) variables. Then $X_1 + 2X_2$ is ______.
 a) sufficient statistic for θ
 b) not sufficient statistic for θ
 c) Minimal sufficient statistic for θ
 d) complete sufficient statistic for θ

3) If T_n is sufficient statistic for θ based on random sample of size n, then $\frac{\partial \log L}{\partial \theta}$ is a function of ______.
 a) θ only b) T_n only
 c) both T_n and θ d) none of the above

4) The denominator of Cramer-Rao inequality gives ______.
 a) lower bound b) upper bound
 c) amount of information d) none of the above

5) Let T_n be an unbiased estimator of θ. Then ______.
 a) T_n^2 is unbiased estimator of θ^2
 b) $\sqrt{T_n}$ is unbiased estimator of $\sqrt{\theta}$
 c) e^{T_n} is unbiased estimator of e^{θ}
 d) $3T_n + 4$ is unbiased estimator of $3\theta + 4$

6) Conditional distribution of random variable θ given $X = x$ is called _____.
 a) Prior distribution b) Posterior distribution
 c) Loss function d) Bayes risk

7) If T_1 is sufficient statistic for θ and T_2 is an unbiased estimator of θ, then an improved estimator of θ in terms of its efficiency is ______.
 a) $E(T_1T_2)$ b) $E(T_1+T_2)$
 c) $E(T_1/T_2)$ d) $E(T_2/T_1)$

8) Let $I(\theta)$ be the Fisher information on θ, supplied by the sample. If T is an unbiased estimator of $\Psi(\theta)$, then the variance of T will be _____.
 a) $\geq \frac{[\Psi'(\theta)]^2}{I(\theta)}$ b) $\leq \frac{[\Psi'(\theta)]^2}{I(\theta)}$
 c) $\geq \frac{1}{I(\theta)}$ d) $\leq \frac{1}{I(\theta)}$

9) Which of the following is a non-informative prior?
   a) $\pi(\theta) = 1$ b) $\pi(\theta) = 1/\theta$
   c) $\pi(\theta) = \sqrt{I/(\theta)}$ d) All the above

10) Let, X_1, X_2 random sample of size 2 from $N(0, \sigma^2)$ then moment estimator of σ^2 is ______.
   a) $\frac{X_1^2 + X_2^2}{2}$ b) $\frac{X_1 + X_2}{2}$
   c) $X_1 + X_2$ d) $X_1 X_2$

B) Fill in the blanks. **06**

1) If $E_\theta(T) \neq \theta$ then T is ______ estimator of θ.
2) A statistic whose distribution does not depend on parameter is called ______ statistic.
3) Bayes estimator of a parameter under squared error loss function is _____ of posterior distribution
4) Method of scoring is used in ______ estimation.
5) If prior and posterior distributions belongs to the same family of distributions then such family is called ______.
6) Based on random sample of size n from $N(0, \sigma^2), \sigma^2 > 0$, MLE of σ^2 is ______.

Q.2 Answer the following **16**

a) Define sufficient statistic and minimal sufficient statistic.
b) Define Fisher information in a single observation and in n iid observations.
c) Define MLE. State and prove the invariance property of MLE.
d) State and prove Basu's theorem. Illustrate its applicability with example.

Q.3 Answer the following. **16**

a) State and prove Neyman-Fisher factorization theorem.
b) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0, \theta), \theta > 0$ distribution. Show that $X_{(n)}$ is sufficient statistic for θ, but $X_{(1)}$ is not sufficient statistic.

Q.4 Answer the following. **16**

a) State and prove Cramer-Rao inequality with necessary regularity conditions.
b) Let $X_1, X_2, \ldots, X_n$ be iid Poisson (λ) random variables. Obtain C-R lower bound for unbiased estimator of λ.

Q.5 Answer the following. **16**

a) Define UMVUE. State and prove Lehmann-Scheffe theorem.
b) Obtain UMVUE of $p(1-p)$ based on a random sample of size n from $B(1, p)$ distribution.

Q.6 Answer the following. **16**

a) Define maximum likelihood estimator (MLE). Describe the method of maximum likelihood estimation for estimating an unknown parameter.
b) Let $X_1, X_2, \ldots, X_n$ be iid $U(0, \theta), \theta > 0$
   Find:
   1) Moment estimator θ
   2) MLE of θ

Q.7 Answer the following. **16**

a) Define Bayes estimator. Describe the procedure of obtaining Bayes estimator.

b) Let $X_1, X_2, \dots, X_n$ be random sample from $B(1, \theta)$ distribution and prior density of θ is $B_1(\alpha, \beta)$. Assuming squared error loss function, find Bayes estimator of θ.

Seat No.	

Set P

M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Statistical Computing

Day & Date: Friday, 17-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

1) Let $X \sim U(0,1)$ and $F_X(.)$ be its cumulative distribution function. If $Y = F_X(.)$ then Y follows _____.
   a) U (0,1) b) Exp (1)
   c) N (0,1) d) Cauchy (0,1)

2) Which of the following method is used for generating random numbers from a statistical distribution?
   a) Acceptance - Rejection method
   b) Inverse transform method
   c) Monte Carlo methods
   d) All the above

3) In Simpson's $\frac{1^{rd}}{3}$ rule, the numbers of intervals are _____.
   a) Odd b) Even
   c) Multiple of 3 only d) At least 6

4) To obtain one observation from bi-variate Poisson distribution, we need to draw _____ Poisson random numbers.
   a) One b) Two
   c) Three d) Four

5) Which of the following numerical methods are used to solve f(x) = 0?
   a) Bisection method b) Trapezoidal rule
   c) Secant method d) Both a) and c)

6) In Bootstrap method, _____ sampling method is used.
   a) SRSWOR b) SRSWR
   c) Stratified sampling d) Systematic sampling

7) Rate of convergence to correct root is very high for _____ method.
   a) Newton-Raphson b) Bisection
   c) Regula False d) Euler's method

8) For $f(x) = -x^2$ with $x_0 = 3$ and $\alpha = 0.5$, the maximum value of the given function using steepest ascent method is _____.
   a) 1 b) 3
   c) 0 d) -3

9) _____ method fits a quadratic equation using three points for finding approximate root of given function
   a) Bisection method b) Secant method
   c) Newton-Raphson method d) Muller's method

10) If $U_1, U_2 \sim U(0,1)$ then $\sqrt{-2\log(U_1)}\cos(2\pi U_2)$ follows _____.

a) Gamma distribution b) Standard Normal distribution
c) Beta distribution d) Gamma distribution

B) Fill in the blanks. **06**

1) If $U \sim U(0,1)$ then $X = -2\log(1-U)$ follows _____.
2) If $Y \sim$ Gamma (n,1) then, $X = 2Y$ follows Chi-square with _____ degrees of freedom.
3) In Bootstrap resampling technique from sample of size $n,$ we get _____ re-samples.
4) EM algorithm is used to find _____.
5) Congruential random number generator gives _____ random numbers
6) Let $X \sim U(0,1)$ and $Y \sim U(0,1)$ then distribution of $Z = X + Y$ can be obtained using _____.

Q.2 Answer the following **16**

a) Describe linear congruential method with suitable example.
b) Find a positive root of $xe^2 = 2$ by the method of False position (correct up to 4 decimal places).
c) Describe Bootstrap method.
d) What is convolution of distribution? Obtain formula for convolution of continuous distribution.

Q.3 Answer the following **16**

a) What is acceptance rejection (A R) method of random number generation? Write an algorithm to generate random numbers from $N(\mu, \sigma^2)$ distribution using A R method.
b) Obtain an algorithm for generating random numbers from χ_n^2 distribution.

Q.4 Answer the following **16**

a) What is EM algorithm? Illustrate using an example.
b) Explain the Bisection method for finding solution to the equation $f(x) = 0$

Q.5 Answer the following **16**

a) State and prove the result for generating random observations from Poisson distribution.
b) Let $X_1, X_2, \dots X_n$ be a random sample of size n from the displaced exponential with pdf $e^{-(x-\theta)}\, I_{[\theta,\infty)}(x)$ then show that, the jackknife estimator is unbiased estimator of θ.

Q.6 Answer the following **16**

a) Let $X \sim U(0,1)$and $Y \sim U(0,1)$. Define $Z = X + Y$, obtain the distribution of Z.
b) Obtain an algorithm to generate n random numbers from Binomial (m, p) distribution.

Q.7 Answer the following **16**

a) Obtain an algorithm to estimate universal constant π using Monte Carlo method.
b) Explain Newton-Raphson method of finding solution of the equation $f(x) = 0$. Write its geometrical meaning.

SLR-GR-7

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Probability Theory

Day & Date: Monday, 20-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) If $\{A_n\}$ is decreasing sequence of sets, then it converges to ______.
   a) $\lim\inf A_n$ b) $\lim sup\ \{A_n\}$
   c) both (a) and (b) d) None of the above

2) If for two independent events A and B, $P(A) = 0.2$, $P(B) = 0.4$, then $P(A \cup B)$ = ______.
   a) 0.68 b) 0.55
   c) 0.52 d) 0.68

3) If $x \in A$ implies $x \in B$, then ______.
   a) $A \subset B$ b) $B \subset A$
   c) $A = B$ d) All of these

4) If events A and B are independent events, then which of the following is correct?
   a) $P(A \cap B) = P(A) + P(B)$
   b) $P(A \cup B) = P(A) + P(B) - P(A) * P(B)$
   c) $P(A \cup B) = P(A) * P(B)$
   d) $P(A \cap B) = P(A) - P(B)$

5) If X_n is a degenerate random variable for all n and X is identical random variable to X_n , then $\{X_n\}$ converges to X in ______.
   a) r^{th} mean and in probability
   b) probability and in distribution
   c) r^{th} mean, in probability and in distribution
   d) r^{th} mean, almost sure, in probability and in distribution

6) A class $\mathfrak{F}$ is said to be closed under finite intersection, if $A, B \in \mathfrak{F}$ implies ______.
   a) $A \cap B \in \mathfrak{F}$, for all $A, B \in \mathfrak{F}$ b) $A^C \in \mathfrak{F}, B^C \in \mathfrak{F}$
   c) both (a) and (b) d) None of these

7) If F is a σ-field, then which of the following is not always correct?
   a) F is a field
   b) F is a class closed under countable unions
   c) F is a class closed under complementation
   d) F is a minimal sigma field

8) The sequence of sets $\{A_n\}$, where $A_n = \left(0{,}2 + \frac{1}{n}\right)$ converges to ______.
   a) (0,2) b) (0,2]
   c) [0,3) d) [0,2]

9) A class F is said to be monotone class, if ______.
   a) It is a field
   b) If it is closed under monotone operations
   c) Both (a) and (b)
   d) Either (a) and (b)

10) Indicator function is a ______.
   a) Simple function
   b) Elementary function
   c) Arbitrary function
   d) All of these

B) Fill in the blanks. **06**

1) If $F(.)$ is a distribution function for some random variable, then $\lim_{n\to-\infty} F(x) =$______.
2) Convergence in probability implies ______ convergence.
3) A class closed under complementation and finite union is called as ______.
4) If $A \subset B$, then $P(A)$ ____ $P(B)$.
5) The convergence in ______ is also called as a weak convergence.
6) Expectation of a random variable X exists, if and only if ______ exists.

Q.2 Answer the following **16**

a) Define mixture of two probability measures. Show that mixture is also a probability measure.
b) Prove or disprove: Arbitrary intersection of fields is a field.
c) Write a note on Lebesgue measure.
d) Write a note on characteristic function of a random variable.

Q.3 Answer the following **16**

a) State and prove monotone convergence theorem.
b) Prove that if $\{B_n\}$ converges to B, then $P(B_n)$ also converges to P(B).

Q.4 Answer the following **16**

a) Discuss limit superior and limit inferior of a sequence of sets. Find the same for sequence $\{A_n\}$, where $A_n = \left(0, 3 + \frac{(-1)^n}{n}\right), n \in N$
b) Prove that an arbitrary random variable can be expressed as a limit of sequence of simple random variables.

Q.5 Answer the following **16**

a) Prove that collection of sets whose inverse images belong to a σ -field, is a also a σ -field.
b) Prove that inverse image of σ -field is also a σ -field.

Q.6 Answer the following **16**

a) Prove or disprove:
   1) Convergence in distribution implies convergence in probability
   2) Convergence in probability implies convergence in distribution
b) Define expectation of simple random variable. If X and Y are simple random variables, prove the following:
   1) $E(X + Y) = E(X) + E(Y)$
   2) $E(cX) = c\,E(X)$, where c is a real number
   3) If $X > 0$ a.s., then $E(X) > 0$

Q.7 Answer the following **16**

a) Prove that expectation of a random variable X exists, if and only if $E|X|$ exists.
b) State and prove Borel-Cantelli lemma.

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Stochastics Processes

Day & Date: Tuesday, 21-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question 1 and 2 are compulsory.
2) Attempt any Three from Q. 3 to Q. 7
3) Figures to the right indicate full marks.

Q.1 A) Choose Correct Alternative. **10**

1) Which of the following are class properties?
a) Persistency b) Periodicity
c) Transientness d) All of these

2) Let $\{X_n, n \geq 0\}$ be a mark chain with state space $\{0, 1, 2\}$ and tpm
$P = \begin{vmatrix} 0.5 & 0 & 0.5 \\ 0.1 & 0.1 & 0.8 \\ 0.8 & 0 & 0.2 \end{vmatrix}$ Which of the following is true?
a) State 1 is ergodic
b) State 0 and 1 are communicative
c) All states are recurrent
d) State 1 is transient

3) To find n-step transition probabilities, _____ are used
a) Newton equations
b) Sterling's equations
c) Chapman-Kolmogorov equations
d) Lebesgue equations

4) If $\{N(t)\}$ is a counting process, then $N(0) =$
a) 0 b) 1
c) 10 d) 2.71

5) The collection of all possible states of a stochastic process is called as _____.
a) State Space b) Time Space
c) Chain space d) All of these

6) A non-null recurrent aperiodic state is also called as _____.
a) Transitive state b) Binomial state
c) Ergodic state d) None of these

7) A column sum of a stochastic matrix is _____.
a) Always one
b) Always equal to the number of states
c) Is always 3
d) May or may not be 1

8) In a Markov chain, if for a state i, $P_{ii} = 1$, then state i is called as _____.
a) finite state b) absorbing state
c) complete state d) None of above

9) If $\{N(t)\}$ is a poisson process, then the inter-arrival times follow _____.
   a) beta distribution of second kind b) Poisson distribution
   c) binomial distribution d) exponential distribution

10) The process $\{X(t), t > 0\}$, where $X(t) =$ number of COVID patients in a city at the end of n^{th} day, is an example of _____ stochastic process.
   a) discrete time continuous state space
   b) discrete time discrete state space
   c) continuous time continuous state space
   d) continuous time discrete state space

B) Fill in the blanks **06**

1) If probability 'p' of positive jump is 0.5 for a random walk, then it is called as _____.
2) If period of a state is one, then the state is called as _____.
3) A finite Markov chain which contains only one communication class is _____.
4) For a persistent state 'i', the ultimate first return probability $F_{ii} =$ ____1
5) In a Markov chain, if for a state i, $P_{ii} = 1$, then state i is called as _____.
6) Yule-Furry process is also called as _____.

Q.2 Answer the following. **16**

a) Discuss first return probability for a state.
b) Write a note on counting process.
c) What is transition probability matrix?
d) Define and illustrate Markov chain. Show that initial distribution and TPM specifies the Markov chain completely.

Q.3 Answer the following.

a) Describe gambler's game. If a gambler starts the game with initial amount, 'i', find his winning probability. **08**
b) Prove or disprove: Periodicity is class property. **08**

Q.4 Answer the following.

a) Define stationary distribution of a Markov chain. Find the same for a Markov chain with state space {1,2,3}, whose tpm is **08**

$$\begin{bmatrix} 1/3 & 2/3 & 0 \\ 1/3 & 1/3 & 1/3 \\ 2/5 & 1/5 & 2/5 \end{bmatrix}$$

b) Prove that persistency is a class property. **08**

Q.5 Answer the following.

a) Define pure birth process and obtain its probability distribution. **08**
b) Define branching process. With usual notations, obtain its mean and variance. **08**

Q.6 Answer the following.

a) If {N(t)} is a Poisson process, then for s < t, obtain the distribution of N(s), if it is already known that N(t)=k. **08**
b) Calculate the extinction probability for branching process. **08**

Q.7 a) Define stochastic process. Discuss its classification based on state space and time space. **08**
b) Establish the equivalence between two definitions of Poisson process. **08**

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov - 2022
(STATISTICS)
Theory of Testing of Hypotheses

Day & Date: Wednesday, 22-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) Let X has a $N(\mu, \sigma^2)$ distribution where both μ and σ^2 are unknown. Then the simple hypothesis is ______.
   a) $H_0: \sigma = 5$ b) $H_0: \mu = 10$
   c) $H_0: \mu = 5, \sigma = 1$ d) $H_0: \mu \neq 5, \sigma = 4$

2) In order to obtain a most powerful test, we ______.
   a) minimize the level of significance
   b) minimize the power
   c) minimize the level of significance and fix the power
   d) fix the level of significance and maximize the power

3) In testing $H_0: \sigma = \sigma_0$ in $N(0, \sigma^2)$ the critical region based on n observations is $\sum_{i=1}^{n} X_i^2 < k$. For which alternative hypothesis does this provide UMP test?
   a) $\sigma \neq \sigma_0$ b) $\sigma = \sigma_0$
   c) $\sigma < \sigma_0$ d) $\sigma > \sigma_0$

4) For testing simple versus simple hypotheses MP and LRT tests are ____.
   a) the same
   b) different
   c) not comparable
   d) equivalent in size but not with respect power

5) If in Wilcoxon's signed-rank test, sample size is large, the statistic T^+ is distributed with mean ______.
   a) $n(n+1)/2$ b) $n(n+1)/4$
   c) $n(2n+1)/4$ d) $n(n-1)/4$

6) If $k = 2$ then Kruskal-Wallis H test reduces to ______.
   a) Kolmogorov-Smirnov test b) Wilcoxon signed-rank test
   c) Mann-Whitney U test d) none of these

7) Family of *Cauchy* $(1, \theta)$distribution ______.
   a) has MLR property
   b) belong to one parameter exponential family
   c) has mean θ
   d) does not have MLR property

8) A nonparametric version of the parametric analysis of variance is ______.
   a) Mann-Whitney test
   b) Kruskal-Wallis test
   c) sign test
   d) Wilcoxon signed-rank test

9) The area of critical region depends _______.
   a) size of type-I error
   b) size of type-II error
   c) value of statistic
   d) number of observations

10) A size α test is said to unbiased if ______.
   a) it has maximum power in the class of all size α tests
   b) size and power are equal
   c) power is smaller than size
   d) size of the test does not exceed its power

B) Fill in the blanks. **06**

1) To obtain a critical region (or cut off point) in testing a statistical hypothesis, we need the distribution of test statistic under ______ hypothesis.
2) In likelihood ratio test, under some regularity conditions on $f(x,\theta)$ the random variable $-2\log\lambda(x)$ (where $\lambda(x)$ is a likelihood ratio is asymptotically distributed as _______.
3) In testing independence in a 2×3 contingency table, the number of degrees of freedom in χ^2 distribution is ______.
4) The statistic H in Kruskal-Wallis test is approximately distributed as ______.
5) Let $X_1, X_2, \ldots, X_n$ be iid $N(\mu,\sigma^2)$, where σ^2 is known. Then pivotal quantity for confidence interval of μ is ______.
6) Generalized NP lemma is used to construct ______ tests.

Q.2 Answer the following **16**

a) Explain the terms: i) Randomized test ii) Nonrandomized test. Give one example for each.
b) Describe Wald-Wolfowitz run test.
c) Describe pivotal quantity method to obtain confidence interval of parameter θ.
d) State one sample U statistic theorem.

Q.3 Answer the following **16**

a) Define most powerful (MP) test. Explain the method of obtaining MP test of size α for testing simple hypothesis against simple alternative.
b) Obtain a most powerful test of size α for testing $H_0: \sigma = \sigma_0$ against $H_1: \sigma = \sigma_1 (> \sigma_0)$ based on a random sample of size n from $N(\mu,\sigma^2)$, where μ is known.

Q.4 Answer the following **16**

a) Show that for a family having MLR property, there exists UMP test for testing one sided hypotheses against one sided alternative.
b) Let $X_1, X_2, \ldots, X_n$ be a random sample drawn from $U(0,\theta)$ distribution. Find UMP size α test for testing $H_0: \theta \le \theta_0$ against $H_1: \theta > \theta_0$.

Q.5 Answer the following **16**

a) Describe likelihood ratio test (LRT). Show that LRT for testing simple hypothesis against simple alternative is equivalent to Neyman-Pearson test.

b) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta,\sigma^2), \sigma^2$ is known. Obtain shortest length confidence interval for θ.

Q.6 Answer the following **16**

a) Define (i) UMA confidence interval and (ii) UMAU confidence interval. State and prove the result useful in obtaining UMA confidence interval using suitable test.

b) Discuss the use of chi-square test in goodness of fit problem.

Q.7 Answer the following **16**

a) Define (i) similar test and (ii) test having Neyman structure. State the result connecting similar test with Neyman structure.

b) Stating the hypothesis, explain two sample Wilcoxon-Mann-Whitney test and state mean of the test statistic.

Seat No.	

Set P

M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Sampling Theory

Day & Date: Thursday, 23-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) The most important factor in determining the sample size is
   a) The availability of resources b) Purpose of survey
   c) Heterogeneity of population d) None of the above

2) In which of the following situation(s) cluster sampling is appropriate?
   a) When the units are situated for apart
   b) When sampling frame is not available
   c) When all the units are not easily identifiable
   d) All the above

3) To have minimum $var(\bar{y}_{st})$, one has to choose a large sample using cost per unit of survey provided
   a) n is fixed, S_j is small and C_j is small
   b) n is fixed, S_j is small and C_j is large
   c) n is fixed, S_j is large and C_j is small
   d) n is fixed, S_j is large and C_j is large

4) Cluster sampling is efficient than simple random sampling without replacement if the intraclass correlation coefficient ρ is,
   a) $\rho > 0$ b) $\rho < 0$
   c) $\rho = 0$ d) $\rho = 1$

5) If the size values associated with 5 population units are 23, 18, 36, 9 and 14 and a random number selected is 44 then which population unit will you select using Cumulative total method in the probability proportional to size sampling?
   a) 4 b) 3
   c) 2 d) 5

6) Systematic random sampling is more efficient than stratified random sampling if,
   a) $\rho_{wst} = 0$ b) $\rho_{wst} > 0$
   c) $\rho_{wst} < 0$ d) $\rho_{wst} = 1$

7) In cluster sampling in usual notations, the relation between S^2, S_w^2 and S_b^2 is,
   a) $(NM-1)S^2 = N(M-1)S_w^2 + (N-1)S_b^2$
   b) $(NM-1)S^2 = M(N-1)S_b^2 + (N-1)S_b^2$
   c) $(N-1)S^2 = M(N-1)S_w^2 + S_b^2$
   d) $(NM-1)S^2 = S_w^2 + (N-1)S_b^2$

8) A population was divided into clusters and it was found that within cluster variation was less than the variation between the clusters. If a random sample of units was selected from each cluster then the sampling procedure used is,

a) Multistage sampling b) Stratified sampling
c) Cluster sampling d) Systematic sampling

9) Probability Proportional to Size sampling is more efficient than simple random sampling with replacement in usual notations if,

a) $COV(X,\frac{y^2}{x}) < 0$ b) $COV(X,\frac{y^2}{x}) > 0$
c) $COV\left(X,\frac{y^2}{x}\right) = 0$ d) $COV(X,\frac{x^2}{y}) > 0$

10) f $\bar{\bar{y}}$ is sample mean per element in two stage sampling, when a simple random sample of n first stage units are drawn from N first stage units and m second stage units drawn from M second stage units with replacement at both stages then $V(\bar{\bar{y}})$ is,

a) $V(\bar{\bar{y}}) = \frac{N-1}{Nn}S_1^2 + \frac{M-1}{M}S_2^2$
b) $V(\bar{\bar{y}}) = \frac{N-1}{N}S_1^2 + \frac{M-1}{Mm}S_2^2$
c) $V(\bar{\bar{y}}) = \frac{N-1}{Nn}S_1^2 + \frac{M-1}{Mm}S_2^2$
d) $V(\bar{\bar{y}}) = \frac{N-1}{Nn}S_1^2 + \frac{M-1}{Mnm}S_2^2$

B) Fill in the blanks or true or false. **06**

1) More heterogeneous is the population _____ is the sample size.
2) In cluster sampling, the variance within clusters is _____ between cluster variance.
3) If an investigator select districts from a state, Panchayat samities from districts and farmers from Panchayat samities, then such sampling procedure is known as _____
4) Estimation of sample size for a stratum subject to the prefixed value of $var\ (\bar{X}_{st})$ in stratified sampling is called _____ allocation
5) If information is not available on certain sampling units then it is called as _____.
6) When the population consists of units arranged in a sequence or deck, one would prefer _____.

Q.2 Answer the following **16**

a) Explain the following concepts with respect to random to sampling.
i) Probability sampling
ii) Population
iii) Sample unit
iv) Sampling frame

b) If a simple random sample without replacement of size n clusters is drawn from the population of N clusters each with same size *M,* then derive an unbiased estimator of population mean with its variance in terms of intra class correlation coefficient.

c) If a simple random sample of size n is drawn from a population of N units. Discuss when regression estimator is more precise than ratio estimator assuming sample size n is large with justification.

d) Explain Midzuno sampling design and obtain first order inclusion probability under Midzuno sampling design.

Q.3 Answer the following **16**

a) In stratified random sampling suppose the total cost of sampling is $= \sum_{h=1}^{k} C_h\, n_h$, where C_h is the average cost of surveying a unit in the h[th] stratum. Determine the sample size, n_h, allocated to h[th] stratum such that n_h is proportional to population stratum size, N_h for a given cost of survey. Is estimator $\bar{y}_{st} = \sum_{h=1}^{k} W_h\, \bar{y}_h$ remains unbiased for population mean? Justify your answer. Obtain the mean square error of $\bar{y}_{st}$ under proportional allocation for given cost function.

b) Explain the Neyman allocation, in case of stratified random sampling. Derive the condition on stratum size for which variance of an unbiased estimator of population mean is minimum when sample size is fixed. Hence obtain the minimum variance of an unbiased estimator of population mean.

Q.4 Answer the following **16**

a) In a linear systematic random sampling of size n show that

$$var\left(\bar{y}_{sys}\right) = \frac{N-n}{nN}\, S_{wst}^2\, [1 + (n-1)\rho_{wst}]$$

where is variance among units that lie in the same stratum and is correlation between units that are in the same systematic sample. Hence show that systematic sampling is more precise than stratified random sampling if $\rho_{wst} < 0$.

b) Explain circular systematic random sampling with an illustration. Show that mean of circular systematic random sample is an unbiased estimator of population mean.

Q.5 Answer the following **16**

a) Explain Lahiri's method to obtain probability proportional to size sample of size n. Show that, inclusion probability of i[th] unit is proportional to its size variable.

b) In case of probability proportional to size (PPS) sample drawn with replacement, show that Hansen-Hurwitz estimator is an unbiased estimator of population total. Obtain variance of Hansen-Hurwitz estimator.

Q.6 Answer the following **16**

a) Define Hartley and Ross estimator of population ratio. Show that it is unbiased

b) Obtain variance of an unbiased estimator $\bar{y}$ of population mean when sample is selected without replacement in first-stage as well as in second-stage. Also obtain estimated variance.

Q.7 Answer the following **16**

a) Determine the optimum allocation value n' of first-phase sampling and n_h^* of second-phase so as to minimize variance of $\bar{y}_{std}$, an unbiased estimator of population mean, for specified cost, in double sampling.

b) What do you mean by unit non-response? Discuss the effect of it on the estimation of population mean.

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Asymptotic Inference

Day & Date: Monday, 13-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative **10**

1) An estimator T_n is said to weakly consistent for θ if _____.
   a) $P_\theta\{|T_n - \theta| > \varepsilon\} = 1$
   b) $\lim_{n\to\infty} P\,\theta\{|T_n - \theta| > \varepsilon\} = 1$
   c) $\lim_{n\to\infty} P\,\theta\{|T_n - \theta| > \varepsilon\} = 0$
   d) all the above

2) Consider the following statements:
   1. Joint consistency implies marginal consistency.
   2. Marginal consistency implies joint consistency.

   Which of the above statements is / are true?
   a) Only 1 b) Only 2
   c) both 1 and 2 d) neither 1 nor 2

3) Which one of the following is true for estimation of θ for $U(0,\theta)$ distribution by the MLE _____.
   a) unbiased but not consistent
   b) consistent but not unbiased
   c) both consistent and unbiased
   d) neither consistent nor unbiased

4) For a distribution belonging to one parameter exponential family, -— estimator based on sufficient statistic is CAN for θ.
   a) maximum likelihood b) Moment
   c) both (A) and (B) d) neither (A) nor (B)

5) In a random sample of size n from $N(\theta, 1)$ distribution, MLE of θ was reported to be 1.5. The variance of the asymptotic normal distribution of case of $\sqrt{n}\,(\bar{X}n^2 - \theta^2)$ is given by _____.
   a) 10 b) 9
   c) 4 d) 1

6) Let $X_1, X_2 \dots, X_n$ be iid with $E(X_i^2 = Var(X_i) = \sigma^2$ then asymptotic distribution of $\bar{X}_n$ is _____
   a) $N(0, \frac{\sigma^2}{n})$ b) $N(0, \sigma^2)$
   c) $N(0,1)$ d) $N(0, \frac{1}{n})$

7) If T_1 and T_2 are consistent estimators of θ then we prefer T_1 to T_2 if ___.
   a) ARE $(T_1, T_2) = 1$ b) ARE $(T_1, T_2) > 1$
   c) ARE $(T_1, T_2) < 1$ d) None of these

8) A sequence of estimators T_1,T_2,…..,T_n of $\emptyset(\theta)$ is said to be best asymptotically normal (BAN) if it satisfies the condition _____.
   a) $\sqrt{n}$ [T_n-$\emptyset(\theta)$]~N (0,σ^2) as n→ ∞
   b) T_n is consistent
   c) T_n has minimum variance as compared to the variance of any other estimator is T_n^*
   d) all the above

9) The variance stabilizing transformation for Poisson population is _____.
   a) square root b) Logarithmic
   c) tanh^{-1} d) sin^{-1}

10) Bartlett's test is used to investigate the significant difference between _____ of normally distributed populations.
   a) proportions b) means
   c) variances d) none of these

B) Fill in the blanks. **06**

1) Let $X_1, X_2 \dots, X_n$ be iid from poisson (θ). CAN estimator of $P_\theta(X = 0)$ is __
2) Exponential family is _______ than the Cramer family.
3) If an estimator T_n is consistent for θ then $\Psi(T_{n)}$ is consistent for $\Psi(\theta)$ if Ψ is _____ function.
4) Asymptotic distribution of LRT statistic is _____.
5) Variance stabilizing transformation was introduced by ______.
6) For *Laplace* $(\theta, 1)$ distribution, the asymptotic variance of $\bar{X}_n$ is _____.

Q.2 Answer the following **16**

a) State Cramer regularity conditions.
b) Show that sample mean is consistent estimator of population mean whenever population mean is finite.
c) Describe Rao's score test. State its asymptotic distribution.
d) Based on random sample of size n from Poisson (θ), obtain variance stabilizing transformation of the estimator.

Q.3 Answer the following **16**

a) Define a consistent estimator for a vector parameter. Show that joint consistency is equivalent to marginal consistency.
b) Let $X_1, X_2 \dots, X_n$ be iid $U(0, \theta)$, computing the actual probability show that $X_{(n)}$ is consistent estimator of $U(0, \theta)$.

Q.4 Answer the following **16**

a) Define CAN estimator for a real parameter θ . State and prove invariance property for a CAN estimator.
b) Let $X_1, X_2 \dots, X_n$ be iid exponential with location θ . Examine whether $X_{(1)}$ is CAN for θ.

Q.5 Answer the following **16**

a) Define BAN estimator. Show that sample distribution function at a given point is CAN for the population distribution function at the same point
b) Let $X_1, X_2 \dots, X_n$ be a random sample of size n from the distribution having $pdf\ f(x; \mu, \lambda) = \frac{1}{\lambda} \exp\left[-\left(\frac{x-\mu}{\lambda}\right)\right], x \geq \mu, \lambda > 0$. Obtain moment estimator of (μ, λ) and its asymptotic variance-covariance matrix.

Q.6 Answer the following **16**

a) What is variance stabilizing transformation? Illustrate an application of variance stabilizing transformation in constructing large sample confidence intervals.

b) Based on random sample of size n from exponential distribution with mean $\theta,$ obtain variance stabilizing transformation for MLE of θ . Obtain $100(1-\alpha)$ % confidence interval for θ based on the transformation.

Q.7 Answer the following **16**

a) Derive Bartlett's test for homogeneity of variances of several normal populations.

b) Let $X_1, X_2 \dots, X_n$ be iid B $(1, \theta)$. Let $\Psi(\theta) = \theta(1-\theta)$. Obtain CAN estimator for $\Psi(\theta)$. Discuss its asymptotic distribution at $\theta = \frac{1}{2}$.

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Multivariate Analysis

Day & Date: Tuesday, 14-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Q. Nos.1and 2 are compulsory.
2) Attempt any Three questions from Q.3 to Q.7
3) Figures to the right indicate full marks.

Q.1 A) Choose Correct Alternative. **10**

1) Let $\underline{X} = (X_1, X_2, X_3)$ is a random vector with $\mathrm{Var}(\underline{X}) = \sum$. Eigen values of $\sum$ are 0.6, 0.4 and 0.2. Then proportion of variation explained by the first principal component is _____.
a) 0.2 b) 0.5
c) 0.3333 d) 0.6666

2) Let $\underline{X}$ be a random vector with covariance matrix Σ. A decrease in variances of p variables in $\underline{X}$ will be lead to _____.
a) increase trace(Σ) b) Decrease trace(Σ)
c) Does not affect trace(Σ) d) Nothing can be said

3) Let p dimensional vector $\underline{X}$ has $N_p(\mu, \Sigma)$ distribution. Let us partition $\underline{X} = (\underline{X}_{(1)}, \underline{X}_{(2)})$ in q and p-q component sub vectors. Then conditional variance-covariance matrix of $\underline{X}_{(2)}$ given $\underline{X}_{(1)}$ is _____.
a) $\Sigma_{11} - \Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21}$
b) $\Sigma_{22} - \Sigma_{21}\,\Sigma_{11}^{-1}\,\Sigma_{12}$
c) $\Sigma_{12} - \Sigma_{12}\,\Sigma_{22}^{-1}\,\Sigma_{21}$
d) $\Sigma_{21} - \Sigma_{21}\,\Sigma_{11}^{-1}\,\Sigma_{12}$

4) Which of the following statistical techniques identifies homogenous subgroups?
a) Factor analysis b) Multivariate analysis of variance
c) Cluster analysis d) Discriminant analysis

5) Let $\underline{X}_1, \underline{X}_2, \_\_\_\_, \underline{X}_n$ be a random sample of size n from p-variate normal distribution with mean vector μ and covariance matrix $\sum$. The distribution of mean vector $\underline{\bar{X}}$ is
a) $N_p(\mu, \frac{1}{n}\sum)$ b) $N_p(\mu, \sum)$
c) $N_p(\mu, \frac{1}{n-1}\sum)$ d) $N_p(\frac{1}{n}\mu, \frac{1}{n}\sum)$

6) _____ is a clustering procedure where all objects start out in one giant cluster
a) Divisive clustering b) Non-hierarchical clustering
c) Agglomerative clustering d) Single linkage clustering

7) If $\underline{X}$ has $N_p(\mu, \sum)$ distribution then linear combination $Z = a\,\underline{X}$ has _____ distribution.
a) $N_p(\mu, \sum)$ b) $N_p(\mu, a\sum a')$
c) $N(a'\mu, a'\sum a)$ d) $N(a'\mu a, a'\sum a)$

8) The mean vector of $(X_1 + X_2, X_1 - X_2)$ is (10,0) then mean vector of $(X_1, 2X_1 - X_2)$ is ______.
   a) (5,10) b) (10,0)
   c) (10,5) d) (5,5)

9) For a multivariate normal random vector, the variance-covariance matrix is always ______.
   a) symmetric b) square matrix
   c) non-negative definite d) All of these

10) Let A has $W_p(n, \sum)$ distribution and B is a $(q \times p)$ matrix then distribution of BAB' is ______.
   a) $W_p(n, \sum)$ b) $W_q(n, \sum)$
   c) $W_p(n, B\sum B')$ d) $W_q(n, B\sum B')$

B) Fill in the blanks **06**

1) A ______ is a graphical device for displaying clustering results.
2) As the distance between two populations increases, misclassification error _______.
3) The ______ principal component explains least variation of the data.
4) At the start of divisive clustering, we assume total ______ clusters.
5) Partitioning clustering uses ______ approach of clustering.
6) With usual notations, Fisher's best discriminant function is given by ______.

Q.2 Answer the following. **16**

1) Describe ECM rule in discriminant analysis.
2) Obtain moment generating function of multivariate normal distribution.
3) Define variance-covariance matrix. State its properties.
4) Show that two p-variate normal vectors $\underline{X_1}$ and $\underline{X_2}$ are independent if and only if cov $\left(\underline{X_1}, \underline{X_2}\right) = 0$

Q.3 Answer the following.

a) State multivariate normal density. Find mean vector and variance-covariance matrix for this density. **08**

b) Find maximum likelihood estimator of $\sum$ based on a random sample from multivariate normal distribution $N_p(\mu, \sum)$. **08**

Q.4 Answer the following.

a) If $\underline{X} \sim N_p(\underline{\mu}, \sum)$, then find the distribution of the following: **08**
   1) $\underline{a}'\underline{X}$, where $\underline{a}$ is a p-dimensional vector of constants.
   2) $A\underline{X}$, where A is matrix of order $m \times p$

b) Differentiate between hierarchical and non-hierarchical clustering methods. Explain, in detail, k-means clustering **08**

Q.5 Answer the following.

a) Explain the idea of discriminant analysis. What are the potential errors involved in it? Obtain the classification rule for the case of two populations with densities $f_1(\underline{x})$ and $f_2(\underline{x})$. **08**

b) Derive the density of multivariate normal distribution **08**

Q.6 Answer the following.

a) Describe canonical variable and canonical correlations. State and prove any two properties of canonical variables. **08**

b) Describe agglomerative clustering in detail. Illustrate with the help of example using complete linkage method. **08**

Q.7 a) Discuss principal components analysis. How it can be used as a dimension reduction technique? **08**

b) Define: **08**

1) Distance matrix
2) Single linkage
3) Complete linkage
4) Average linkage

SLR-GR-14

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Planning and Analysis of Industrial Experiments

Day & Date: Wednesday, 15-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) The error degrees of freedom in one-way classification model $Y_{ij} = \mu + \alpha_i + \epsilon_{ij};\ i = 1,2,\ldots\ldots v,\ j = 1,2,\ldots\ldots n_i$ and assumptions on errors are followed; are ______.
   a) $n-1$ b) $v-1$
   c) $n-v$ d) $v-n$

2) In a 2^2 factorial experiment with, the contrast due to interaction effect AB is ______.
   a) [(a) + (ab) + (1) - (b)] b) [(ab) + (a) - (1) - (b)]
   c) [(1) + (ab) - (a) - (b)] d) [(a) + (ab) + (1) + (b)]

3) A BIBD is always ______.
   a) disconnected b) connected
   c) orthogonal d) complete

4) Which of the following is two-way ANOCOVA model with single covariate?
   a) $Y_{ij} = \mu + \alpha_i + \gamma z_{ij} + \epsilon_{ij}$
   b) $Y_{ij} = \mu + \alpha_i * \beta_j + z_{ij} + \epsilon_{ij}$
   c) $Y_{ij} = \mu + \alpha_i + \beta_j + \gamma z_{ij} + \epsilon_{ij}$
   d) $Y_{ij} = \mu + \alpha_i + \beta_j - z_{ij} + \epsilon_{ij}$

5) In a 2^6 experiment, number of two - factor interaction effects are _____.
   a) 6 b) 32
   c) 15 d) 63

6) In a RBD with 5 treatments and 4 blocks, the number of experimental units in each block are ______.
   a) 6 b) 4
   c) 5 d) 19

7) In resolution IV design all main effects are ______.
   a) Strongly Clearly estimable
   b) Clearly estimable
   c) not clearly estimable
   d) not estimable

8) The rank of C matrix is _____.
   a) v + 1 b) v - 1
   c) v d) v - 2

9) In one - way ANOVA model, the error sum of squares are estimates of ______.
   a) within treatment variation b) between treatment variation
   c) total variation d) none of these

10) In a 3^2 experiment, main effects has _____ degrees of freedom.
   a) 2 b) 4
   c) 6 d) 9

B) Fill in the blanks: **06**

1) Replication, randomization and ______ are three basic principles of design of experiments.
2) In a factorial experiment _____ and_____ effects are more important.
3) In single replicate design error has _____ degrees of freedom.
4) In total confounding _____ effect is confounded in _____ replicates.
5) In a 2^{5-1} fractional factorial experiment with defining relation I = ABCD, it resolution ______ design.
6) The total number of effect in 2^5 experiment are ______.

Q.2 Answer the following **16**

a) Define a connected block design. Show that RBD is a connected design.
b) Write down lay out of 2^3 experiment in two replicates.
c) Define BIBD. Show that in a $\mathrm{BIBD}\ (\mathrm{v}, \mathrm{b}, \mathrm{r}, \mathrm{k}, \lambda), \mathrm{NN}' = [\{\mathrm{r} - \lambda)\mathrm{I}_{\mathrm{v}} + \lambda \mathrm{E}_{\mathrm{vv}}]$.
d) In one-way ANOVA model; obtain the least square estimates of parameters.

Q.3 Answer the following. **16**

a) Obtain reduced normal equations for estimating treatment effects in general block design.
b) Obtain the least square estimates of parameters in the following model -
$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_{ij} + \varepsilon_{ijk}, i = 1,2, \ldots\ldots v, j = 1,2, \ldots\ldots b. k = 1,2, \ldots r. \varepsilon_{ijk} \sim N(0, \sigma^2)$

Q.4 Answer the following. **16**

a) Derive the necessary and sufficient condition for orthogonality of a given block design.
b) In general block design state and prove the properties of Q, where
$\mathrm{Q} = \mathrm{T} - \mathrm{NK}^{-\delta}\ \mathrm{B}$

Q.5 Answer the following. **16**

a) Obtain two blocks of 2^5 factorial experiments using suitable generator
b) Derive 1/4 fraction of 2^6 experiment and write its consequences

Q.6 Answer the following. **16**

a) Define resolution of design and minimum aberration design. Give illustrative example of each.
b) In a connected block design prove that rank (C) = v - 1 and hence rank of estimation space is v + b - 1.

Q.7 Answer the following. **16**

a) Prove that dual of a symmetric BIBD is also a symmetric BIBD.
b) Derive the test for testing hypothesis of equality of all treatment effects in one-way classification model.

SLR-GR-15

Seat No.	

Set P

M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Regression Analysis

Day & Date: Thursday, 16-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

Instructions: 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

Q.1 A) Multiple choice questions. **10**

1) In simple linear regression model $Y = \beta_0 + \beta_1 X + \epsilon$, β_0 and β_1 are respectively _____.
   a) slope and intercept b) error and slope
   c) intercept and slope d) intercept and error

2) In a multiple linear regression model with $\varepsilon \sim N(0, \sigma^2 I)$, the distribution of residual vector e is ______.
   a) $N(0, H\sigma^2)$ b) $N(0, (I-H)\sigma^2)$
   c) $N(0, \sigma^2 I)$ d) $N(0, (X'X)^{-1}\sigma^2)$

3) Which one of the statement is true regarding residuals in regression analysis?
   a) Mean of residuals is always zero
   b) Mean of residuals is always less than zero
   c) Mean of residuals is always greater than zero
   d) There is no such rule for residuals

4) To test significance of an individual regression coefficient in multiple linear regression model ______ is used.
   a) F test b) t test
   c) Z test d) $\mathcal{X}^2$ test

5) The coefficient of determination (R^2) is the square of correlation coefficient between (where Y is response) ______.
   a) Y and hat matrix b) Y and its predicted value
   c) regressors d) none of these

6) Backward elimination process begins with the assumption that _____.
   a) no regressors are in the model
   b) some regressors are in the model
   c) all regressors are in the model
   d) none of these

7) Multicollinearity is concerned with ______.
   a) correlation among predictors
   b) correlation among error terms
   c) correlation between response and predictors
   d) none of these

8) Orthogonal polynomials are used to fit a polynomial model of _____.
   a) first order in one variable
   b) second order in two variables
   c) any order in two variables
   d) any order in one variable

9) Logistic regression model is an appropriate model when response variable is distributed as ______.
   a) Poisson b) Binomial
   c) Normal d) Gamma

10) If a response variable in a GLM follows Binomial distribution, then _____ link function is suitable.
   a) $\log\theta$ b) $-\log\theta$
   c) $1/\theta$ d) $\log(\theta/1-0)$

B) Fill in the blanks. **06**

1) In a multiple linear regression model with k regressors, the distribution of $(SS_{\mathrm{Reg}}/\sigma^2)$ is ______.
2) The proportion of variation explained by the regression model is measured by ______.
3) $\mathrm{E}(\mathrm{C_p}\ /\ \mathrm{Bias}=0)=$ ______.
4) The largest condition index of (X' X) is defined as ______.
5) In usual notations, $\mathrm{a}(\emptyset)$ for normal distribution is always equal to _____.
6) The joint points of pieces in polynomial fitting are usually called _____.

Q.2 Answer the following **16**

a) Discuss variance stabilizing transformation and its use.
b) With usual notations, show that $\mathrm{Cov}\ (\hat{y}, e) = 0$
c) Discuss Variance inflation factor (VIF) method for detection of multicollinearity.
d) Discuss the logistic regression model. Give a real-life situation where this model is appropriate.

Q.3 Answer the following. **16**

a) Define multiple linear regression model and obtain the least squares estimates of its parameters.
b) In the usual notations, outline the procedure of testing a general linear hypothesis $\mathrm{T}\beta = 0$.

Q.4 Answer the following. **16**

a) Explain the concept of non-linear regression model. Discuss least squares method for estimation of parameters for non-linear regression model.
b) Describe forward selection method for variable selection and state its limitations.

Q.5 Answer the following. **16**

a) State the autocorrelation problem. Explain Durbin-Watson test for detecting autocorrelation. What are its limitations?
b) Explain the residual plots. Outline the procedure of construction of normal probability plot and procedure for checking normality assumption.

Q.6 Answer the following. **16**

a) Define k^{th} order polynomial regression model in one variable. Describe orthogonal polynomial to fit the polynomial model in one variable.

b) Define a one parameter natural exponential family. Show that the $B(n, \theta), \theta \in [0,1]$ is member of natural exponential family.

Q.7 Answer the following.

a) Derive the maximum likelihood estimators of parameters of a logistic regression model with one covariate. **16**

b) Define 'Deviance statistic'. Find it when data comes from normal distribution with mean μ and variance σ^2.

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Data Mining

Day & Date: Monday, 20-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternatives from the options. 10

1) The part of the entire data, which is used for building the model is called as ______.
   a) Training data b) Testing data
   c) Irrelevant data d) Residual data

2) Which one is example of case based learning?
   a) Decision Tree b) K-Nearest neighbor
   c) Genetic algorithm d) Neural networks

3) A 100% efficient decision tree has 3 independent variables and 1 class label, each with two categories, then maximum possible number of end nodes (at extreme bottom) is ______.
   a) 2 b) 4
   c) 8 d) 16

4) Looking for combinations of items purchased together is called _.
   a) market data analysis
   b) market basket analysis
   c) marketing data analysis
   d) Combo analysis

5) Which of the following is a type of activation function?
   a) Linear b) Non-Linear
   c) Sigmoid d) All of these

6) Naive Bayesian classifier uses _______ tool.
   a) information gain b) Probability
   c) Both (a) and (b) d) None of these

7) Task of inferring a model from unlabeled training data is called ______.
   a) supervised learning
   b) unsupervised learning
   c) Reinforcement learning
   d) None of these

8) The things that customers actually purchase are known as _____.
   a) Items b) Support
   c) Transaction d) Function

9) Support vector machine is ______.
   a) unsupervised learning
   b) supervised learning
   c) reinforcement learning
   d) genetic algorithm

10) Which one is non-hierarchical clustering algorithm?
   a) Agglomerative clustering
   b) Divisive clustering
   c) k-means clustering
   d) All of these

B) Fill in the blanks. **06**

1) _____ is the type of machine learning in which machines are trained using well "labelled" training data.
2) The ______ algorithm of supervised learning is known as ‘Lazy learning algorithms.
3) In data mining, ANN stands for ______.
4) SVM is an abbreviation for _______.
5) With usual notations, the formula for accuracy is $\frac{\cdots}{P+N}$
6) Unlike regression problem, the type of class label in classification problem is ______.

Q.2 Answer the following. **16**

a) Discuss, with illustration, the concept of supervised learning.
b) What are the advantages of unsupervised learning?
c) Discuss accuracy and precision of a classifier.
d) Describe the problem of imbalanced data.

Q.3 Answer the following. **16**

a) Discuss Bayesian classifier. Also explain why it is called as naive classifier.
b) Describe decision tree classifier in detail.

Q.4 Answer the following. **16**

a) Discuss the working mechanism of ANN.

b) Discuss logistic regression classifier in detail.

Q.5 Answer the following. **16**

a) What are the advantages and disadvantages of supervised learning?

b) Discuss confusion matrix in detail.

Q.6 Answer the following. **16**

a) Discuss the different metrics for Evaluating Classifier Performance.

b) Describe supervised learning method. Also explain SVM classifier.

Q.7 Answer the following. **16**

a) Discuss characteristics of kNN classifier.

b) Describe-

1) Sensitivity of a model
2) Specificity of a model

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Industrial Statistics

Day & Date: Tuesday, 21-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. **10**

1) The control limits of the p and np charts are based on the assumption that the number of nonconforming items follows ______ distribution.
   a) normal b) binomial
   c) Poisson d) geometric

2) Usually 2σ limits are called as ______.
   a) specification limits b) control limits
   c) warning limits d) action limits

3) Assignable causes are ______.
   a) not as important as natural causes
   b) within the limits of control chart
   c) also referred to as chance causes
   d) causes of variation that can be identified and removed

4) To determine location of a defect, which of the following tool is used?
   a) Defect concentration diagram
   b) Check sheet
   c) Scatter diagram
   d) Pareto chart

5) Producer's risk is the probability of _______.
   a) accepting a good lot b) rejecting a good lot
   c) rejecting a bad lot d) accepting a bad lot

6) Which of these is not a part of magnificent seven of SPC?
   a) Single sampling plan b) Pareto chart
   c) Check Sheet d) Scatter Diagram

7) The capacity index C_p involves ______ parameter(s) to be estimated.
   a) only μ b) only σ
   c) both μ and σ d) none of these

8) Tabular method is used to implement ______ chart.
   a) CUSUM b) EWMA
   c) Moving average d) CRL

9) In demerit system, the unit will not fail in service but has minor defects in finish or appearance is classified as ______ defects.
   a) class A b) class B
   c) class C d) class D

10) In most acceptance sampling plans, when a lot is rejected, the entire lot is inspected and all defective items are replaced. When using this technique, the AOQ ______.
    a) becomes a larger fraction
    b) becomes a smaller fraction
    c) is not affected
    d) none of these

B) Fill in the blanks. **06**

1) The control chart designed to deal with the defects or nonconformities of a product is called ______.
2) An out-of-control signal given by a control chart when the process is actually in-control is called ______.
3) The concept of Six-Sigma was developed by ______ company.
4) For the centered process, the relation between capability index C_p and probability of nonconformance p is given by ______.
5) V-mask procedure is used to implement ______ chart.
6) In the development of $\bar{X}$ and S charts, the distribution of quality characteristic X is assumed to be ______.

Q.2 Answer the following **16**

a) Explain chance and assignable causes of variation.
b) Explain Ishikawa diagram with suitable example.
c) Write a short note on Conforming run length (CRL) chart.
d) Explain the following terms:
1) Acceptance Quality Level (AQL)
2) Lot Tolerance Percentage Defective (LTPD)

Q.3 Answer the following **16**

a) Discuss various definitions of 'Quality' and various dimensions of quality.
b) Discuss the various steps involved in the construction of $\bar{X}$ and S charts.

Q.4 Answer the following **16**

a) Explain the variable sampling plan when upper specification is given and standard deviation is known.
b) Define OC function and ARL of a control chart. Obtain the same for $\bar{X}$ chart assuming normality of process with known standards.

Q.5 Answer the following **16**

a) Describe double sampling plan for attributes. Obtain AOQ and ASN for the same.
b) Stating the underlying assumptions, explain the construction and operation of a sequential sampling plan for attributes.

Q.6 Answer the following **16**

a) Stating the assumptions clearly, define index C_p. Interpret $C_p = 1$. Obtain $(1-\alpha)$ level confidence interval for C_p.
b) What is CUSUM chart? Explain its construction and operation.

Q.7 Answer the following **16**

a) Describe the development and operation of Hotelling's T^2 chart to monitor process mean vector.
b) Explain SIX SIGMA methodology and DMAIC cycle in detail.

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
(STATISTICS)
Reliability and Survival Analysis

Day & Date: Wednesday, 22-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Choose the correct alternative. **10**

1) The i^{th} component of a system is relevant if ______.
   a) $\phi(1_i, \underline{x}) = 1 \; and \; \phi(0_i, \underline{x}) = 1$
   b) $\phi(1_i, \underline{x}) = 0 \; and \; \phi(0_i, \underline{x}) = 0$
   c) $\phi(1_i, \underline{x}) = 1 \; and \; \phi(0_i, \underline{x}) = 0$
   d) $\phi(1_i, \underline{x}) = 0 \; and \; \phi(0_i, \underline{x}) = 1$

2) A vector $\underline{X}$ is called path vector if ______.
   a) $0 \leq \phi(\underline{X}) \leq 1$ b) $\phi(\underline{X}) = 1$
   c) $\phi(\underline{X}) = 0$ d) $\phi(\underline{X}) = 0.5$

3) If distribution F is $IFRA$ then ______ is star shaped.
   a) hazard function b) $\log R\,(t)$
   c) $-\log R\,(t)$ d) $1/R(t)$

4) The minimal path sets of structure ϕ are ______ for its dual.
   a) minimal path vectors b) minimal cut sets
   c) minimal path sets d) none of the above

5) Suppose $R(t)$ is the reliability of series system of two components having reliabilities $R_1(t)$ and $R_2(t)$ respectively then ______.
   a) $R(t) = Max\{R_1(t), R_2(t)\}$ b) $R(t) = Min\{R_1(t), R_2(t)\}$
   c) $R(t) \leq Min\{R_1(t), R_2(t)\}$ d) $R(t) < Max\{R_1(t), R_2(t)\}$

6) In type II censoring ______.
   a) the number of failures is fixed
   b) the time of an experiment is fixed
   c) both time and number of failures is fixed
   d) none of these

7) Which of the following distribution has no ageing property?
   a) lognormal b) exponential
   c) gamma d) none of these

8) A distribution function $F(t)$ said to have new better than used (NBU) if ______.
   a) $\overline{F}(t+x) \geq \overline{F}(t)\overline{F}(x)$ b) $\overline{F}(t+x) \leq \overline{F}(t)\overline{F}(x)$
   c) $\overline{F}(t+x) = \overline{F}(t)\overline{F}(x)$ d) none of the above

9) In survival analysis, the outcome variable is ______.
   a) continuous b) discrete
   c) dichotomous d) none of the above

10) The censoring time is identical for every censored observation in ___.
   a) right random censoring
   b) type I censoring
   c) type II censoring
   d) both type I and type II censoring

B) Fill in the blanks. **06**

1) As the number of components n increases, the reliability of parallel system ______.
2) Series system of n components has ______ minimal path sets.
3) IFRA property is preserved under ______.
4) The hazard function ranges between ______.
5) To find exact confidence interval for mean of exponential distribution under no censoring, the pivotal quantity has ______ distribution.
6) A sequence of (2x2) contingency tables is used in ______ test.

Q.2 Answer the following. **16**

a) Define:
   1) Structure function
   2) Coherent structure. Illustrate giving one example each.

b) Write a short note on star shaped function.

c) Explain the following terms:
   1) Survival function
   2) Random censoring

d) Write a short note on empirical survival function and its properties.

Q.3 Answer the following. **16**

a) If failure time of an item has gamma distribution obtain the failure rate function.

b) Define Poly function of order 2 (PF_2). Prove that if $f \in PF_2$ then $F \in IFR.$

Q.4 Answer the following. **16**

a) Define NBU and NBUE classes of distributions. Prove that $F \in IFRA \Rightarrow F \in NBU.$

b) If failure time of an item has the distribution
$f(t) = \frac{\lambda^\alpha}{\Gamma\alpha} t^{\alpha-1} e^{-\lambda t}, t > 0, \;\; \lambda, \alpha > 0$
Examine whether it belongs to IFR or DFR.

Q.5 Answer the following **16**

a) Obtain the nonparametric estimator of survival function based on complete data. Also obtain confidence band for the same using Kolmogorov-Smirnov statistic.

b) Obtain maximum likelihood estimate of mean of the exponential distribution under type II censoring.

Q.6 Answer the following **16**

a) Describe Kaplan-Meier estimator and derive an expression for the same.

b) Describe Mantel's technique of computing Gehan's statistics for a two-sample problem for testing equality of two life distributions.

Q.7 Answer the following **16**

a) Derive Greenwood's formula for an estimate of variance of actuarial estimator of survival function.

b) Define:
1) k-out-of- n system.
2) Dual of a structure function.

Obtain the dual of k-out-of- n system

SLR-GR-20

Seat No.	

Set P

M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov - 2022
(STATISTICS)
Optimization Techniques

Day & Date: Thursday, 23-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

Instructions: 1) Q. Nos. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

Q.1 A) Fill in the blanks by choosing correct alternatives given below. 10

1) Which of the following is correct?
   a) A linear programming problem with only one decision variable restricted to integer value is not an integer programming problem.
   b) An integer programming problem is an LLP with all decision variable are restricted to integers.
   c) Pure IPP is one where all decision variable are restricted to integers.
   d) None of the above

2) The zero -one programming problem requires ______.
   a) Decision variables to have values either 0 or 1.
   b) The decision variables have coefficients between 0 and 1.
   c) All constraints have coefficients between 0 and 1.
   d) All of the above

3) Given a system of m simultaneous linear equations with n unknowns (m<n). The number of basic variables will be ______.
   a) n b) m
   c) n-m d) none of the above

4) If $X'QX$ is positive semi definite then, it is ______.
   a) Strictly convex b) Strictly concave
   c) Convex d) Concave

5) In two person zero sum game is said to be fair if ______.
   a) The upper value and lower value of the game are not equal
   b) The upper value is more than lower value of the game
   c) The upper value and lower value of the game are same and equal to zero.
   d) None of the above

6) To maintain optimality of current optimum solution for a change Δc_k in the coefficient c_K of non basic variable, we must have ______.
   a) $\Delta c_k = z_K - c_k$ b) $\Delta c_k \geq z_K - c_k$
   c) $\Delta c_k \leq z_K - c_k$ d) $\Delta c_k = z_K$

7) If we delete one of the constraint from LP problem then ______.
   a) Feasible solution space is enlarged
   b) Feasible solution space is reduced
   c) It may or may not affect on feasible solution space
   d) None of these

8) Integer linear programming problem means ______.
   a) It linear programming problem with additional constraint only one decision variable is integer
   b) It linear programming problem with additional constraint all or some of the decision variables are integers
   c) Decision variables takes only 0 or 1 value
   d) Coefficients in objective function are integers

9) In Gomory's cutting plane method each cut involves introduction of ______.
   a) An equality constraints
   b) Less than equal to constraint
   c) Greater than equal to constraint
   d) An artificial variable

10) When all the players of the game follow their optimal strategies, then the expected pay off of the game is called ______.
   a) Gain of the game   b) Loss of the game
   c) Value of the game   d) None of these

B) Fill in the blanks. **03**
1) Feasible solution to an LPP must satisfied ______.
2) Simplex method starts with ______.
3) Saddle point of the pay off matrix is a point satisfied ______.

C) State whether following statements are true or false. **03**
1) Primal problem has optimal solution then dual problem has optimal solution.
2) Basic feasible solution to LLP is not unique.
3) Necessary and sufficient condition for a basic feasible solution to a minimization LPP to an optimum is (for all j) $z_j - c_j \leq 0$.

Q.2 Answer the following **16**
a) Describe graphical method to solve LPP.
b) Describe two phase method to solve LPP.
c) Define the following terms
1) Convex set
2) Convex combinations
3) Convex polyhedral
d) Write a note on non linear programming problem.

Q.3 a) State and prove fundamental theorem on duality. **08**
b) Use simplex method to solve the following LPP **08**

$$
\begin{aligned}
&Max\ Z = 5X_1 + 3X_2\\
&sub\ to \quad X_1 \leq 4,\\
&X_2 \leq 3,\\
&X_1 + 2X_2 \leq 18,\\
&X_1 + X_2 \leq 9,\\
&X_1, X_2 \geq 0;
\end{aligned}
$$

Q.4 a) Describe simplex method in detail to solve LPP. **08**
b) Solve the following LPP using two phase method. **08**

$Maximize\ z = 5X_1 + 3X_2$

subject to

$$
\begin{aligned}
2X_1 + X_2 &\leq 1\\
X_1 + 4X_2 &\geq 6\\
X_1, X_2 &\geq 0
\end{aligned}
$$

Q.5 **a)** Explain Gomory's cutting plane method to solve all IPP. **08**

b) Explain in brief the Wolfe's method to solve QPP. **08**

Q.6 **a)** With usual notations for rectangular game problem prove that **08**

$$\max\min \bar{v} \leq \min\max \underline{v}$$

b) Describe how $m\ X\ n$ rectangular game problem is converted into a linear programming problem. **08**

Q.7 **a)** Solve the game with payoff matrix using graphical method. **08**

$$\begin{bmatrix} 2 & 7 \\ 3 & 5 \\ 11 & 2 \end{bmatrix}$$

b) Solve the following LPP using Big M method **08**

$$Max\ Z = 2x_1 + 5x_2$$

Subject to the constraints:

$$3x_1 + 2x_2 \geq 6,$$
$$2x_1 + x_2 \leq 6,$$
$$x_1, x_2 \geq 0.$$