# SLR-GO-1

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**Set P**

## M.Sc. (Semester-I) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Number Theory

Day & Date: Monday, 13-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) For positive integers a and b, lcm(a, b) = a.b iff _____.
   a) $a \nmid b$ b) $b \nmid a$
   c) $\gcd(a, b) = 1$ d) $\gcd(a, b) = ab$

2) If $ca \equiv cb \pmod{n}$ and $\gcd(c, n) = d$ then _____.
   a) $a \equiv b \pmod{n}$ b) $a \equiv b \pmod{d}$
   c) $a \equiv b \pmod{nd}$ d) $a \equiv b \left(\mathrm{mod}\ \frac{n}{d}\right)$

3) For n > 1, the sum of positive integers less than 'n' and relatively prime to 'n' is _____.
   a) $\frac{n.\varphi(n)}{2}$ b) $n.\varphi(n)$
   c) $\frac{\varphi(n)}{2}$ d) $\varphi(n)$

4) Which of the following is true?
   a) $\varphi(n)$ is even for only finitely many values of n
   b) $\varphi(n)$ is always an odd number
   c) $\varphi(n)$ is even for infinitely many values of n
   d) $\varphi(n)$ is always an even number

5) If p is a prime and $d|p-1$ then the congruence $x^d - 1 \equiv 0 (mod\ p)$ has _______ solutions.
   a) exactly p b) exactly d
   c) more than d d) pd

6) Which of the following is not a square free integer?
   a) 105 b) 30
   c) 40 d) 10

7) Consider the statements:
   I) If $a^k \equiv b^k \pmod{m}$ then $a \equiv b \pmod{m}$ for all $k \geq 1$
   II) If $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ then $a + c \equiv b + d \pmod{m}$
   a) only I is true b) only II is true
   c) both I and II are true d) both I and II are false

8) Which of the following is a perfect square?
   a) (10000)! b) (95)!
   c) (40)! d) none of these

9) If ‘p’ is a prime number and ‘a’ be an integer such that p|a then which of the followings are true?
   a) $a^p \equiv a \pmod p$  b) $a^p \equiv 0 \pmod p$
   c) $a \equiv 0 \pmod p$  d) all of these

10) For any positive integer $n$, $\varphi(n) =$ ______.
   a) $n\sum_{d|n} \frac{\mu(d)}{d}$  b) $n\sum_{d|n} \mu(d)$
   c) $\sum_{d|n} \frac{\mu(d)}{d}$  d) $d\sum_{d|n} \frac{\mu(d)}{n}$

**B) Fill in the blanks.** **06**

**1)** The largest integer value of $[\pi]$ is _____.

**2)** If ‘a’ has order k (mod n) then $a^h$ has order k (mod n) iff _____.

**3)** The system of linear congruences $ax + by \equiv r \pmod n$ and $cx + dy \equiv s \pmod n$ has a unique solution $\pmod n$, whenever _____.

**4)** If gcd(1769,2378)=1769x+2378y then by Euclidean algorithm the values of x and y are ______.

**5)** The last two digits in decimal representation of $3^{256}$ are _____.

**6)** The congruence $x^2 \equiv -1 \pmod p$, p is a prime, has a solution if and only if _____.

**Q.2 Answer the following** **16**

**a)** If $f(n) = n^2 + 2$ and $n = 6$ then show that $\sum_{d|6} f(d) = \sum_{d|6} f\left(\frac{6}{d}\right)$

**b)** Show that the product of any three consecutive integers is divisible by $3!$.

**c)** Solve the congruence $x^{17} \equiv 7 \pmod{19}$.

**d)** Prove that $a \equiv b \pmod n$ iff $a$ and $b$ have the same remainders with respect to $n$.

**Q.3 Answer the following.** **16**

**a)** If $m$ and $n$ are relatively prime then prove that $\varphi(mn) = \varphi(m)\varphi(n)$ and find $\varphi(5040)$.

**b)** State and prove Fermat’s theorem.

**Q.4 Answer the following.** **16**

**a)** Explain Fermats Factorization method and Factorize 340663.

**b)** Find the gcd and lcm of 527 and 765. Express the gcd as a linear combination of 527 and 765.

**Q.5 Answer the following.** **16**

**a)** If ‘a’ is a primitive root modulo $n$ and $b, c$ and $k$ are any integers then show that
   1) If $b \equiv c \pmod n$ then $\text{ind.}\, b \equiv \text{ind.}\, c \left(\text{mod}\ \varphi(n)\right)$
   2) $\text{ind.}(bc) \equiv \text{ind.}\, b + \text{ind.}\, c \ (\text{mod}\ \varphi(n))$
   3) $\text{ind.}\, b^k \equiv k\, \text{ind.}\, b (\text{mod}\ \varphi(n))$
   4) $\text{ind.}\, 1 \equiv 0 (\text{mod}\ \varphi(n))$

**b)** If a and b are any two integers not both zero then show that there exists integers x and y such that gcd(a,b) = ax + by

**Q.6 Answer the following.** **16**

**a)** State and prove Fundamental theorem of Arithmetic.

**b)** Show that the integer $2^n$ has no primitive root for $n \geq 3$

**Q.7 Answer the following.** **16**

**a)** Solve the system of linear congruence's.

$x \equiv 5 \pmod{6}, x \equiv 4 \pmod{11}, x \equiv 3 \pmod{17}$

**b)** If $f$ and $F$ be two number theoretic functions related by the formula $F(n) = \sum_{d|n} f(d)$ then show that

$$f(n) = \sum_{d|n} \mu(d) F\left(\frac{n}{d}\right) = \sum_{d|n} \mu\left(\frac{n}{d}\right) F(d)$$

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# M.Sc. (Semester-I) (New) (CBCS) Examination: Oct/Nov-2022
# (MATHEMATICS)
# Object Oriented Programming Using C++

Day & Date: Monday, 13-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) Conditional operator (?:) is a handy operator which acts as a shortcut for ______.
   a) if-else statement b) switch statement
   c) break statement d) goto statement

2) A derived class with only one base class, is called _____ inheritance.
   a) multiple b) single
   c) multilevel d) hierarchical

3) The mechanism of giving special meaning to an operator is known as ______ overloading.
   a) operator b) function
   c) pointer d) constants

4) _____ are operators that are used to format the data display.
   a) Bitwise b) Denominator
   c) Relational d) Manipulators

5) The _____ operator can be used to create objects of any type.
   a) create b) start
   c) new d) initialize

6) _____ means the ability to take more than one form.
   a) Inheritance b) Abstraction
   c) Polymorphism d) None of these

7) A _____ is a collection of objects of similar type.
   a) object b) class
   c) polymorphism d) inheritance

8) _____ is used to declare integer data type.
   a) int b) integer
   c) Integer d) INT

9) _____ is the process by which objects of one class acquire the properties of another class.
   a) Encapsulation b) Abstraction
   c) Polymorphism d) Inheritance

10) Destructor has a same name as the constructor and it is preceded by?
   a) ! b) ?
   c) ~ d) $

**B) Write True or False.** **06**

1) Polymorphism means the ability to take more than one form.
2) Inheritance is a collection of objects of similar type.
3) Float is used to declare float data type.
4) Objects are the basic run-time entities.
5) An inline function is a function that is expanded in multiple lines when it is invoked.
6) Constructors should be declared in the public section.

**Q.2 Answer the following** **16**

**a)** What is Algorithm? Explain with suitable example.
**b)** Explain the use of static data member with example.
**c)** What is Arrays of Objects? Explain memory representations of arrays of objects.
**d)** What is Constructor? Explain the use of constructors with default arguments.

**Q.3 Answer the following.** **16**

**a)** What is operator? Explain different types of operators used in C++.
**b)** What is inline function? Explain importance of inline function with example.

**Q.4 Answer the following.** **16**

**a)** What is function overloading? Explain with syntax and example.
**b)** What is inheritance? Explain different types of inheritances.

**Q.5 Answer the following.** **16**

**a)** What is Template? Explain different types of templates.
**b)** Write a C++ program to implement function overloading (assume your own data).

**Q.6 Answer the following.** **16**

**a)** What is Manipulator? Explain different types of manipulators.
**b)** What is meant by C++ stream classes? Explain C++ stream classes.

**Q.7 Answer the following.** **16**

**a)** Explain the use of following statements with syntax and example.
   1) width()
   2) precision()
   3) fill()
   4) setf()

**b)** Write a C++ program to implement multiple Inheritance. (assume your own data).

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# M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
# (MATHEMATICS)
# Algebra - I

Day & Date: Tuesday, 14-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) Consider the following statements.
P : Every normal series is subnormal
Q : Every composition series is normal series
Then,
a) P is true but Q is false b) P is false but Q is true
c) Both P and Q is true d) Both P and Q is false

2) Which of the following is true in a commutative ring with unity R?
a) Every maximal ideal is prime
b) R is an integral domain
c) R has no zero divisors
d) Every prime ideal is maximal

3) < 2Z, +, * > is not an integral domain because ______.
a) it has zero divisors b) it has unit element
c) it has no unity d) none of these

4) If G is a group then which of the following necessarily imply that G'= {e} ______.
a) G is non abelian b) G is abelian
c) G is cyclic d) None of these

5) If a group G is infinite cyclic group, then number of generators of G is ____.
a) 0 b) 1
c) 2 d) infinite

6) If D is a Unique Factorization Domain, then ______.
a) D[x] is Unique Factorization Domain
b) D[x] is need not be Unique Factorization Domain
c) D[x] is Euclidean domain
d) D[x] is Principal ideal domain

7) If $\mathbb{F}$ is a field, then ______.
a) $\mathbb{F}$ [x] is Field
b) $\mathbb{F}$ [x] is not an integral domain
c) $\mathbb{F}$ x] is not Unique Factorization Domain
d) $\mathbb{F}$ [x] is never field

8) In Z [x], content of $4x^2+6x-8$ is ______.
a) 1 b) -1
c) -2 d) 2

9) If a group G is solvable iff the $n^{th}$ derived subgroup of G is _____.
   a) { } b) G
   c) {e} d) none of these

10) Any group of order $p^n$ where p is prime then G is ______.
   a) Abelian b) Non abelian
   c) Nilpotent d) None of these

**B) Fill in the blanks.** **06**

**1)** If G is abelian group of order n then class equation of G is ______.
**2)** Two sylow p-subgroup of a group G are _____ to each other.
**3)** If G is finite group and p| O (G), if r is the no. of sylow p-subgroup in G then r | o(G) and ______.
**4)** Let R be a ring and S is said to be ideal in R then S is said to be prime ideal of $\mathrm{R}$ if $\mathrm{ab} \in \mathrm{R}$ $\mathrm{a}, \mathrm{b} \in \mathrm{R}$ implies that _____.
**5)** Let a,b,c be any element in Euclidean domain R & gcd (a,b) = 1 if a|bc then _____.
**6)** A non-zero element in an integral domain D having proper divisors are called _____.

**Q.2 Answer the following** **16**

**a)** If H is a normal in G then prove that H is maximal in G iff quotient group $\frac{G}{H}$ is simple group.
**b)** If O(G) = $P^2$ where P is prime then prove that G is abelian.
**c)** Define zero of the polynomial and find all zero's of $f(x) = x^2 - 5x + 6$ in $Z$.
**d)** State the Eisenstein's criteria of irreducibility over Q and check the irreducibility of $f(x) = x^3 + x^2 - 2x - 1 \in Z[x]$ Over $\mathrm{Q}$.

**Q.3 Answer the following.** **16**

**a)** Show that "There exist at least one composition series for every finite group G".
**b)** Give the isomorphic refinement of following two subnormal series $< \mathrm{Z}, + >$.
$\{0\} \lhd 60\mathrm{Z} \lhd 20\mathrm{Z} \lhd \mathrm{Z}$ and $\{0\} \lhd 245\mathrm{Z} \lhd 49\mathrm{Z} \lhd \mathrm{Z}$

**Q.4 Answer the following.** **16**

**a)** State and prove $1^{st}$ Sylow theorem.
**b)** Solve
   1) How many Sylow 3-subgroup for a group whose order 255?
   2) If O(G) = 45 then check whether it is simple or not?

**Q.5 Answer the following.** **16**

**a)** Show that: Subgroup of solvable group is solvable.
**b)** Show that: If D is an integral domain then the polynomial ring D[x] is also an integral domain.

**Q.6 Answer the following.** **16**

**a)** If F be a field and $\mathrm{f(x)} \in \mathrm{F[X]}$ be a non-zero polynomial of degree n then prove that, f(x) has at most n zero's in F.
**b)** Prove that The ring of integer is principal ideal domain.

**Q.7 Answer the following.** **16**

**a)** If R be a Euclidean ring & a,b be any non-zero element in R then prove that.
   1) If b is unit in R then d(ab) = d(a)
   2) If b is not unit in R then d(ab) >d(a)
**b)** Prove that: Every principal ideal domain is Unique Factorization Domain".

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## M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Real Analysis – I

Day & Date: Wednesday, 15-02-2023 — Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) Riemann - Stieltje's integral reduces to Riemann integral if $\alpha(x)$ _____.
   a) $1$ b) $x$
   c) $x^2$ d) $0$

2) If $f: R \to R$ than Total derivative is ______.
   a) Real number b) Gradient vector
   c) Real matrix d) None of these

3) A necessary and sufficient condition for integrability of a bounded function is ______.
   a) $\lim_{\mu(P)\to\infty}\big(U(p,f) - L(P,f)\big) = 0$
   b) $\lim_{\mu(P)\to\infty}\big(U(p,f) + L(P,f)\big) = 0$
   c) $\lim_{\mu(P)\to 0}\big(U(p,f) + L(P,f)\big) = 0$
   d) $\lim_{\mu(P)\to 0}\big(U(p,f) - L(P,f)\big) = 0$

4) If $f_1$ and $f_2$ are bounded and integrable functions on [a,b] then the following function/functions is/are integrable.
   a) $f_1 - f_2$ b) $f_1^2$
   c) $|f_1|$ d) All of the above

5) The value of M and m for $f(x) = x$ on [1,2] are M= _____, m=______.
   a) 1, 2 b) 2, 1
   c) 1, 1 d) 1, 0

6) For any partition P, the norm of partition is defined as $\mu(p) =$ ______.
   a) $\max P$ b) $\min P$
   c) $\min \Delta x_i$ d) $\max \Delta x_i$

7) The directional derivative of $f$ at $c$ in the direction $u$ denoted by $f'(c,u)$ is defined as _____.
   a) $\lim_{h\to 0}\dfrac{f(c+hu)+f(c)}{h}$ b) $\lim_{h\to 0}\dfrac{f(c-hu)-f(c)}{h}$
   c) $\lim_{h\to 0}\dfrac{f(c+hu)-f(c)}{h}$ d) $\lim_{h\to 0}\dfrac{f(c-hu)+f(c)}{h}$

8) If we plot P points in between a and b of [a,b] then number of sub intervals created are _____.
   a) p b) p+1
   c) 2p d) none of these

9) If a continuous function $f$ is Riemann intergable with respect to $\alpha$ on $[a,b]$ then there exists a number $\xi \in [a,b]$ such that $\int_a^b f(x)d\alpha =$ ____.

a) $f(\xi)\big(f(a)+f(b)\big)$
b) $f(\xi)\big(\alpha(b)-\alpha(a)\big)$
c) $f(\xi)(b-a)$
d) $f(\xi)(b+a)$

10) If $f$ and $|f|$ are bounded and integrable on $[a,b]$ then, $|\int_a^b f(x)dx|$ _____.

a) $\geq \int_a^b |f|dx$
b) $\leq \int_a^b |f|dx$
c) $= \int_a^b |f|dx$
d) none of these

**B) Write True or False.** **06**

**1)** A bounded function $f$ is intergrable on $[a,b]$ if the set of points of discontinuity has _____ limit points.

**2)** If $f(x) = x$ on [0,1] and divide the interval into two equal sub intervals then $L(P,f) =$ ______.

**3)** A function $f = (f_1, f_2, \dots \dots f_n)$ has continuous partial derivative on an open set $S$ in $R^n$ and the Jacobian determinant in non zero at some point $a$ in $S$ then there is an n-ball $B(a)$ on which $f$ is _______.

**4)** The directional derivative of $f(x,y) = x^2y$ at point (1,2) in the direction (1,1) is _____.

**5)** If $P_1$ and $P_2$ are two partitions of $[a,b]$ the their common refinement is given by $P^*$ = ______.

**6)** The mean value of $\int_0^1 x^2\,dx$ in $[0,1]$ is ______.

**Q.2 Answer the following** **16**

**a)** Define: Upper sum, Lower sum, Upper Integral, Lower Integral.

**b)** Write Short note on Total derivative of function.

**c)** Examine whether the function $f(x) = x^2 + 4x + 3$ on $[-10, 10]$ have local extrema or not.

**d)** Check the integrability of a function.

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

**Q.3 Answer the following.** **16**

**a)** If and $f_1$ and $f_2$ are two bounded and integrable functions on $[a,b]$ then prove that their product $f_1.f_2$ is also bounded and integrable.

**b)** If $P^*$ is a refinement of a partition $P$ then for a bounded function $f$ prove that

1) $L(P^*f) \geq L(P,f)$

2) $U(P^*f) \leq U(P,f)$

**Q.4 Answer the following.** **16**

**a)** If $B = B(a;r)$ is an n-ball in R$^n$, $\partial$B denotes its boundary, $\partial B = \left\{\frac{x}{||x-a||} = r\right\}$ and $\overline{B} = B \cup \partial B$ denote its closure, $f = f_1, f_2, \dots \dots . f_n$ be continuous on $\overline{B}$ and assume that all partial derivatives $D_j f_i(x)$ exist if $x \in B$. Assume further that $f(x) \neq f(a)$ if $x \in \partial B$ and that the Jacobian $J_f(x) \neq 0$ for each $x \in B$ prove that $f(B)$ the image of $B$ under $f$ contains an n-ball with center at $f(a)$.

**b)** If $f$ is differentiable function at $c$ with total derivative $T_c$ then prove that the directional derivative $f'(c;u)$ exists for every $u$ in $R^n$ and also prove that $T_c(u) = f'(c;u)$

**Q.5 Answer the following.** **16**

**a)** Solve $\int_0^5 (4x+5)dx$

**b)** Prove that : A function $f$ is bounded and integrable on $[a,b]$ and there exists a function $F$ such that $F' = f$ on $[a,b]$ then prove that $\int_a^b f(x)dx = F(b) - F(a)$

**Q.6 Answer the following.** **16**

**a)** If $S$ is an open subset of $R^n$ and $f: S \to R^m$ is differentiable at each point of $S$, $x$ and $y$ are two points in $S$ such that $L(x,y) \subseteq S$ then prove that for every vector a in $R^m$ there is a point $z$ on $L(x,y)$ such that,
$a.\{f(y) - f(x)\} = a.\{f'(z)(y-x)\}$

**b)** Prove that : A function $f$ is integrable with respect to $\alpha$ on $[a,b]$ iff for every $\in > 0$ there a partition $P$ of $[a,b]$ such that $U(P,f,\alpha) - L(P,f,\alpha) < \in$.

**Q.7 Answer the following.** **16**

**a)** Find directional derivative of

$$f(x) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2} & if\ (x,y) \neq (0,0) \\ 0, & if (x,y) = (0,0) \end{cases}$$

**b)** Prove that : The oscillation of a bounded function $f$ on an intervbal $[a,b]$ is the supremum of the set $\{|f(x_1) - f(x_2)| x_1, x_2 \in [a,b]\}$ of numbers.

**SLR-GO-5**

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**Set P**

# M.Sc. (Semester-I) (New) (CBCS) Examination: Oct/Nov-2022
# (MATHEMATICS)
# Differential Equations

Day & Date: Thursday, 16-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) The two linearly independent solutions of $y" - 36y = 0$ are $\emptyset_1(x) =$ ____ and $\emptyset_2(x) =$ ______.
   a) $e^{-6ix}, e^{6ix}$ b) $e^{6x}, e^{-6x}$
   c) $e^{6}, xe^{6x}$ d) none

2) Two functions $x, |x|$ are ______.
   a) Linearly independent b) Linearly dependent
   c) Constant function d) none of these

3) Initial value problem for second order differential equation is denoted by ______.
   a) $L(y) = 0, y(x_0) = 0, y'(x_0) = 0$
   b) $L(y) = 0$
   c) $L(y) = 0, y(x_0) = \alpha, y'(x_0) = \beta$
   d) none of these

4) Which of the following is the form of Bessel's equation ______.
   a) $x^2y" + xy' + (x^2 - \alpha^2) = 0$
   b) $x^2y" + xy' + (x^2 - \alpha^2) = 1$
   c) $x^2y" - y' + (x^2 - \alpha^2) = 0$
   d) None of these

5) For a linear differential equation $a_0(x)y^n + a_1(x)y^{(n-1)} + \cdots + a_n(x)y = b(x)$ the points $x$ where $a_0(x)$ are called _____.
   a) singular points b) ordinary point
   c) regular singular point d) none of these

6) General non-homogeneous linear differential equation of order n with variable coefficients is an equation of the form _____.
   a) $L(y) = y^n + a_1(x)y^{n-1} + a_2(x)y^{n-2} + \cdots + a_n(x)y = b(x), b(x) \neq 0$
   b) $L(y) - y^n + a_1y^{n-1} + a_2y^{n-2} + \_\_\_\_ a_ny = b(x), b(x) = 0$
   c) Both a and b
   d) None

7) If $r_1$ and $r_2$ are distinct roots of characteristic polynomial of $L(y) = y" + a_1y' + a_2y = 0$ then solution of $\mathrm{L(y) = 0}$ is ______.
   a) $e^{r_1x}$ b) $e^{r_2x}$
   c) $e^{r_1x} + e^{r_2x}$ d) All of these

8) If function g is analytic at $x_0$ then g can be expressed in power series about $x_0$ which has _____ radius of convergence.
   a) Positive  b) Negative
   c) Zero  d) none

9) A solution of the differential equation is said to be general solution if number of arbitrary constant is ____ order of the differential equation.
   a) Unequal to  b) Less than
   c) More than  d) Equal to

10) The regular singular point of $x^2y" + \sin xy' + \cos xy = 0$ is _____.
   a) 1  b) -1
   c) 0  d) None of these

**B) Fill in the blanks.** **06**

**1)** The regular singular point of $xy'' + 4y = 0$ is _____.
**2)** The solutions of $y'' - 4y = 0$ are ______.
**3)** The Wronskian of $x^2$ and $(x^2 \log x)$ is _____.
**4)** The order of differential equation whose solution is $(a \sin x + b \cos x)$ is _____.
**5)** Regular singular point of Bessel's equation is $x$ = ______.
**6)** On an interval I containing $x_0$ there exists _____ solution of the initial value problem $L(y) = 0$

**Q.2 Answer the following** **16**

**a)** Show that if $\varphi_1$ and $\varphi_2$ are two solution of second order differential with constant coefficient $L(y) = y" + a_1y' + a_2y = 0$ then there linear combination is also solution of $L(y) = 0$.

**b)** Define
   1) Singular point
   2) Regular singular point

**c)** Find the singular point of the following equations and check whether they are regular singular or not.
$x^2y" + (x + x^2)y' - y = 0$

**d)** Show that the following function satisfies lipschitz condition $f(x, y) = 4x^2 + y^2$ on
$S = \{(x, y)/|x| \leq 1, |y| \leq 1\}$

**Q.3 Answer the following.** **16**

**a)** If the given differential equation is $y'' + y' - 6y = 0$ then,
   1) Compute the solution φ satisfies φ (0) = 1, φ' (0) =0
   2) Compute the solution ψ satisfies ψ (0) = 0, ψ' (0) =1
   3) Find ψ (1) and ψ (1)

**b)** If $x_0$ any real number and α, β are constant then prove that there exists a solution φ of IVP $L(y) = y" + a_1y' + a_2y = 0$ such that $y(x_0) = \alpha, y'(x_0) = \beta$ where $a_1$, $a_2$ are constant.

**Q.4 Answer the following.** **16**

**a)** If φ be the solution of $L(y) = y" + a_1y' + a_2y = 0$ on an interval I containing the point $x_0$ then prove that, for all $x \in$ I.
$||\varphi(x_0)||.e^{-k|x-x_0|} \leq ||\varphi(x)|| \leq ||\varphi(x_0)||.e^{k|x-x_0|}$
Where, $k = 1 + |a_1| + a_2|$ and $||\varphi(x)|| = [|\varphi(x)|^2 + |\varphi'(x)|^2]^{1/2}$

**b)** Show that Two solutions $\varphi_1$ and $\varphi_2$ of equation $L(y) = y'' + a_1y' + a_2y = 0$ are linearly dependent iff $W(\varphi_1, \varphi_2)(x) = 0$ for all $x \in I$.

**Q.5 Answer the following.** **16**

**a)** If the given differential equation is $y" + \frac{1}{x}y' - \frac{1}{x^2}y = 0 (x > 0)$ then,

1) Show that there is solution is of the form $x^r$. where $r$ is constant
2) Find the two linearly independent solution for $x > 0$.

**b)** Verify that the function $\varphi_1 (x) = e^{x^2}$ ) satisfies the equation and find the second independent solution of.

$$y" - 4xy' + (4x^2 - 2)y = 0$$

**Q.6 Answer the following.** **16**

**a)** Find two linearly independent power series solution of following equation.

$$y" + y = 0$$

**b)** Show that, A function φ is the sol ution of IVP $y' = f(x, y), y(x_0) = y_0$ on an interval I iff it is solution of the integral equation.

$$y = y_0 + \int_{x_0}^{x} f(t, \varphi(t))dt \text{ on I}$$

**Q.7 Answer the following.** **16**

**a)** If the initial value problem $y' = 3y + 1$ , $y(0) = 2$ then,

1) Compute the first four approximate solutions
2) Compute the exact solution
3) Compare the exact and approximate solution

**b)** If $\varphi 1, \varphi 2, \ldots, \varphi_n$ be the $n$ solutions of $L(y) = y^n + a_1 y^{n-1} + a_2 y^{n-2} + \cdots + a_n y = 0$ on an interval I containing a point $x_0$ then prove that $\varphi 1, \varphi 2, \ldots \ldots, \varphi_n$ are linearly independent iff $W(\varphi 1, \varphi 2, \ldots \ldots, \varphi_n)(x_0) \neq 0$

# SLR-GO-6

| Seat No. | |
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**Set P**

## M.Sc. (Semester - I) (New) (CBCS) Examination: Oct/Nov - 2022
## (MATHEMATICS)
## Classical Mechanics

Day & Date: Friday, 17-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

**Q.1 A) Fill in the blanks by choosing correct alternatives given below.** **10**

1) Determinant value of an orthogonal matrix is _____.
   a) 1 b) -1
   c) Either 1 or -1 d) Neither 1 nor -1

2) The rotation matrix in 2-dimension has ______ degrees of freedom.
   a) One b) Two
   c) Three d) Zero

3) If $q_k$ is cyclic in Lagrangian then the corresponding momentum represents ______.
   a) Variation in motion b) Constant of motion
   c) Variation of energy d) Constant of energy

4) Number of Cartesian coordinates require to describe configuration of double pendulum is/are ______.
   a) 1 b) 2
   c) 3 d) 4

5) Lagrangian is defined as ______.
   a) $\mathrm{L} = \mathrm{T} - \mathrm{V}$ b) $\mathrm{L} = \mathrm{T} + \mathrm{V}$
   c) $2\mathrm{T} + \mathrm{V}$ d) $\mathrm{L} = 2\mathrm{T} - \mathrm{V}$

6) Hamiltonian H is independent of ______.
   a) Generalized coordinates b) Generalized velocity
   c) Generalize momentum d) Time

7) Which of the following does not represents a rotation?
   a) orthogonal matrix with determinant -1
   b) orthogonal matrix with determinant +1
   c) Eulerian angles
   d) Both b and c

8) Geodesic on the surface of sphere is ______.
   a) parabola b) arc of great circle
   c) cycloid d) hyperbola

9) The curve of shortest length constrained to lie on surface in space is ______.
   a) Geodesic b) Circle
   c) Hyperbola d) Ellipse

10) Rheonomic constraint depends on _____.
    a) Co-ordinates  b) Time
    c) Momentum  d) Both a) and b)

**B) Fill in the blanks.** **06**

1) Bead sliding in moving wire is ______ constraint.
2) Euler - Lagrange's differential equations are _____ conditions for extremum of a functional.
3) Gravitational force is an example of ______.
4) Scleronomic constraint are not depending on ______.
5) Brachistochrone problem deals with ______.
6) Shortest distance between any two points is a ______.

**Q.2 Answer the following.** **16**

**a)** Explain four types of constraints.
**b)** If q is cyclic in L then show that it is cyclic in H.
**c)** Write short note on virtual work.
**d)** State modified Hamilton's principle.

**Q.3 Answer the following.**

**a)** Obtain Lagranges's equation of motion for simple pendulum. **08**
**b)** Show that: The path followed by a particle in sliding from one point to another under the influence of gravity is a cycloid. **08**

**Q.4 Answer the following.**

**a)** Derive Lagranges equation of motion from Hamilton's principle. **08**
**b)** Given the Lagrangian function $L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + \frac{K}{r}$. Find the Hamiltonian function H and Routhian function R. Also find Routhian equation of motion. **08**

**Q.5 Answer the following.**

**a)** Obtain Hamilton's canonical equation of motion from variational principle. **08**
**b)** Describe Routh's procedure to solve the problem involving both cyclic and non-cyclic co-ordinate. **08**

**Q.6 Answer the following.**

**a)** State and prove the principle of least action. **08**
**b)** Prove that: The product of two linear orthogonal transformations is again a linear orthogonal transformation and hence show that finite rotations of a rigid body about the fixed point of body are not commutative. **08**

**Q.7 Answer the following.**

**a)** Derive Hamilton's canonical equation of motion. **08**
**b)** Find the extremal for an isoperimetric problem $I[Y(x)] = \int_0^1 (y'^2 + x^2)dx$ subject to condition $\int_0^1 (y^2)dx = 2, y(0) = 0, y(1) = 0$. **08**

**SLR-GO-7**

| Seat No. | |
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**Set P**

# M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov-2022
# (MATHEMATICS)
# Algebra - II

Day & Date: Monday, 20-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) The non-zero element of a field form an _____ group w.r.t. multiplication.
   a) Abelian b) Non abelian
   c) Cyclic d) None of these

2) If F is field then the dimension of F (F) is ______.
   a) 1 b) 2
   c) 3 d) 0

3) The degree of extension of $Q(\sqrt{2}, \sqrt{3})$ over $Q$ is ______.
   a) 2 b) 4
   c) 5 d) 6

4) The extension K of a field F is called simple extension of F if ____ for some a in K.
   a) K = F (a) b) F = K (a)
   c) F (a) = F d) None of these

5) Which of the following is not algebraic over Q?
   a) $\sqrt{2}$ b) $\sqrt{3}$
   c) e d) None of these

6) The Splitting field of $x^2 + 1 \epsilon R\ [x]$ over $R$ is _____.
   a) Q b) R
   c) C d) None of these

7) Any finite extension of a field F of characteristic _____ is simple extension.
   a) 0 b) 1
   c) 2 d) 3

8) If K is finite extension of a field F and G (K, F) is finite group then which of the following is true _____.
   a) O(G(K,F)) = [K,F] b) O(G(K,F)) < [K,F]
   c) O(G(K,F)) > [K,F] d) $O(G(K, F)) \leq [K, F]$

9) The number of automorphism of field of real number is / are _____.
   a) 1 b) 2
   c) 3 d) 0

10) If $[K: F] = m$ then each element in $K$ is algebraic over $F$ of degree _____
    a) Equal to m b) less than m
    c) greater than m d) at most m

**B) State true or false.** **06**

1) If 'a' is constructible then $\sqrt{a}$ is also constructible.
2) For every prime number p and every integer m there exists a field having $p^m$ elements.
3) Any two field having same number of element are isomorphic.
4) The field C of complex number is a finite extension of the field of real number R.
5) The irrational number 'e' is algebraic over Q.
6) If F is field then it is integral domain.

**Q.2 Answer the following** **16**

**a)** Prove that: Every finite extension is algebraic extension.

**b)** Define:
1) Splitting field
2) Multiple root of the polynomial

**c)** Write a note on Galois group.

**d)** Prove that: There exists a splitting field for every $f(x) \in F[x]$.

**Q.3 Answer the following.** **16**

**a)** Prove that: If L is a finite extension of K and K is finite extension of F then L is finite extension of F and [L: F] = [L: K]. [K: F]

**b)** Prove that: Let K be an extension of field F then the element a ∈ K is algebraic over F iff F(a) is finite extension of F.

**Q.4 Answer the following.** **16**

**a)** If a, b in K are algebraic over F then prove that $a \pm b,\ ab,\ \frac{a}{b}\ (b \neq 0)$ are all algebraic over F, where K is extension of F.

**b)** If F be a field of rational numbers then determine the degree of spitting field of the polynomial $x^3 - 2$ over F.

**Q.5 Answer the following.** **16**

**a)** If $p(x)$ is an irreducible polynomial in $F[x]$ of degree $n \geq 1$ then prove that there is an extension $E$ of $F$ such that $[E{:}\,F] = n$ in which $p(x)$ has a root.

**b)** If the complex number Z is a root of $p(x)$ having real coefficients then prove that $\overline{Z}$ is also root of $p(x)$.

**Q.6 Answer the following.** **16**

**a)** If K be the field of complex number and F be the field of real number then show that K is normal extension of F.

**b)** If K be an extension of field of rational number F then show that any automorphism of K must leave every element of F is fixed.

**Q.7 Answer the following.** **16**

**a)** Find the elements of Galois group of $x^3 - 2$ over the field of rational number.

**b)** If $a, b$ are constructible then show that $a \pm b,\ ab,\ \frac{a}{b}\ (b \neq 0), \sqrt{a}$ are constructible.

# SLR-GO-8

| Seat No. | |
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**Set P**

**M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov - 2022**
**(MATHEMATICS)**
**Real Analysis - II**

Day & Date: Tuesday, 21-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

**Q.1 A) Fill in the blanks by choosing correct alternatives given below.** **10**

1) A property is said to be hold almost everywhere if there exists a set of points where it fails to hold is of measure ______.
   a) $> 0$ b) $< 0$
   c) $= 0$ d) None of these

2) A non-negative measurable function $f$ is said to be integrable over the measurable set $E$ if ______.
   a) $\int_E f(x)dx < \infty$ b) $\int_E f(x)dx > \infty$
   c) $\int_E f(x)dx = \infty$ d) None of these

3) If $f$ is a function of bounded variation on $[a,b]$ then ______.
   a) $T_a^b(f) = p_a^b(f) - N_a^b(f)$ b) $T_a^b(f) = p_a^b(f) + N_a^b(f)$
   c) $T_a^b(f) = p_a^b(f)/N_a^b(f)$ d) $T_a^b(f) = p_a^b(f) \times N_a^b(f)$

4) If $\langle E_i \rangle$ is the sequence of disjoint measurable sets and $A$ is any set, them $m^*(A \cap \cup_{i=1}^{n} E_i)$ = ______.
   a) $\sum_{i=1}^{n} m^*(A \cap E_i)$ b) $\sum_{i=1}^{n} m^*(A \cup E_i)$
   c) $\sum_{i=1}^{n} m^*(A - E_i)$ d) $m^*(A)$

5) We say that $f = g$ a.e., if $f$ and $g$ have the same domain of the definition and ______.
   a) $m\{x: f(x) = g(x)\} = 0$ b) $m\{x: f(x) \neq g(x)\} = 0$
   c) $m\{x: f(x) \neq g(x)\} \neq 0$ d) $m\{x: f(x) \neq g(x)\} \geq 0$

6) If $m^*$ is an outer measure then $m^*[a,b] =$ ______.
   a) $b - a$ b) $a - b$
   c) $a + b$ d) $a.b$

7) If $\{E_i\}$ is an infinite increasing sequence of measurable sets then $m(\cup_{i=1}^{\infty} E_i)$= ______.
   a) $\bigcup_{i=1}^{\infty} m(E_i)$ b) $\lim_{n\to\infty} m(E_n)$
   c) $\bigcap_{i=1}^{\infty} m(E_i)$ d) $\lim_{n\to 0} m(E_n)$

8) Countable union of collection of measurable sets is ______.
   a) Need not be measurable
   b) Uncountable
   c) Measurable
   d) Finite

9) Consider the two statements: ______.
   I) Outer measure $m^*$ is translation invariant
   II) The collection of all measurable sets is $\sigma$- algebra
   a) Only I is true
   b) Only II is true
   c) Both are true
   d) Both are false

10) If $A$ is countable set then $m^*(A) =$ ______.
   a) 0
   b) 1
   c) Non-zero
   d) None of these

**B) Fill in the blanks.** **06**

1) $A$ set which is countable intersection of open sets is called ______.
2) Cantor set is an uncountable set with outer measure _____.
3) A function ∅ is called simple function if it is measurable and assumes only _____ values.
4) By Fatou's lemma, if $f_n$ is a sequence of non negative measurable functions and $f_n \to f$ a.e. on $E$ then $f_E f$ ______.
5) For a function $f$, positive part $f^+(x)$ is defined as _____.
6) If $A$ and $B$ are disjoint measurable sets then $\int_{A \cup B} f =$ ______.

**Q.2 Answer the following.** **16**

**a)** Prove that outer measure $m^*$ is monotone. i.e. $A \subseteq B \Rightarrow m^*(A) \leq m^*(B)$

**b)** If $E_1$ and $E_2$ are measurable sets then prove that

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$$

**c)** If $f$ is measurable function and $f = g$ a.e., then prove that $g$ is measurable.

**d)** Define Outer measure, Lebesgue measure and give one example each.

**Q.3 Answer the following.**

**a)** If $E_1$ and $E_2$ are measuable sets then prove that $E_1 \cup E_2$ is measurable. **08**

**b)** If $\{E_i\}_{i=1}^{\infty}$ be a sequence of measurable sets then prove that **08**

$$m\left(\bigcup_i E_i\right) \leq \sum_i m\,(E_i)$$

Also deduce that if sets $E_i$'s are pairwise disjoint then

$$m\left(\bigcup_i E_i\right) = \sum_i m\,(E_i)$$

**Q.4 Answer the following.**

**a)** State and prove Bounded Convergence Theorem. **08**

**b)** If $f$ and $g$ are two integrable functions over $E$ then. **08**

a) $cf$ is integrable over $E$, and $\int_E cf = c\int_E f$

b) $f + g$ is integrable over $E$, and $\int_E f + g = \int_E f + \int_E g$

c) $f \leq g$ a.e. then $\int_E f \leq \int_E g$

d) If $A$ and $B$ are disjoint measurable sets contained in $E$, then $\int_{A \cup B} f = \int_A f + \int_B f$

**Q.5 Answer the following.**

**a)** Prove that: A function $f$ is of bounded variations on $[a, b]$ if and only if $f$ is difference of two monotone real valued functions on $[a, b]$. **08**

**b)** If $\emptyset = \sum_{i=1}^{n} a_i \chi_{E_i}$ where $E_i \cap E_j = \emptyset$ for $i \neq j$ and each $E_i$ is measurable set with finite measure then prove that

$$\int \emptyset = \sum_{i=1}^{n} a_i m(E_i)$$

**Q.6 Answer the following.**

**a)** Prove that outer measure of Cantor set is zero. **08**

**b)** If $\{A_n\}$ be a countable collection of sets of real numbers then **08**

$$m^*\left(\bigcup_n A_n\right) \leq \sum_n m^*(A_n)$$

**Q.7 Answer the following.**

**a)** Prove that interval $(a, \infty)$ is measurable. **08**

**b)** Given any set $A$ and any $\epsilon > 0$, there is an open set $O$ such that $A \subset O$ and $m^*(O) \leq m^*(A) + \in$ then prove that there is a set $G \in G_\delta$ such that $A \subseteq G$ and $m^*(A) = m^*(G)$. **08**

Where $G_\delta$ is set which is Countable intersection of open sets.

# SLR-GO-9

| Seat No. | |
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**Set P**

## M.Sc. (Semester - II) (New) (CBCS) Examination: Oct/Nov - 2022
## (MATHEMATICS)
## General Topology

Day & Date: Wednesday, 22-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

**Q.1 A) Fill in the blanks by choosing correct alternatives given below.** **10**

1) Which of the following statement is Correct?
   I) Topology define on any set $X$ need not be unique.
   II) Every Subset of $P(X)$ is topology on $X$.
   a) Both I and II are correct b) Both I and II are incorrect
   c) Only I is correct d) Only II is correct

2) Every singleton set in $T$ ______ -space is open.
   a) 1 b) 2
   c) Indiscrete d) Both a) and b)

3) If $X = \{a, b, c\}$ $\tau = \{\emptyset, X\{a\}, \{a\}, \{b\}, \{a, b\}\}$ $E = \{a, b\}$ then derived set of $E, d(E) =$ ______.
   a) $\{c\}$ b) $\emptyset$
   c) $\{a\}$ d) $\{a, c\}$

4) Derived set of each subset of discrete topological space is ______.
   a) Singleton set b) Empty set
   c) Compact d) Connected

5) Which of the following statement is correct?
   I) $d(A \cup B = d(A) \cup d(B)$ II) $A^\circ \cup B^\circ \subseteq (A \cup B)^\circ$
   a) Both I and II are correct b) Both I and Ii are incorrect
   c) Only I is correct d) Only II is correct

6) Consider $X = \{a, b\}, \tau = \{\emptyset, X, \{a\}, \{b\}\}$ then $< X, \tau >$ is _____.
   a) Discrete b) Indiscrete
   c) Not a topological space d) None of these

7) If $X = \{a, b, c\}$ then which of the following is a topology?
   a) $\tau = \{\emptyset, \{a\}, \{b\}\}$ b) $\tau = \{\emptyset, X, \{a\}, \{b\}\}$
   c) $\tau = \{\emptyset, \{a\}\}$ d) $\tau = \{\emptyset, X\}$

8) Which of the following is not true?
   a) A second countable space is always first countable
   b) A first countable space is always second countable
   c) Every subspace of second countable space is second countable
   d) A second countable space is always separable

9) Every topological space $< X, \tau >$ is compact if _____.
   a) $X$ is not finite
   b) $\tau$ is not finite
   c) Every open cover of $X$ is reducible to finite subcover
   d) None of these

10) Let $< X, \tau > be\ T_3$ – space, then which one is true?
   a) $< X, \tau >$ is $T_{31/2}$    b) $< X, \tau >$ is $T_1$
   c) $< X, \tau >$ is $T_4$    d) None of these

**B) Fill in the blanks.** **06**

1) If $X = \{a, b, c\}$ $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ then exterior of $\{a, c\} =$ ______.
2) Let $A =< X, \tau >$ where $\tau = \{\emptyset, X\}$, then _____.
3) Neighbourhood of any point $x$ ______ in T-space.
4) Subspace of first countable space is ______.
5) If $X$ is topological space. A covering of $X$ is called open covering, if _____.
6) _____ statement is correct.
   statement I: Every Topological space is $T_o$ space.
   statement II: Collection of all $T_o$ spaces is proper subset of T-space

**Q.2 Answer the following.** **16**

**a)** If $\beta$ is base for $\tau$ of $X$ then show that $B_y = \{B \cap Y | B \in \beta\}$ is base for subbase topology on $Y$.

**b)** If $A \subseteq B$ then prove that derived set $A$ is subset of derived set of $B$ i.e. $d(A) \subseteq d(B)$

**c)** Define:
   1) Base for Topology
   2) $T_{3/2}$ Space.

**d)** Prove that intersection of any number of closed compact set of T- space is closed compact set.

**Q.3 Answer the following.**

**a)** In any Topological space, prove that $d(A \cup B) = d(A) \cup d(B) \forall A, B \subseteq X$ **06**

**b)** Define: a) Connected set b) Separated set **10**
Let $< X, \tau >$ be topological space and let $A$ and $B$ a nonempty subsets of $X$. Then prove that the following are equivalent.
1) $X = A|B$
2) $X = A \cup B,\ \bar{A} \cap \bar{B} = \emptyset$
3) $X = A \cup B,\ A \cap B = \emptyset$ $A$ and $B$ both are closed in $X$
4) $B = X - A$ and $b(A) = \emptyset$
5) $X = A \cup B\ \ A \cap B = \emptyset$ $A$ and $B$ both are open in $X$

**Q.4 Answer the following.**

**a)** Prove that every convergent sequence in $T_2$ space has unique limit. **08**

**b)** If $< X, \tau >$ is $T$- space, then $< X, \tau >$ is $T_1$ space if and only if singleton sets are closed. **08**

**Q.5 Answer the following.**

**a)** Show that being first axiom space is topological property. **08**

**b)** Prove that subspace of Regular space is Regular space. **08**

**Q.6 Answer the following.**

**a)** Prove that a metric space is lindelof space if and only if it is second countable. **08**

**b)** If $< X, \tau >$ and $< X^*, \tau^* >$ be any closed T-spaces then $f: X \to X^*$ is continuous mapping on $X$ if and only if inverse image of any closed set in $X^*$ is closed in $X$. **08**

**Q.7 Answer the following.**

**a)** If $< X, \tau >$ and $< X^*, \tau^* >$ be two T-spaces then $f: X \to X^*$ is continuous mapping on $X$ if and only if $f[c(E)] \subseteq c^*[f(E)]$ for any $E \subseteq X$. **08**
Where $c(E)$ is closure of $E$ in $< X, \tau >$
$c^*(E)$ is closure of $E$ in $< X^*, \tau^* >$

**b)** If $< X, \tau >$ be any T-space. **08**
$X^* = \{a, b, c\}$ $\tau^* = \{\emptyset, X^*, \{b\}, \{b, c\}\}$
be defined by $f(x) = a \ \forall x \in X$ discuss the continuity of $f$ on $X$.

# SLR-GO-10

| Seat No. | |
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**Set P**

## M.Sc. (Mathematics) (Sem-II) (New) (CBCS) Examination: Oct/Nov-2022
## Complex Analysis

Day & Date: Thursday, 23-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) The product of two conjugate complex numbers is _____.
   a) Real b) Purely imaginary
   c) Rational d) Integer

2) $\int_c \frac{f(z)}{z-a}\,dz$ is equal to _____.
   a) $2\pi i\, f(a)$ b) $2\pi i\, Im\, f(a)$
   c) $2\pi i\, res\, f(a)$ d) $-2\pi i\, res\, f(a)$

3) Critical points of $w = \frac{\alpha z+\beta}{\gamma z+\delta}, \alpha\delta - \beta\gamma \neq 0$ are ______.
   a) $-\frac{\delta}{\gamma}$ b) $-\frac{\delta}{\gamma}$ and $0$
   c) $-\frac{\delta}{\gamma}$ and ∞ d) ∞ and $0$

4) A non-constant analytic function maps open set to a ______.
   a) Circle b) Straight line
   c) Open set d) Closed set

5) The radius of convergence of the power series $\sum_{n=0}^{\infty}(n+2i)^n z^n$ is _____.
   a) 0 b) 1
   c) ∞ d) $n^2+4$

6) Which of the following function does represent the series $\sum_{n=0}^{\infty}\frac{z^n}{n!}\; for\; |z| < \infty$
   a) sin z b) cos z
   c) $e^z$ d) $\frac{e^z}{n!}$

7) The simple pole of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ is _____.
   a) at $z = -2$ b) at $z = 1$
   c) at $z = 0$ d) at $z = 2$

8) If z is any complex number then $|z+5|^2 + |z-5|^2 = 75$ represents _____.
   a) a circle b) an ellipse
   c) a triangle d) straight line

9) Which of the following mapping does not change the shape of the figure but it changes size of the figure?
   a) Rotation b) Translation
   c) Magnification d) Bilinear Transformation

10) If f is an entire function then _______.
   a) f has power series expansion
   b) f has not a power series expansion
   c) f is constant
   d) f is polynomial

**B) Fill in the blanks.** **06**

**1)** If $z = a$ is a singularity of $f(z)$ such that $f(z)$ is analytic at each point in its neighbourhood then $z = a$ is called as _____.

**2)** A polygon with three sides is called ______.

**3)** The value of $\int_C \frac{dz}{z^2-1}$, where $C$ is the circle $|z|$ = 4 is equal to

**4)** The magnification factor of the mapping $w = \sqrt{2}e^{\frac{\pi i}{4}}z + (1 - 2i)$ is _____.

**5)** If $T_1(z) = \frac{z+2}{z+3}$ and $T_2(Z) = \frac{z}{z+1}$, then $T_2T_1(z)$ is _____.

**6)** If $f(z) = \frac{1-e^z}{1+e^z}$ then at $z = \infty, f(z)$ have _____ singularity.

**Q.2 Answer the following** **16**

**a)** Find Laurent series expansion of $\frac{1}{z^2-3z+2}$ for $|z| > 2$.

**b)** Prove that every non-constant polynomial has a root in *C*.

**c)** Define the following terms with one example of each.
   1) Removable singularity
   2) Residue of an analytic function

**d)** Find the fixed points of $f(z) = \frac{3z+z}{2-4z}$

**Q.3 Answer the following.** **16**

**a)** If $f$ be analytic in the disk $B(a, R)$ and suppose that $\gamma$ is a closed rectifiable curve in $B(a, R)$ then prove that $\int_\gamma f = 0$.

**b)** Find the Mobius transformation which maps the given points $z_1 = -1, z_2 = 0$ and $z_3 = 1$ onto the points $w_1 = i, w_2 = 0$ and $w_3 = \infty$.

**Q.4 Answer the following.** **16**

**a)** Show that the set of all bilinear transformation forms a non-abelian group under composition.

**b)** If $G$ be an open set and $f: G \to C$ be a differentiable function then prove that $f$ is analytic on $G$.

**Q.5 Answer the following.** **16**

**a)** Evaluate $\int_{|z|=\frac{3}{2}} \frac{3z^2-z+1}{2z^3-z^2+2z+1} dz$.

**b)** If $f$ has an isolated singularity at $z = a$ then prove that the point $z = a$ is removable singularity iff $\lim_{z\to a}(z - a)f(z) = 0$.

**Q.6 Answer the following.** **16**

**a)** Explain Laurent series development.

**b)** Prove that all the roots of equation $z^7 + 10z^3 + 14 = 0$ lie within annulus $1 < |z| < 2$.

**Q.7 Answer the following.** **16**

**a)** If $z_1, z_2, z_3, z_4$ be the four distinct points in $C_\infty$, then prove that the cross ratio $(z_1; z_2, z_3, z_4)$ is real iff all four points lie on a circle or straight line.

**b)** If $G$ be an open subset of the complex plane $C$ and $f: G \to C$ be an analytic function. Let $\gamma$ is a closed rectifiable curve in $G$ such that, $\eta(\gamma; w) = 0$; $\forall\, w \in C - G$ then for a $a \in G - \{\gamma\}$ prove that,

$$f(a), \eta(\gamma; a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(w)}{w - a} dw$$

# SLR-GO-12

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**Set P**

## M.Sc. (Semester-III) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Functional Analysis

Day & Date: Monday, 13-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) In a quotient space $N/M$ , the addition is defined as $(x+M)+(y+M) =$ ______.
   a) $x+y+M$ b) $x+y+2M$
   c) $M$ d) none of these

2) In a Hilbert space $H$, the two vectors $x, y \in H$ are said to be orthogonal if _____.
   a) $\langle x, y\rangle \neq 0$ b) $\langle x, y\rangle = \text{finite}$
   c) $\langle x, y\rangle = 1$ d) $\langle x, y\rangle = 0$

3) If $T: X \to Y$ be linear transformation then T is continuous iff _____.
   a) T is bounded
   b) T is continuous at origin
   c) T is continuous at any point of X
   d) All of the above

4) The set of all continuous linear transformations on a normed linear space N into normed linear space N' is denoted by _____.
   a) B(N) b) B(N')
   c) B(N, R) d) B(N, N')

5) If N and N' are normed linear spaces and $T: N \to N'$ then graph of T is gives as $T_G =$ ______.
   a) $\{(x, T(x))/x \in N'\}$ b) $\{(x, T(x))/x \in N\}$
   c) $\{(x, T(x))/x \in T\}$ d) $\theta$

6) The norm on $N \times N'$ is defined as $\| (x, y) \| =$ _____ for all $x \in N, y \in N'$
   a) $\| x \| + \| y \|$ b) $\max \| x, y \|$
   c) $(\| x \|^p + \| y \|^p)^{\frac{1}{p}}$ d) all of the above

7) If $x$ and $y$ are two vectors in a Hilbert space then by Parallelogram law, $(\| x+y \|^2 + (\| x-y \|)^2 =$ _____.
   a) $2(\| x \|^2 + \| y \|^2)$ b) $2(\| x \|^2 - \| y \|^2)$
   c) $0$ d) $2 \| x \| . \| y \|$

8) If Y is proper closed subspace of normed linear space X then for any $x_1 \in X - Y, d(x_1, Y)=$ _______.
   a) $\inf\{\| x_1 - y \| / y \in Y\}$ b) $\sup\{\| x_1 - y \| / y \in Y\}$
   c) $\inf\{\| x_1 - y \| / y \in X\}$ d) $\sup\{\| x_1 - y \| / y \in X\}$

9) Two projections P and Q on are orthogonal if _____.
   a) $PQ = 0$ b) $PQ = 1$
   c) $P = Q$ d) $P + Q = 1$

10) If $\frac{1}{p} + \frac{1}{q} = 1$ then the conjugate space of $l_p^n$ is _____.
   a) $l_q^n$ b) $l_p^\infty$
   c) $l_p^n$ d) $l_q^\infty$

**B) Fill in the blanks.** **06**

**1)** An idempotent linear transformation on a linear space N is called _____.
**2)** A self adjoint operator T is said to be positive if _____.
**3)** A projection E on a linear space L determines two linear subspaces M and N such that L = _______.
**4)** A subset S of a normed linear space (X, ||.||) is bounded if there exist a positive constant K such that _____ for all $x \in S$.
**5)** In a normed linear space, the triangular inequality property is given as _____.
**6)** If $T: X \to Y$ is a linear transformation and T is bounded then T maps bounded sets in X into_____ sets in Y.

**Q.2 Answer the following** **16**
**a)** Show that $| \parallel x \parallel - \parallel y \parallel | \leq \parallel x - y \parallel, \forall x, y \in V$
**b)** State and prove Pythagorean theorem.
**c)** Prove that: Every complete subspace of normed linear space is closed.
**d)** Define: Inner Product and Norm

**Q.3 Answer the following.** **16**
**a)** If $T: X \to Y$ be any linear transformation then prove that T is continuous on X if and only if T is bounded X.
**b)** If $x$ and $y$ are two vectors in a Hilbert space H then prove that
$4 < x, y > = \parallel x + y \parallel^2 - \parallel x - y \parallel^2 + i \parallel x + iy \parallel^2 - i \parallel x - iy \parallel^2$

**Q.4 Answer the following.** **16**
**a)** Prove that $B(X, Y)$ is normed linear space, where,
$\parallel T \parallel = \sup\{\parallel T(x) \parallel : x \in X, \parallel x \parallel \leq 1\}$
**b)** State and prove Riesz Lemma.

**Q.5 Answer the following.** **16**
**a)** Prove that : All norms on finite dimensional space are equivalent.
**b)** If M be a linear subspace of a Hilbert space H then prove that M is closed if and only if $M = M^{\perp\perp}$.

**Q.6 Answer the following.** **16**
**a)** If X is a complex IPS then Prove that following.
   1) $< ax - by, z > = a < x, z > - b < y, z >$
   2) $< x, ay + bz > = \overline{a} < x, y > + \overline{b} < x, z >$
   3) $< x, ay - bz > = \overline{a} < x, y > - \overline{b} < x, z >$
   4) $< x.0 > = 0$ and $< 0, x > = 0, \forall x \in X$

**b)** If $H$ is a Hilbert space and $f$ is an arbitrary functional in $H^*$ then prove that there exists a unique vector $y \in H$ such that $f(x) = < x, y >$ for every $x \in H$ and $\parallel f \parallel = \parallel y \parallel$

**Q.7 Answer the following.** **16**

**a)** If $N$ and $N'$ be two normed linear spaces and $D$ a subspace of $N$ then prove that a linear transformation $T : D \to N'$ is closed if and only if its graph $T_G$ is closed.

**b)** If $S(x, r)$ be an open sphere in $B$ with centre at $x$ and radius $r$, $S_r$ is the open sphere with centre at origin and radius $r$ then prove the following results.

1) $S(x, r) = x + S(0, r)$ or $x + S_r$

2) $S_r = r.S_1$ or $S(0, r) = rS(0, r)$

# SLR-GO-13

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**Set P**

## M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATCIS)
## Advanced Discrete Mathematics

Day & Date: Tuesday, 14-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) A lattice $(\mathrm{L}, \vee, \wedge)$ is called a distributive latticeif for any $\mathrm{a, b, c} \in \mathrm{L}$ we have _____.
   a) $\mathrm{a} \wedge (\mathrm{b} \vee \mathrm{c}) = (\mathrm{a} \wedge \mathrm{b}) \vee (\mathrm{a} \wedge \mathrm{c})$
   b) $\mathrm{a} \wedge (\mathrm{b} \vee \mathrm{c}) = (\mathrm{a} \vee \mathrm{b}) \wedge (\mathrm{a} \vee \mathrm{c})$
   c) $\mathrm{a} \wedge (\mathrm{b} \vee \mathrm{c}) = (\mathrm{a} \wedge \mathrm{b}) \vee (\mathrm{a} \vee \mathrm{c})$
   d) $\mathrm{a} \wedge (\mathrm{b} \vee \mathrm{c}) = (\mathrm{a} \wedge \mathrm{b}) \vee (\mathrm{a} \wedge \mathrm{c})$

2) The only complete bipartite graph which is complete is _____.
   a) $\mathrm{K}_{1,2}$ b) $\mathrm{K}_{3,1}$
   c) $\mathrm{K}_{3,3}$ d) $\mathrm{K}_{1,1}$

3) For any connected graph G, ______.
   a) $rad\,(G) \leq 2\,rad\,(G)$ b) $rad\,(G) \leq diam\,(G)$
   c) $diam\,(G) \leq 2\,rad\,(G)$ d) All of these

4) There are 5 different algebra books, 6 different complex analysis books and 8 different classical mechanics books. Then the number ways to pick an unordered pair of two books not both of the same course are _____.
   a) 118 b) 88
   c) 240 d) 19

5) The explicit formula for the sequence defined by the recurrence relation $a_n = a_{n-1} + 4;\ \forall\, n \geq 2$ with $a_1 = 2$ is _____.
   a) $a_n = 4n$ b) $a_n = 4n + 1$
   c) $a_n = 4n - 2$ d) $a_n = 4n + 2$

6) Consider the statements:
   I) Every distributive lattice is modular
   II) Every modular lattice is distributive
   a) Only I is true b) Only II is true
   c) Both I and II are true d) Both I and II are false

7) Every tree has _____ centres.
   a) one b) exactly one
   c) at most two d) two

8) If G is a acyclic graph with n vertices and k connected components then G has _____ edges.
   a) $n + k$ b) $n - k$
   c) $n.k$ d) $k$

9) The degree of $V_i^{th}$ vertex is equal to the sum of the entries in _____ of adjacency matrix.
   a) $\mathrm{i^{th}}$ row  b) $\mathrm{i^{th}}$ column
   c) Both a and b are true  d) $\mathrm{i^{th}}$ row and $\mathrm{i^{th}}$ column

10) An edge 'e' of a graph G is called a bridge if the subgraph G - e has ______ connected components than G has.
   a) more  b) less
   c) equal  d) both b and c

**B) Fill in the blanks.** **06**

**1)** An expression for geometric series $\frac{1}{(1-x)^n}$ is ______.

**2)** If $\mathrm{A} = (\{1,2,3,4,---,10\},/)$ is a Poset then the least upper bound of the subset $\{1,2,4\}$ is _____.

**3)** The different arrangements which can be made out of a given set of things, by taking some or all of them at a time is called as _____.

**4)** The coefficient of $x^{12}$ in $(x^3 + x^4 + x^5 + ---)^3$ is ______.

**5)** Every tree with 'n' $(\mathrm{n} \geq 2)$ vertices has at least two ______.

**6)** A simple graph which is isomorphic to its own complement is called _____.

**Q.2 Answer the following** **16**

**a)** If in a distributive lattice $(\mathrm{L}, \preccurlyeq)$ if $\mathrm{a} \wedge \mathrm{b} = \mathrm{a} \wedge \mathrm{c}$ & $\mathrm{a} \vee \mathrm{b} = \mathrm{a} \vee \mathrm{c}$ then prove that b = c.

**b)** Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

**c)** Define isomorphism of graph with two examples.

**d)** Show that an acyclic graph with n vertices is tree iff it contains precisely (n-1) edges.

**Q.3 Answer the following.** **16**

**a)** Determine the sequence corresponding to each of the following generating functions.

i) $\dfrac{1}{5 - 6x + x^2}$

ii) $\dfrac{x^5}{5 - 6x + x^2}$

**b)** If G be a graph with n vertices $v_1, v_2, v_3, \ldots v_n$ & A denote the adjacency matrix of G with respect to this listing of vertices. Let $B = [b_{i,j}]$ be the matrix $B = A + A^2 + A^3 + \cdots + A^{n-1}$ .Then show that G is connected graph iff for every pair of distinct indices $i, j$ we have $b_{i,j} \neq 0$.

**Q.4 Answer the following.** **16**

**a)** Show that in a complemented distributive lattice, the followings are equivalent:

1) $\mathrm{a} \preccurlyeq \mathrm{b}$
2) $\mathrm{a} \wedge \mathrm{b}' = 0$
3) $\mathrm{a}' \vee \mathrm{b} = 1$
4) $\mathrm{b}' \leq \mathrm{a}'$

**b)** Show that a graph G is connected if and only if it has a spanning tree.

**Q.5 Answer the following.** **16**

**a)** Write a short note on matrix representation of graph with two examples.

**b)** Among the integers 1 to 1000. Find how many of them are not divisible by 3, nor by 5, nor by 7.

**Q.6 Answer the following.** **16**

**a)** If $(L, \preccurlyeq)$ be a lattice then for any $a, b, c, d \in L$ Show that,

i) $a \preccurlyeq b \Longrightarrow a \vee c \preccurlyeq b \vee c$

ii) $a \preccurlyeq b \Longrightarrow a \wedge c \preccurlyeq b \wedge c$

iii) $a \preccurlyeq b \text{ and } c \preccurlyeq d \Longrightarrow a \vee c \preccurlyeq b \vee d$

iv) $a \preccurlyeq b \text{ and } c \preccurlyeq d \Longrightarrow a \wedge c \preccurlyeq b \wedge d$

**b)** Prove that an edge e of a graph G is a bridge if and only if e is not a part of any cycle in G.

**Q.7 Answer the following.** **16**

**a)** Write a short note on Hasse diagram of the Poset. Draw the Hasse diagram of the Poset $(P(S), \subseteq)$ where $P(S)$ is the power set on $S = \{a, b, c, d\}$.

**b)** Solve the recurrence relations.

1) $y_{n+2} - y_{n+1} - 2y_n = n^2$

2) $y_n - 4y_{n-1} + y_{n-2} = n + 4^n$

# SLR-GO-14

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**Set P**

## M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Linear Algebra

Day & Date: Wednesday, 15-02-2023 — Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) If $f: \mathbb{R}^3 \to \mathbb{R}$ is a linear functional defined by $(f(x,y,z) = x + y + z, \forall (x,y,z) \in \mathbb{R}^3$ Then $rank\ (f) =$
   a) 0 b) 1
   c) 2 d) 3

2) If $\mathrm{V}$ is a finite dimensional vector space and $\mathrm{V}^*$ is its dual space then _____.
   a) $\dim \mathrm{V} = \dim \mathrm{V}^*$ b) $\dim \mathrm{V} < \dim \mathrm{V}^*$
   c) $\dim \mathrm{V} > \dim \mathrm{V}^*$ d) None of these

3) If $A$ is any $m \times n$ matrix over the field $\mathbb{F}$, then consider the following two statements.
   Statement (1): rank (A) = row rank (A)
   Statement (2): rank (A) = column rank (A)
   a) Statement (1) is true and (2) is false
   b) Statement (1) is false and (2) is true
   c) Statements (1) and (2) both are true
   d) Statements (1) and (2) both are false

4) If A is any matrix of order $n$ over the field $\mathbb{F}$ and let $f(x),\ m(x)$denote its characteristic and minimal polynomial respectively, then which of the following is annihilating polynomial for A.
   a) $f(x)$ b) $m(x)$
   c) $g(x)$ such that $f(x)|g(x)$ d) all of the above

5) Which of the following matrix of order 2 over $\mathbb{R}$ is diagonalizable?
   a) Matrix with 2 distinct eigenvalues
   b) Matrix with both eigenvalues equal
   c) Nilpotent matrix
   d) Sum of two nilpotent matrix of order 2

6) Characteristic values of $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ are _____.
   a) $0, 0$ b) $1, 0$
   c) $1, -1$ d) $i\ , -i$

7) If V is a vector space and $\mathrm{W}_1, \mathrm{W}_2$ are two subspaces of V such that $\mathrm{V} = \mathrm{W}_1 + \mathrm{W}_2$ Then ______.
   a) $\mathrm{W}_1 + \mathrm{W}_2 = \{0\}$ b) $\mathrm{W}_1 \cap \mathrm{W}_2 = \{0\}$
   c) $\mathrm{W}_1 = \mathrm{W}_2$ d) None of these

8) The minimal polynomial for null matrix $A$ of order $n$ over the field $\mathbb{F}$ is _____.
   a) $m(x) = x^2$  b) $m(x) = x^n$
   c) $m(x) = x$  d) $m(x) = x^k, k > 1$

9) Which of the following are invariant subspace for V under a linear transformation $\mathrm{T: V \rightarrow V}$?
   a) $\mathrm{N(T)}$  b) $\{0\}$
   c) $\mathrm{R(T)}$  d) All of the above

10) If $\mathrm{T}: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a projection on X – axis. Then R(T) =
   a) {0}  b) $\mathbb{R}^2$
   c) X – axis  d) Y – axis

**B) Fill in the blanks.** **06**

1) If $\mathrm{T : V \rightarrow V}$ is a linear transformation and $W$ is a subspace of $V$ then $W$ is called an invariant subspace of $V$ if _____.
2) Let V be an inner product space over the field of complex numbers, then $< c\alpha + \beta \mid \gamma >=$ ______.
3) A finite dimensional real inner product space is called ______.
4) If V is an inner product space, if $\{\alpha_1, \alpha_2, \ldots, \alpha_3\}$ is a orthogonal set of non-zero vectors in an inner product space V. If β is any vector in V, the Bessel's inequality is given by______.
5) Let V be an inner product space and S = {0} is a subset of V. Then $\mathrm{S}^{\perp} =$ ______.
6) If E is a projection defined on V, then $(\mathrm{I - E})^2 =$ ______.

**Q.2 Answer the following** **16**

**a)** Let V be a finite dimensional vector space over the field $\mathbb{F}$ and W be a subspace of V, then prove that dim W + dim W° = dim V

**b)** Let V be a finite dimensional vector space. Let $\mathrm{W_1, W_2, \ldots\ldots, W_k}$ be subspaces of V and let $\mathrm{W = W_1 + W_2 + \cdots + W_k}$. Then prove that following are equitant.
   1) $\mathrm{W_1, W_2, \ldots\ldots, W_k}$ are indpeendant.
   2) For each $j, 2 \le j \le k, W_j \cap \left(W_1 + W_2 + \cdots + W_{j-1}\right) = \{0\}$

**c)** Let V be an inner product space and W be a finite dimensional subspace and E the orthogonal projection of V on W. Then prove that the mapping $\mathrm{T : V \rightarrow V}$ given by $\mathrm{T(\beta) = \beta - E\beta}$ is the orthogonal projection on $\mathrm{W}^{\perp}$.

**d)** Define:
   1) Minimal for a linear transformation
   2) Characteristic polynomial for a linear transformation
   3) Hermitian form
   4) Self-adjoint linear transformation

**Q.3 Answer the following.** **16**

**a)** Let V be an inner product space and let $\beta_1, \beta_2, \ldots, \beta_n$ be any linearly independent vectors in V. Then prove that an orthogonal set of vectors $\alpha_1, \alpha_2, \ldots, \alpha_n$ may be constructed such that for $\mathrm{k} = 1,2, \ldots, \mathrm{n},$ the set $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a basis for the subspace spanned by $\beta_1, \beta_2, \ldots, \beta_n$.

**b)** Let V be a finite dimensional vector space over the field $\mathbb{F}$ and $\mathrm{B} = \{\alpha_1, \alpha_2, \ldots\ldots, \alpha_n\}$ be a basis of V. Then prove that there is a unique dual basis $\mathrm{B}^* = \{f_1, f_2, \ldots\ldots, f_n\}$ for $\mathrm{V}^*$ such that $f_i(\alpha_j) = \delta_{ij}$. Further prove that for each linear functional $f$ on V. $f = \sum_{i=1}^{n} f(\alpha_i) f_i$, and each vector $\alpha \in \mathrm{V}$, $\alpha = \sum_{i=1}^{n} f_i(\alpha)\alpha_i$.

**Q.4 Answer the following.** **16**

**a)** Let V be a finite dimensional vector space over the field $\mathbb{F}$ and let T be a linear operator on V. Then prove that T is triangulable iff the minimal polynomial for T is a product of linear polinomals over $\mathbb{F}$.

**b)** Let $V = W_1 \oplus W_2 \oplus \ldots \oplus W_k$, then prove that there exist $k$ linear operators $E_1, E_2, \ldots., E_k$ on $V$ such that

1) each $E_i$ is a projection
2) $E_i E_j = 0$ if $i \neq j$
3) $I = E_1 + E_2 + \cdots + E_k$
4) the range of $E_i$ is $W_i$.

**Q.5 Answer the following.** **16**

**a)** Let $V$ be a finite dimesnional inner product space. If $T, U$ are linear operators on $V$ and $c$ is a scalar, then prove that

1) $(T + U)^* = T^* + U^*$
2) $(cT)^* = \bar{c}T^*$
3) $(TU)^* = U^*T^*$
4) $(T^*)^* = T$

**b)** Define normal operator. Let V be an inner product space and T is a self adjoint linear operator on V. Then prove that each characteristic value of T is real and characteristic vectors of T associated with distinct characteristic vectors are orthogonal.

**Q.6 Answer the following.** **16**

**a)** Let $V$ be a finite dimensional inner product space and $f$ be a form on $V$. Then prove that there is a unique linear operator $T$ on $V$ such that $f(\alpha, \beta) = < T\alpha | \beta >, \forall \alpha, \beta \in V$ and the map $f \to T$ is an isomorphism of the space of forms onto $L(V, V)$.

**b)** Orthonormalize the set {(1,0,1), (0,1,1), (1,3,3)} in $\mathbb{R}^3$ equipped with standard inner product.

**Q.7 Answer the following.** **16**

**a)** Consider the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$. Prove that A is diagnoalizable over $\mathbb{R}$ and find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix.

**b)** Find the Jordan canonical form for the matrix $A = \begin{bmatrix} 3 & 1 & -2 \\ -1 & 0 & 5 \\ -1 & -1 & 4 \end{bmatrix}$

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**Set P**

# M.Sc. (Semester - III) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Differential Geometry

Day & Date: Thursday, 16-02-2023 Max. Marks: 80
Time: 11:00 AM To 02:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) If $U_1, U_2, U_3$ are natural frame fields at $p$, then $U_i[f]$= ______.
   a) $\frac{df}{dx}$ b) $\frac{df}{dx_i}$
   c) $\frac{\partial f}{\partial x_i}$ d) Does not exist

2) The osculating plane to unit speed curve β at a point β$(s)$ is spanned by ______.
   a) N and T b) T and B
   c) N and B d) T, N, B

3) If $\alpha$ is a curve then its arc length function $s(t)$ of the curve $\alpha$ from $t = 0$ to any point $t$ on the curve is given by _____.
   a) $s(t) = \int_0^t \parallel \alpha'(u) \parallel du$ b) $s(t) = \int_0^t \parallel \alpha(u) \parallel du$
   c) $s(t) = \int_0^t \parallel \alpha''(u) \parallel du$ d) None of these

4) A mapping $X : D \to E^3$ is regular iff ______.
   a) $X$ is one-one b) $X$ is onto
   c) $X_u . X_v = 0 \forall (u, v) \in D$ d) $X_u \times X_v \neq 0 \forall (u, v) \in D$

5) If $\sum$ is a sphere of radius 'a' with center at origin of $E^3$ , then for any spherical curve β on the sphere, the curvature of β is _____.
   a) at most $\frac{1}{a}$ b) at least $\frac{1}{a}$
   c) exactly $\frac{1}{a}$ d) exactly a

6) If $f : \mathbb{R}^3 \to \mathbb{R}$ is a differentiable real valued function, $\overline{v}_p$ is a tangent vector to$\mathbb{R}^3$, then the formula for directional derivative $\overline{v}_p[f]$= ______.
   a) $\frac{d}{dt}[f(t)]_{t=0}$ b) $\frac{d}{dt}[f(p + \overline{v}t)]_{t=0}$
   c) $\frac{d}{dt}[f(p + \overline{v}t)]_{t=a}$ d) $\frac{d}{dt}[f(p + \overline{v}t)]_{t=p}$

7) A surface $M$ in $\mathbb{R}^3$ is called a flat surface if its _____.
   a) mean curvature is zero
   b) Gaussian curvature is zero
   c) positive Gaussian curvature
   d) positive mean curvature

8) If $\overline{F}$ is an isometry, $\overline{r}$ is a rotation, and $\overline{T}$ is a translation, then _____.
   a) $\overline{F} = \overline{T} + \overline{r}$ b) $\overline{F} = \overline{T} - \overline{r}$
   c) $\overline{F} = \overline{T}\overline{r}$ d) $\overline{F} = \overline{r}\overline{T}$

9) Every orthogonal transformation _____.
   a) is linear b) preserves dot product
   c) isometry d) all of the these

10) A curve $\alpha$ is called an unit speed curve if _____.
   a) $\| \alpha \| = 1$ b) $\| \alpha \|^2 = 1$
   c) $\| \alpha' \| = 1$ d) $\| \alpha'' \| = 1$

**B) Fill in the blanks.** **06**

1) Let $X: D \to E^3$ be a coordinate patch. Then for each $(u_0, v_0)$ in $D$, the velocity vector at $u_0$ of u- parameter curve $v = v_0$ is denoted by _____.
2) If $T$ is a translation such that for some point $\overline{p} \in E^3, T(\overline{p}) = \overline{p},$ then $T =$ ___.
3) A curve $\alpha: I \to E^3$ is said to be regular if _______.
4) Let $Y$ and $V$ be two vector fields on $E^3$ and $f: E^3 \to R$, then $\nabla_v(aY + bZ) =$ _______.
5) If $\alpha: I \to E^3$ us a regular curve with arbitrary speed then its torsion $\tau =$ _______.
6) If $\frac{\tau}{k}$ is constant for a curve $\alpha$, then $\alpha$ is a ______.

**Q.2 Answer the following** **16**

**a)** Define:
   1) directional derivative $v_p[f]$
   2) Covariant derivative of a vector field

**b)** Show that the rotation is an orthogonal transformation.

**c)** Find the directional derivative $\overline{v}_p[f]$ when $\overline{v} = (2, -1, 3), p = (2, 0, -1)$ for
   1) $f = y^2 z$
   2) $f = e^x \cos y$

**d)** For a patch $X: D \to E^3$, if $E = X_u . X_u, F = X_u . X_v, G = X_u . X_v$, then prove that $X$ is regular iff $EG - F^2 \neq 0$.

**Q.3 Answer the following.** **16**

**a)** If $\alpha : 1 \to E^3$ is a regular arbitrary speed curve in $E^3$, then derive expressions for Frenet apparatus $T, N, B, \tau$ and $k.$

**b)** Find the unit speed reparametrization of a curve helix.

**Q.4 Answer the following.** **16**

**a)** Let $V, W, Y$ and $Z$ be vector fields on $E^3$, then prove that.
   1) $\nabla_v(aY + bZ) = a\nabla_v Y + b\nabla_v Z$
   2) $\nabla_{fv+gw} Y = f\, \nabla_v Y + g\nabla_v Y\; \forall f, g$
   3) $\nabla_v(fY) = f\nabla_v Y + V[f]Y$
   4) $V[Y.Z] = Y.\nabla_v Z + \nabla_v Y.Z$

**b)** Prove that every isometry of $E^3$ can be uniquely described as orthogonal transformation followed by translation.

**Q.5 Answer the following.** **16**

**a)** Define Gaussian and Mean curvature for a surface. Prove that if $k_1$ and $k_2$ are principal curvatures at a point $P \in M$ then show that the Guassian curvature and mean curvature are respectively given by $K(P) = k_1, k_2$ and $H(P) = \frac{k_1+k_2}{2}$ $\forall P \in M$

**b)** If $\bar{F}$ is an isometry of $\mathbb{R}^3$ such that $\overline{F}(0) = 0$ , then show that $\overline{F}$ is an orthogonal transformation.

**Q.6 Answer the following.** **16**

**a)** Show that $M: Z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is a surface and $X(u,v) = (aucosv, businv, u^2)$ is a parametrization of $M$.

**b)** Compute the Frenet apparatus for the curve $\alpha(t) = (e^t cost, e^t sint, e^t)$.

**Q.7 Answer the following.**

**a)** Prove that a mapping $X: D \to \mathbb{R}^3$ is regular iff $X_u \times X_v \neq 0, \forall (u,v) \in D$. **16**

**b)** If $X$ is patch in $M \subset \mathbb{R}^3$ and $\overline{\mathrm{U}}$ is a unit normal vector field to $M$, then prove that $l = \overline{U}.X_{uu}, m = \overline{U}.X_{uv}$ and $n = \overline{U}.X_{vv.}$

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**Set P**

# M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
# (MATHEMATICS)
# Measure & Integration

Day & Date: Monday, 20-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Q. Nos. 1 and. 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

**Q.1 A) Fill in the blanks by choosing correct alternatives given below.** **10**

1) If $f$ and $g$ are integrable functions and $E$ is measurable set. If $f \geq g$ a.e on $E$ then ______.
   a) $\int_E f \leq \int_E g$  b) $\int_E f = \int_E g$
   c) $\int_E f \geq \int_E g$  d) $\int_E f < \int_E g$

2) If $f$ be any function then the sets $\{x|f(x) < a\}$ are called _____ sets for $f$.
   a) Dense  b) Ordinate
   c) Open  d) Closed

3) If $E$ is measurable subset of $X \times Y$ then ______.
   a) $(U_i E_i)_x = U_i (E_i)_x$  b) $(U_i E_i) \leq \Sigma_i (E_i)_x$
   c) $(U_i E_i)_x = \Sigma_i (E_i)_x$  d) $(U_i E_i)_x \neq \Sigma_i (E_i)_x$

4) If $A, B \in \mathcal{B}$ with $\mu^*(A - E) < \infty$ and $\mu^*(B - E) < \infty$ if $A \subseteq B$ then ______.
   a) $\mu(A) - \mu^*(A - E) \leq \mu(B) - \mu^*(B - E)$
   b) $\mu(A) \leq \mu(B) - \mu^*(B - E)$
   c) $\mu(A) - \mu^*(A - E) \leq 0$
   d) $\mu^*(A - E) = \mu^*(B - E)$

5) If $E_1$ and $E_2$ are measurable set's then $\mu(E_1 \cup E_2) + \mu(E_1 \cap E_2) =$ _____.
   a) $\mu(E_1) + \mu(E_2)$  b) $\mu(E_1).\mu(E_2)$
   c) $\mu(E_1)$  d) $\mu(E_2)$

6) An outer measure $\mu^*$ is said to be regular it given any subset $E$ of $X$ and $\in > 0$, there is a $\mu^*$- measurable set $A$ with $E \subset A$ and _____.
   a) $\mu^*(E) < 0$  b) $\mu^*(E) = 0$
   c) $\mu^*(A) \leq \mu^*(E) + \in$  d) $\mu^*(E) \leq \mu^*(A) + \in$

7) Which of the following is incorrect?
   a) Continuous functions are measurable
   b) The characteristic function $\chi_A$ of the set $A$ is measurable iff $A$ is measurable
   c) Let $f$ be a continuous function and $g$ be measurable function then the composition function $f_o g$ is measurable
   d) If $| f |$ is measurable then $f$ is also measurable

8) Which of the following is true?
   I) If a set has finite positive measure then it must be a positive set
   II) Every measurable subset of a positive
   a) Only I  b) Only II
   c) Both I and II  d) Neither I nor II

9) If $\{f_n\}$ be a increasing sequence of non-negative measurable functions on $E$ and let $f_n \to f$ almost everywhere pointwise on $E$, then _____.
   a) $\int_E f \le \lim inf f \int_E f_n$ b) $\int_E f = \lim inf f \int_E f_n$
   c) $\int_E f \le \lim \int_E f_n$ d) $\int_E f = \lim \int_E f_n$

10) If $\{A_i\}$ be a disjoint sequence of sets in $Q$, then _____.
   a) $\mu_*(E \cap \cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$
   b) $\mu_*(E \cap \cup_{i=1}^{\infty} A_i) \le \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$
   c) $\mu_*(E \cap \cup_{i=1}^{\infty} A_i) \ge \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$
   d) $\mu_*(E \cap \cup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mu^*(E \cap A_i)$

**B) Fill in the blanks.** **06**
1) The subset of a measurable set is _____
2) A set that is positive and negative with respect to signed measure $\gamma$ is called _____.
3) Two measures $\gamma_1$ and $\gamma_2$ on $(X, \mathcal{B})$ are said to be mutually singular if there are disjoint measurable sets $A$ and $B$ with $X = A \cup B$ such that _____.
4) Define, $\delta_x(A) = \begin{cases} 1; if\ x \in A \\ 0; otherwise \end{cases}$. This measure is called _____ at $x$.
5) Fubini's theorem is defined on _____.
6) If $\gamma \ll \mu$ and $\mu \ll \gamma$ then $\left[\frac{d\gamma}{d\mu}\right]$ = _____.

**Q.2 Answer the following.** **16**
**a)** Show that Hahn decomposition is unique except for null sets.
**b)** State and prove monotone convergence theorem.
**c)** Show that $E$ is negative set iff $\vartheta^+(E) = 0$.
**d)** If $A \in \mathcal{A}$ then show that $A$ is measurable set with respect to $\mu^*$.

**Q.3 Answer the following.**
**a)** Define an inner measure of a set. Show that for any set $E$ we have $\mu_*(E) \le \mu^*(E)$. Further if $E \in \mathcal{A}$, $\mu_*(E) = \mu^*(E)$. **08**
**b)** If $f$ and $g$ are non- negative measurable fuctions and $a, b$ are non-negative constants then prove that **08**
   1) $\int af + bg = a \int f + fg$
   2) $\int f = 0$ then $f = 0\ a.e.$

**Q.4 Answer the following.**
**a)** If $X = \{a, b, c, d, e, f\}$ and $\mathfrak{B} \in P(X)$ then $(X, \mathfrak{B})$ be a measurable space. If $\gamma: \mathfrak{B} \to R$ defined by $\gamma(\varphi) = \gamma(c) = \gamma(d) = 0, \gamma(a) = 1, \gamma(b) = -1, \gamma(e) = -2$, $\gamma(f) = 2$. Then show that $\gamma$ is a signed measure and compute the $\gamma$ measure of **10**
   1) $A_1 = \{a, b, c\}$
   2) $A_2 = \{a, b, c, d, f\}$
   3) $A_3 = \{a, b, c, d, e\}$
**b)** If $x \in X$ be any element. Then for $E \in R_{\sigma\delta}$, $E_x$ is measurable subset of $Y$. **06**

**Q.5 Answer the following.**
**a)** State and prove Jordan decomposition theorem. **10**
**b)** If $\gamma$ is a signed measure and $\mu$ is measure such that $\gamma \perp \mu$ and $\gamma \ll \mu$ then prove that $\gamma = 0$ **06**

**Q.6 Answer the following.**

**a)** If $E_i \in \mathfrak{B}$ then prove that $(\bigcup_{i=1}^{\infty} E_i) \leq \sum_{i=1}^{\infty} \mu(E_i)$. **08**

**b)** If $A \in \mathcal{A}$, then prove that $\mu(A) = \mu_*(A \cap E) + \mu^*(A \cap E^c)$. **08**

**Q.7 Answer the following.**

**a)** If $\gamma$ be a signed measure on a measurable space $(X, \mathcal{B})$. Let $E$ be a measurable set such that $0 < \gamma(E) < \infty$ then prove that there is a positive set $A$ contained in $E$ with $\gamma(A) > 0$. **08**

**b)** If $X$ be an uncountable set and let **08**

$\mathfrak{B} = \{A \subseteq X | A$ is countable or $A = E^c$, Where $E$ is countable$\}$

Define $\mu: \mathfrak{B} \rightarrow [0, \infty) \cup \{\infty\}$ by

$$\mu(A) = \begin{cases} 0; & if\ A\ is\ countable \\ 1; & if\ A = E^c\ is\ countable \end{cases}$$

then show that $(X, \mathfrak{B}, \mu)$ is a measure space.

# SLR-GO-18

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**Set P**

## M.Sc. (Sem-IV) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Partial Differential Equations

Day & Date: Tuesday, 21-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) Number of arbitrary constant is equal to number of independent variable then elimination of arbitrary constant shall give rise to _____.
   a) Unique partial differential equation of order 1
   b) More than one partial differential equation of order 1
   c) Unique partial differential equation of order 2
   d) None

2) A necessary and sufficient condition that there exist relation between two functions u(x,y) and v(x,y) a relation F(u,v)=0 or u=H(v) not involving x or y explicitly is that,
   a) $\frac{\partial(u,v)}{\partial(x,y)} = 0$  b) $\frac{\partial(u,v)}{\partial(x,y)} \neq 0$
   c) $\frac{\partial(u,v)}{\partial(x,v)} = 0$  d) $\frac{\partial(x,v)}{\partial(u,y)} = 0$

3) General integral is envelope of ______ parameter subfamily of the family of solutions.
   a) 1  b) 2
   c) both a and b  d) none

4) The complete integral of partial differential equation $z = px + qy + 2pq$ is given by,
   a) $z = ax + 2by + 2ab$  b) $z = ax + by + ab$
   c) $z = ax + by + 3ab$  d) $z = ax + by + 2ab$

5) A first order partial differential equation is said to be semi linear equation if it is linear in ______.
   a) p, q and z  b) p, q and x
   c) q, z, x and y  d) p and q

6) The canonical form of the differential equation $x^2u_{xx} - y^2u_{yy} = 0$ is _____.
   a) Hyperbolic  b) Parabolic
   c) Elliptical  d) None of these

7) The vibration of a string is described by the second order partial differential equation is given by ______.
   a) $y_{xx} = (1/c^2)y_{tt}$  b) $y_x = (1/c^2)y_{tt}$
   c) $y_{xx} = (1/c^2)y_t$  d) $y_x = (1/c^2)y_t$

8) The given second order partial differential equation
$\mathrm{u_{xx}} - 2\sin x\ \mathrm{u_{xy}} - \cos 2x\ \mathrm{u_{yy}} - \cos x\ \mathrm{u_y} = 0$ is of the form ______.
   a) Elliptical  b) Parabolic
   c) Hyperbolic  d) None

9) The solution of wave equation ______.
   a) is always exist  b) do not exist
   c) if it exist then it is unique  d) None of these

10) If u(x,y) be harmonic function in bounded closed region D and continuous in D∪B, where B is boundary of region D , then 'u' attains its minimum ______.
   a) on D
   b) on B
   c) on inside D and outside D
   d) on DUB

**B) Write True or False.** **06**

**1)** The complete integral of partial differential equation pq=c is given by, z = ax+ cy/a + b
**2)** For n=1 the given equation $(n-1)^2\, u_{xx} - y^{2n}\, u_{yy} = n.y^{2n-1}\, u_y$ reduces to the hyperbolic canonical form.
**3)** If the discriminant $\mathrm{S}^2 - 4\mathrm{RT} = 0$ of the quadratic equation $\mathrm{R}\lambda^2 + \mathrm{S}\lambda + \mathrm{T} = 0$ then roots are real and distinct.
**4)** Compatible system of first order partial differential equation has one parameter family of common solutions.
**5)** Singular integral is envelope of two parameter family.
**6)** The solution of Dirichlet problem is not unique.

**Q.2 Answer the following** **16**

**a)** Find the partial differential equation which represent all surfaces of revolution with z- axis as the axis of revolution.
**b)** Find a complete integral of the equation $zpq - p - q = 0$.
**c)** Prove that the solution of Dirichlet's problem if it is exist then it is unique.
**d)** Describe Jacobi's method of solving a first order partial differential equation.

**Q.3 Answer the following.** **16**

**a)** Show that the singular integral is obtained by eliminating $p$ and $q$ from the equations

$$f(x,y,z,p,q) = 0, f_p(x,y,z,p,q) = 0, \& \ f_q(x,y,z,p,q) = 0$$

**b)** Verify that the Pfaffian differential equation is integrable and find the corresponding integral of,

$$yz\,dx + xz\,dy + xy\,dz = 0$$

**Q.4 Answer the following.** **16**

**a)** If $h_1 = 0$ and $h_2 = 0$ are compatible with $f = 0$ then show that $h_1$ and $h_2$ satisfies.

$$\frac{\partial(f,h)}{\partial(x,u_x)} + \frac{\partial(f,h)}{\partial(y,u_y)} + \frac{\partial(f,h)}{\partial(z,u_z)} = 0$$

**b)** Solve the following non-linear partial differential equation in two variable by Jacobi's method.

$$zu_z\left(u_x + u_y\right) + x + y = 0$$

**Q.5 Answer the following.** **16**

**a)** Find the integral surface of the following partial differential equation.

$$(2xy-1)p+(z-2x^2)q=2(x-yz)$$

Which passes through the curve $x_0(s)=1, y_0(s)=0, z_0(s)=s$

**b)** Find the complete integral of $(p^2+q^2)x=pz$ and also find the integral surface through the curve $x=0, z^2=4y$

**Q.6 Answer the following.** **16**

**a)** Reduce the equation $u_{xx}+x^2u_{yy}=0$ to a canonical form.

**b)** Obtain the D-Alembert's solution of the one dimensional wave equation which describe the vibrations of an infinite string when both the end points not fixed.

**Q.7 Answer the following.** **16**

**a)** Show that surfaces $x^2+y^2+z^2=r^2, r>0$ forms a family of equipotential surfaces and find the general form of corresponding potential function.

**b)** If $u(x,y)$ be a harmonic function in bounded closed region $D$ and continuous in $\overline{D}=D\cup B$ then prove that $u(x,y)$ attains it's maximum in the boundary $B$ of $D$.

# SLR-GO-19

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**Set P**

## M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Integral Equation

Day & Date: Wednesday, 22-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Q. Nos.1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7
3) Figure to right indicate full marks.

**Q.1 A) Choose the correct alternatives from the options.** **10**

1) The kernel $K(x,t) = \sin(x+t)\ is$
   a) Separable b) Symmetric
   c) both (a) and (b) d) inseparable

2) Which of the following is convolution type kernel?
   a) $K(x,t) = cos(xt)$ b) $K(x,t) = cos(x + t)$
   c) $K(x,t) = cos(2x - 3t)$ d) $K(x,t) = cos\,(t - x)$

3) An initial value problem gets converted into ______.
   a) Volterra integral equation
   b) Fredholm integral equation
   c) Fredholm integral equation
   d) Singular integral equation

4) Which of the following integral equation can have eigenvalues and eigen functions?
   a) Volterra integral equation
   b) homogeneous Fredholm integral equation of the second kind.
   c) Non-homogeneous integral equation
   d) homogeneous Volterra integral equation

5) The formula for nth iterated kernel $K_n(x,t)$ for Volterra integral equation is ______.
   a) $$K_n(x,t) = \int_t^x K(x,z)K_1(z,t)dz$$
   b) $$K_n(x,t) = \int_t^x K(x,z)K_{n-1}(z,t)dz$$
   c) $$K_n(x,t) = \int_t^x K(x,z)K_{n-2}(z,t)dz$$
   d) $$K_n(x,t) = \int_t^x K_{n-1}(x,z)K_n(z,t)dz$$

6) Eigenvalues of symmetric kernel of a Fredholm integral equation are _______.
   a) always positive  b) always negative
   c) always real  d) purely imaginary

7) If $\mathrm{K}(\mathrm{x},\mathrm{t}) = 1$ for an Volterra integral equation then $K_2(x,t) =$
   a) 1  b) $(x-t)$
   c) $\frac{(x-t)^2}{2}$  d) $e^{\lambda(x-t)}$

8) Solution of $y(x) = 1 + \int_0^x y(t)\, dt$ is _______.
   a) 1  b) $x$
   c) $e^x$  d) 0

9) An integral equation can be solved by using Laplace transform if the kernel is ______.
   a) Convolution type  b) symmetric
   c) degenerate  d) constant

10) If the order of differential equation in the boundary value problem is 5 then ______.
   a) $G\,(x,t)$ is continuous
   b) $G'(x,t)$ is continuous
   c) $G''(x,t)$ is continuous
   d) all of the above

**B) State whether True or False.** **06**

1) Homogeneous Volterra integral equation always have an eigen value.
2) Eigen values of symmetric kernel are always real.
3) $y(x) = 1$ is solution of $y(x) = \alpha + \int_0^1 y\,(t)dt \; if \; \alpha = 1.$
4) The Green's function of the boundary value problem, considered as a function of t is a solution of the given differential equation.
5) The integral equation $\int_0^x e^{x-t} y(t)dt = x^2$ is Volterra integral equation of the first kind.
6) The integral equation $y(x) - \lambda \int_0^1 (3x - 2)y(t)dt = 0$ does not have eigen value.

**Q.2 Answer the following.** **16**

a) Show that $y(x) = \frac{1}{2}$ is solution of the integral equation, $\int_0^x \frac{y(t)}{\sqrt{x-t}} dt = \sqrt{x}$.

b) Convert the following differential equation into an integral equation:

$$y'' + \lambda xy = f(x), \qquad y(0) = 1, \qquad y'(0) = 0$$

c) Define Green's function.

d) Using the method of successive approximations, solve the integral equation $y(x) = 1 + x - \int_0^x y(t)dt,\ y_0(x) = 1$.

**Q.3 Answer the following.**

a) Convert $y'' - \sin x\, y' + e^x y = x, y(0) = 1, y'(0) = -1$ to an integral equation. Conversely, derive the original differential equation with the initial conditions from the integral equation obtained. **10**

b) Solve: $Y(t) = e^{-t} - 2\int_0^t cos(t - x)Y(x)dx$ **06**

**Q.4 Answer the following.**

a) Define: Iterated kernel and Resolvent kernel. If $R(x, t; \lambda)$ is the resolvent kernel of a Fredholm integral equation, **08**

$y(x) = f(x) + \lambda \int_0^b K(x, t)y(t)dt,$ then prove that the resolvent kernel satisfies the integral equation

$R(x, t; \lambda) = K(x, t) + \lambda \int_0^b K(x, z)R(z, t;\ \lambda)\, dt$

b) Find the eigenvalues and eigen functions of the integral equation **08**

$y(x) = \lambda \int_0^1 (2xt - 4x^2)y(t)dx$

**Q.5 Answer the following.**

a) Solve: $y(x) = (1 + x)^2 + \int_{-1}^1 (xt + x^2t^2)y(t)dt.$ **08**

b) Prove that the eigenfunctions of a symmetric kernel, corresponding to different eigen values are orthogonal. **08**

**Q.6 Answer the following.**

a) Examine whether the Green's function exists for the boundary value problem, **08**

$y'' = 0; y(0) = y'(1), y'(0) = y(1).$ If exists, then construct the Green's function.

b) Solve $y(x) = x + \int_0^x (t - x)y(t)dt$ with the help of resolvent kernel. **08**

**Q.7 Answer the followings.**

**a)** Transform $\frac{d^2y}{dx^2} + xy = 1, y(0) = y(1) = 0$ into an integral equation. **08**

**b)** Solve by the method of successive approximations. **08**

$y(x) = x^2 + \lambda \int_0^1 e^{x-t} y(t)dt$

**SLR-GO-20**

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**Set P**

# M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHEMATICS)
## Operations Research

Day & Date: Thursday, 23-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) A solution to a linear programming problem ______.
   a) Must satisfy all the constraints of the problem simultaneously
   b) Need not satisfy all of the constraints, only some of them
   c) Must be a corner point of the feasible region
   d) Must optimize the value of the objective function

2) For any primal problem and its dual _____.
   a) Optimal value of objective function is same
   b) Dual will have an optimal solution iff primal does too
   c) Primal will have an optimal solution iff dual does too
   d) All of these

3) If any value in $X_B$ column of final simplex table is negative, then the solution is _____.
   a) Feasible b) Infeasible
   c) Bounded d) No solution

4) If at least one $\Delta_j$ is negative then the solution of linear programming problem is ______.
   a) Not optimal b) not feasible
   c) not bounded d) not basic

5) A set of feasible solution to a Linear Programming Problem is _____.
   a) Triangle b) Polygon
   c) Convex d) Square

6) If the primal problem has n constraints and m variables then the number of constraints in the dual problem is _____.
   a) m b) m+n
   c) m-n d) m/n

7) The right hand side constant of a constraint in a primal problem appears in the corresponding dual as _____.
   a) A coefficient in the objective function
   b) a right hand side constant of a function
   c) An input output coefficient a left hand side constraint
   d) Coefficient variable

8) When the sum of gains of one player is equal to the sum of losses to another player in a game, this situation is known as ______.
   a) two-person game b) two-person zero-sum game
   c) non-zero-sum game d) None of these

9) Any solution to a Linear Programming Problem which also satisfies the non-negative restriction of the problem has _____.
   a) solution  b) basic solution
   c) basic feasible solution  d) feasible solution

10) A game is said to be strictly determinable if ______.
   a) Maximin value equal to minimax value
   b) Maximin value is less than or equal to minimax value
   c) Maximin value is greater than or equal to minimax value
   d) Maximin value is not equal to minimax value

**B) Write True or False.** **06**

**1)** The value of the non-basic variables is _____.
**2)** In a Linear Programming Problem functions to be maximized or minimized are called ______.
**3)** Key element is also known as ______.
**4)** The coefficient of slack\surplus variables in the objective function are always assumed to be ______.
**5)** Beal's method is used to solve ______programming problem.
**6)** The method used to solve Linear Programming Problem without use of the artificial variable is called _____.

**Q.2 Answer the following** **16**

**a)** Show that: A hyperplane in $R^n$ is a convex set.
**b)** Define the following terms:
   1) Convex hull
   2) Convex function
**c)** Describe the algorithm of Big-M method.
**d)** Check whether the following game has saddle point or not and find also optimal strategy.

$$\begin{bmatrix} 1 & 1 \\ 4 & 3 \end{bmatrix}$$

**Q.3 Answer the following.** **16**

**a)** If the convex set of the feasible solution AX=b, b ≥ 0 is the convex polyhedron then prove that at least one extreme point gives an optimal solution also if the optimal solution occurs at more than one extreme point then prove that the values of the objective function will be the same for all convex combination of these extreme points.
**b)** Solve the linear programming problem by simplex method.

$$Max. Z = 3x_1 + 2x_2$$

Subject to condition,
$$x_1 + x_2 \leq 4$$
$$x_1 - x_2 \leq 2$$
$$and\ x_1, x_2 \geq 0$$

**Q.4 Answer the following.** **16**

**a)** Solve the linear programming problem.

$$Min\ Z = x_1 + x_2$$

Subject to condition,
$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

**b)** Write the algorithm of Two Phase method.

**Q.5 Answer the following.** **16**

**a)** Show that: The dual of dual of a given primal is the primal.

**b)** If $X_0$ is an optimal solution to the primal then prove that there exist a feasible solution $W_0$ to the dual such that $CX_0 = b^T W_0$.

**Q.6 Answer the following.** **16**

**a)** Apply Wolfe's method and solve the following quadratic programming problem.

$$Max\ Z_x = 2x_1 + x_2 - x_1^2 \text{ Such that,}$$
$$2x_1 + 3x_2 \leq 6,\ \ 2x_1 + x_2 \leq 4,\ \ \ x_1, x_2 \geq 0$$

**b)** Write the algorithm of Beale's method for solving a quadratic programming problem.

**Q.7 Answer the following.** **16**

**a)** Solve the 3*3 game by simplex method of linear programming problem whose payoff matrix is given by,

$$\begin{bmatrix} 3 & -1 & -3 \\ -3 & 3 & -1 \\ -4 & -3 & 3 \end{bmatrix}$$

**b)** Solve the game by arithmetic method whose payoff matrix is given by,

1) $\begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix}$

2) $\begin{bmatrix} 6 & -3 \\ -3 & 0 \end{bmatrix}$

# SLR-GO-21

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**Set P**

## M.Sc. (Semester-IV) (New) (CBCS) Examination: Oct/Nov-2022
## (MATHIMATICS)
## Numerical Analysis

Day & Date: Friday, 24-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) Question no. 1 and 2 are compulsory.
2) Attempt any three questions from Q. No. 3 to Q. No. 7.
3) Figure to right indicate full marks.

**Q.1 A) Multiple choice questions.** **10**

1) In Gauss elimination method the coefficient matrix is reduced to ______.
   a) Diagonal matrix b) Zero matrix
   c) Upper triangular matrix d) None of these

2) Householder method consist of converting real symmetric matrix to _____ Matrix.
   a) Upper triangular matrix b) Lower triangular matrix
   c) Orthogonal matrix d) Tridiagonal matrix

3) Euler's method is used solve ______.
   a) Numerical integration b) Transcedental equation
   c) Numerical differentiation d) None of the above

4) The method of False position is also known as ______.
   a) Regular falsi method b) Secant method
   c) Newton raphson method d) None of the above

5) The linear polynomial which passes through points (0, 2) and (1, 5) is _____.
   a) $y = 2x + 3$ b) $y = 3x - 2$
   c) $y = 3x + 2$ d) $y + 3x = 2$

6) The effect of error _____ with number of iterations.
   a) constant b) decreases
   c) increases d) none of these

7) Rounded off value of 3.14159 correct to four significant figure is _____.
   a) 3.141 b) 3.1416
   c) 3.142 d) 3.1425

8) The correct relation between Percentage error and Relative error is _____
   a) $E_p = E_r/100$ b) $E_p = 100E_r$
   c) $E_r = 100/E_r$ d) $E_r = E_p/100$

9) Convergence of bisection method is ______.
   a) Quadratic b) Cubic
   c) Very slow d) None of these

10) If A is upper triangular then $A^{-1}$ is ______.
   a) Lower triangular  b) Upper triangular
   c) Constant  d) None of these

**B) Fill in the blanks.** **06**

1) _____ theorem applies to the bisection method used for finding interval in which root lies?
2) The convergence of _____ method is sensitive to starting value?
3) Significant digits in 2.24 are ______.
4) Order of convergence of Regula-Falsi method is ______.
5) The Bisection method is also known as ______.
6) Every polynomial equation of the nth degree has _____ roots.

**Q.2 Answer the following** **16**

**a)** What is error in series approximation?

**b)** Evaluate the sum $s = \sqrt{3} + \sqrt{5} + \sqrt{7}$ to 4 significant digits and find its absolute and relative errors.

**c)** If 0.333 is the approximate value of 1/3, find absolute, relative and percentage error.

**d)** Perform three iteration of Secant method to find root of $x^3 - 5x + 1 = 0$ in the interval (0,1).

**Q.3 Answer the following.** **16**

**a)** Solve the system $2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$ using Gauss elimination method.

**b)** Reduce the matrix $\begin{bmatrix} 1 & 3 & 4 \\ 3 & 2 & -1 \\ 4 & -1 & 1 \end{bmatrix}$ to tridiagonal form using Householder's method.

**Q.4 Answer the following.** **16**

**a)** Solve by Euler method $dy/dx = x + y, y(0) = 0,$ choose $h = 0.2$ and compute $y(0.4)$.

**b)** Solve the equation $x_1 + x_2 + x_3 = 1, 4x_1 + 3x_2 - x_3 = 6, 3x_1 + 5x_2 + 3x_3 = 4$ using LU decomposition method.

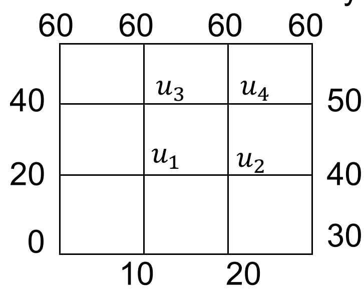
**Q.5 Answer the following.** **16**

**a)** Prove that Newton Raphson method converges quadratically.

**b)** Find the root of equation $x^3 - 2x - 5 = 0$ using Regula-Falsi Method.

**Q.6 Answer the following.** **16**

**a)** If function $u(x, y)$ satisfies Laplace's equation at all points within the square given below and has boundary values as indicated.




Compute solution correct up to two decimal places by finite difference method.

**b)** Explain Alternating direction implicit (ADI) method for numerical solution of Partial differential equation.

**Q.7 Answer the following.** **16**

**a)** Solve Initial value problem $y' = 3x + y/2$ with condition $y(0) = 1$ using Runge-kutta method (take h=0.05).

**b)** If $\frac{dy}{dx} = \frac{1}{x^2+y}$ where $y(4) = 4$ compute $y(4.1)$ and $y(4.2)$ by Taylor's series method.

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**Set P**

# M.Sc. (Semester - IV) (New) (CBCS) Examination: Oct/Nov-2022
# (MATHEMATICS)
# Probability Theory

Day & Date: Friday, 24-02-2023 Max. Marks: 80
Time: 03:00 PM To 06:00 PM

**Instructions:** 1) All questions are compulsory.
2) Figures to the right indicate full marks.
3) Draw neat labeled diagrams wherever necessary.

**Q.1 A) Choose the correct alternatives from the options.** **10**

1) If a random variable $X$ is integrable, then _______.
 a) $X^+$ is integrable b) $X^-$ is integrable
 c) $|X|$ is integrable d) all of these

2) If $\{A_n\}$is decreasing sequence of sets, then the sequence $\{A_n^c\}$ is _____.
 a) Decreasing
 b) Increasing
 c) Need more information
 d) None of these

3) The sequence of sets $\{(0, \mathrm{n}), \mathrm{n} = 1,2,3, \dots\}$ is______.
 a) Convergent b) Divergent
 c) Oscillatory d) None of these

4) If $\mathrm{F}$ is a $\sigma$ –field, then which of the following is not always correct?
 a) $\mathrm{F}$ is a field.
 b) $\mathrm{F}$ is a class closed under countable unions
 c) $\mathrm{F}$ is a class closed under complementation
 d) $\mathrm{F}$ is a minimal sigma field

5) If $F_1$ and $F_2$ are two fields, then _______ is always a field.
 a) $F_1 \cap F_2$ b) $F_1 \cup F_2$
 c) Both (a) and (b) d) Neither a) nor (b)

6) The sequence of sets $\{A_n\}$, where $A_n = \left(0, 2 + \frac{1}{n}\right)$ converges to _____.
 a) (0,2) b) (0,2]
 c) [0,3) d) [0,2]

7) If events A and B are independent events, then which of the following is correct?
   a) $\mathrm{P(A \cap B) = P(A) + P(B)}$
   b) $\mathrm{P(A \cup B) = P(A) + P(B) - P(A) * P(B)}$
   c) $\mathrm{P(A \cup B) = P(A) * P(B)}$
   d) $\mathrm{P(A \cap B) = P(A) - P(B)}$

8) If P is a probability measure defined on $(\Omega, \mathrm{A})$, then $\mathrm{P}(\varphi) =$ _____.
   a) Zero  b) One
   c) 0.5  d) 0.3325

9) Which of the following is not correct?
   a) Every sigma field is a field.
   b) Every sigma field is closed under countable intersection.
   c) Every sigma field is closed under countable union.
   d) None of these.

10) $If\ x \epsilon A$ implies $x \epsilon B$, then _______.
   a) $\mathrm{A\ C\ B}$  b) $\mathrm{B\ C\ A}$
   c) $\mathrm{A = B}$  d) All of these

**B) Fill in the blanks.** **06**

1) A monotonic decreasing sequence of sets converges to _______.
2) Lebesgue measure of a singleton set $\{k\}$ is ______.
3) The sequence of sets $\{A_n\}$, where $A_n = \left(0{,}5 - \frac{1}{n}\right)$ converges to ______.
4) If $X$ and $Y$ are independent variables, then $E(X + Y) =$ _______.
5) The largest field of subsets of $\Omega$ is called as _______.
6) If for two independent events $A$ and $B$, $P(A) = 0.3,\ P(B) = 0.5$, then $P(AUB) =$ ______.

**Q.2 Answer the followings.** **16**

**a)** Define field and $\sigma - \mathrm{field}$. Give an example of field which is not a $\sigma - field$.

**b)** Write a short note on Probability measure.

**c)** Define Pairwise and mutual independence of events. State the relationship between them.

**d)** State
   1) Liapouniv's CLT
   2) Lindeberg-Feller CLT

**Q.3 Answer the followings.** **16**

**a)** Define probability measure. State and prove monotone property of probability measure

**b)** Define monotone field. Prove that every monotone field is a $\sigma - \mathrm{field}$.

**Q.4 Answer the followings.** **16**

**a)** Prove that $P\left(\lim_{n\to\infty} A_n\right) = \lim_{n\to\infty} P(A_n)$

**b)** Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P\ (A_n) = \infty.$

Show that $P\ (\overline{\lim} A_n) = 1.$

**Q.5 Answer the followings.** **16**

**a)** With usual notations prove that,

1) $E(X+Y) = E(X) + E(Y).$

2) $E\ (XY) = E(X)\ E(Y),$ when $X$ and $Y$ are independent.

**b)** Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

**Q.6 Answer the followings.** **16**

**a)** Let $\{X_n\}$be a sequence of random variables such that $X_n \xrightarrow{L} X$ and $c$ be a constant. Show that.

1) $X_n + c \xrightarrow{L} X + c$

2) $c\, X_n \xrightarrow{L} c\, X, c \neq 0$

**b)** Define expectation of simple random variable and expectation of an arbitrary random variable.

If $X \geq 0$ a.s. that show that $E(X) \geq 0$.

**Q.7 Answer the followings.** **16**

**a)** Find lim inf and lim sup of following sequence of sets.

1) $A_n = \left[3, 3 + \frac{5}{n}\right]$

2) $A_n = \left(0, b + \frac{(-1)^n}{n}\right), b > 0$

**b)** Consider the function defined by

$$X(\omega) = \begin{Bmatrix} C_0, \text{if } \omega \in A_0 \\ C_1, \text{if } \omega \in A_1 \\ C_2, \text{if } \omega \in A_2 \end{Bmatrix}$$

Where $C_0, C_1$ and $C_2$ are distinct. Obtain minimum $\sigma -$ field induced by X.