Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-I Sem-II
Paper HCT2.1: PROBABILITY THEORY
Question Bank

SHORT ANSWERS

1 Give two definitions of field and establish their equivalence.

2 Define a field. Prove that every field must contain ϕ and Ω.

3 Prove or disprove: Every field is a field.

4 Define i) Field ii) σ-Field iii) Monotone Field. Give one example of each.

5 Define field and $\sigma-$field of subsets of a set Ω.

6 Prove that every σ-field is a field

7 Define field and $\sigma-\text{field}$. Give an example of field which is not a $\sigma-\text{field}$.

8 Define $\lim\inf$ and $\lim\sup$ of sequence of sets $\{A_n\}$.

9 Define limit of sequence of sets. Show that if $\lim A_n$ exists then $\lim A_n^C$ also exists.

10 If P and Q are probability measures then prove that

$P^*(A) = \alpha\, P(A) + (1-\alpha)\, Q(A),\ 0 \le \alpha \le 1$ is a probability measue.

11 Define i) Random variable ii) Positive and negative part of random variable.

12 Show that any simple function X can be written $X = \sum_{k=1}^{n} x_k\, I_{A_K}$

13 If X is random variable. Prove that 1-X is also a random variable.

14 Define indicator function. With usual notations show that

$I_{A \cap B}(\omega) = \text{Min}\,\{I_A(\omega), I_B(\omega)\}$

15 If X and Y are independent random variables then show that $E(XY) = E(X)E(Y)$.

16 For a non-negative random variable X, prove that $E(X) = \int_0^\infty [1 - F(x)]dx$

17 For a non-negative discrete random variable X, show that $E(X) = \sum_{K=0}^{\infty} P(X > K)$

18 Show that X is integrable if and only if |X| is integrable.

19 Let $|X_n| \le Y$, Y is integrable and $X_n \xrightarrow{a.s.} X$. Show that $E(X_n) \to E(X)$.

20 If $A_n \to A$ show that $A_n^c \to A^c$.

21 If $X \ge Y$ a.s. prove that $E(X) \ge E(Y)$.

22 Define Pairwise and mutual independence of events. State the relationship between them.

23 State and prove dominated convergence theorem.

24 Define convergence in probability and convergence in distribution.

25 If $X \geq Y$ a.s. then show that $E(X) \geq E(Y)$

26 Define characteristic function. Show that it is real iff X is symmetric about origin.

27 If ϕ is characteristic function of random variable X then find characteristic function of $5X+4$.

28 If ϕ is characteristic function of random variable X then find characteristic function of $4X+3$.

29 If ϕ is characteristic function of random variable X then find characteristic function of 2 X-1.

30 Obtain characteristic function when the distribution random variable of X is Binomial with parameters n and p.

31 Define: (i) Weak law of large numbers
(ii) Strong law of large numbers.
(iii) Central limit theorem

32 State i) Chebychev's WLLN and ii) Bernoulli WLLN.

33 State Kolmogorov's three series criterion for almost sure convergence.

34 State i) Liapouniv's CLT ii) Lindeberg-Feller CLT

SHORT NOTES

1 De Morgan's rules for finite union and intersection.

2 Probability measure

3 Mixture of probability measures.

4 Conditional probability measure.

5 Lebesgue measure.

6 Positive and negative part of random variable.

7 Indicator function.

8 Characteristic function of $N(\mu, \sigma^2)$ distribution.

9 Convergence in distribution.

10 Dominated convergence theorem.

11 Strong law of large numbers (SLLN).

12 Weak law of large numbers (WLLN).

13 Bernoulli's weak law of large numbers.

14 Central limit theorem (CLT).

15 Pairwise and mutual independence of events.

16 Kolmogrov three series criteria for almost sure convergence.

17 Dominated convergence theorem.

LONG ANSWERS

1. Define a field. Examine for the class of finite or co-finite sets to be a field.

2. If F_1 and F_2 are fields. Show that

   i) $F_1 \cap F_2$ is a field. ii) $F_1 \cup F_2$ is not a field.

3. Prove or disprove:

   (i) Union of two fields is a field.

   (ii) Intersection of two fields is a field.

4. Define a field and a σ-field. Prove or disprove: Every field is a σ-field.

5. Define a field and a $\sigma-$field. Give an example of a field which is not a $\sigma-$field. Prove that $\sigma-$field is a monotone field.

6. Define $\sigma-$field. Prove that an arbitrary intersection of $\sigma-$fields is also $\sigma-$field.

7. Define a field and $\sigma-$field of subsets of a set Ω. Show that a $\sigma-$field is a field. Is the converse true? Justify.

8. Prove that a non-empty set which is closed under complementation and countable union is a $\sigma-$field.

9. Define monotone field. Prove that every monotone field is a $\sigma-$field.

10. Define monotone decreasing sequence of sets. Prove that if A_n is decreasing sequence of sets then A_n^c is increasing sequence.

11. Define monotone increasing sequence of sets. Prove that if A_n is increasing sequence of sets then A_n^c is decreasing sequence.

12. Define lim inf and lim sup of a sequence of sets. With usual notations show that

    $\overline{\lim}\,(A_n \cup B_n) = \overline{\lim}\,A_n \cup \overline{\lim}\,B_n$.

13. Define a monotone sequence of sets. Prove that its limit exists.

14. Define limit of sequence of sets. Prove or disprove: If $\lim A_n$ exists then $\lim A_n^c$ also exists.

15. Let $\{A_n\}$ be a sequence of sets such that $\lim A_n^c = A^c$. Then show that $\lim A_n = A$.

16. Let $\{A_n\}$ be a sequence of sets such that $\lim A_n = A$. Show that $\lim A_n^c = A^c$

17. Prove that $P\,(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$

18. With usual notations prove that

    $\overline{\lim}\,(A_n \cup B_n) = \overline{\lim}\,A_n \cup \overline{\lim}\,B_n$

19. Let $\{A_n\}$ and $\{B_n\}$ are two sequence of sets such that $A_n \subset B_n$. Prove that

    (i) $\underline{\lim}\,A_n \subset \underline{\lim}\,B_n$ (ii) $\overline{\lim}\,A_n \subset \overline{\lim}\,B_n$

20. Let $\{A_n\}$ and $\{B_n\}$ are two sequence of sets. Prove that

$\overline{\lim}\ (A_n \cup B_n) = \overline{\lim}\ A_n \cup \overline{\lim}\ B_n$

21 Define $\underline{\lim}$ and $\overline{\lim}$ of sequence of sets $\{A_n\}$. Prove that $\underline{\lim}\, A_n \subseteq \overline{\lim}\, A_n$.

22 Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$. Show that $P(\overline{\lim}\, A_n) = 0$.

23 Let $\{A_n\}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) = \infty$. Show that $P(\overline{\lim}\, A_n) = 1$.

24 Define probability measure. State and prove monotone property of probability measure.

25 Define probability measure and conditional probability measure. Show that conditional probability measure satisfies properties of a probability measure.

State and prove continuity property of probability measure.

26 State and prove monotone property of probability measure.

27 Define probability measure. Prove that $P(\lim A_n) = \lim P(A_n)$.

28 Define probability measure. Prove that if P and Q are probability measures then

$P^*(A) = \alpha\ P(A) + (1-\alpha)\ Q(A),\ 0 \le \alpha \le 1$ is a probability measure.

If P_1 and P_2 are probability measures, prove that $P = \alpha P_1 + (1-\alpha) P_2$ is also a probability measure $0 \le \alpha \le 1$.

29 If $A_n \uparrow A$ prove that $P(A_n) \uparrow P(A)$, where P is probability measure.

30 Define a measurable function. Examine for indicator function of a set to be measurable.

31 Define measurable function. Show that an indicator function defined on (Ω, F) is F-measurable.

32 Define a measurable function. Examine for a constant function defined on (Ω, IF) is measurable.

33 Define mapping. Let X be a mapping defined on sample space Ω. Let A and $B \subset \Omega$ such that $A \cap B = \phi$. Prove or disprove: $X(A) \cap X(B) = \phi$.

34 Prove that inverse mapping preserves all the set relations.

35 Define a random variable. Show that Borel function of a random variable is also a random variable.

36 Define a random variable. If X and Y are random variables then prove that $\text{Max}\,\{X, Y\}$ and $\text{Min}\,\{X, Y\}$ are also random variables.

37 Define a random variable. If X is a random variable, examine whether 1-X is also a random variable.

38 Define expectation of simple random variable and arbitrary random variable.

If $X \geq 0$ a.s. then show that $E(X) \geq 0$.

39 If X and Y are arbitrary independent random variables then show that
E (XY) = E (X) E (Y).

40 For two random variables X and Y show that E (X+Y) = E (X) +E (Y).

41 If X and Y are simple random variables then prove that $E(X+Y) = E(X) + E(Y)$.

42 With usual notations prove that,

i) $E(X+Y) = E(X) + E(Y)$.

ii) $E(XY) = E(X)E(Y)$, when X and Y are independent.

43 Define indicator function $I_A(\omega)$. Prove that $I_{A \cap B}(\omega) = I_A(\omega) \cap I_B(\omega)$.

44 Define characteristic function of a random variable x. Show that characteristic function is real iff X is symmetric about origin.

45 Define characteristic function of a random variable X. State uniqueness theorem and inversion formula.

46 Define characteristic function of a random variable X. State and prove its continuity property.

47 Define characteristic function and establish its continuity.

48 Define characteristic function of a random variable X. Examine the effect of change of origin and scale on characteristic function.

49 Define characteristic function and prove any three properties of characteristic function.

50 Let $\phi_X(t)$ be a characteristic function. Prove that

i) $\phi_X(t)$ is continuous.

ii) $\phi_X(t)$ is real if distribution is symmetric about zero.

51 If ϕ_1 and ϕ_2 are characteristic functions prove that for $0 \leq \alpha \leq 1$, $\alpha\phi_1 + (1-\alpha)\phi_2$ is also characteristic function.

52 State inversion formula and obtain the distribution of random variable corresponding to characteristic function (i) $e^{-|t|}$ (ii) e^{-it} and (iii) $\frac{1}{1+t^2}$.

53 Obtain characteristic function when the distribution random variable of X is:
i) Poisson with parameter λ ii) Binomial with parameters n and p.

54 State inversion formula and obtain the probability distribution of random variable corresponding to characteristic function .

55 State and prove monotone convergence theorem.

56 State and prove Fatou's theorem.

57 State and prove Slustky's theorem.

58 State and prove dominated convergence theorem.

59 State and prove Borel-Cantelli lemma.

60 Define various modes of convergence of sequence of random variables.

61 Define convergence in probability. State and prove necessary and sufficient condition for convergence in probability.

62 Define convergence in probability and convergence in distribution. Prove that convergence in probability implies convergence in distribution.

63 Define convergence in probability and convergence in distribution. Establish inter-relationship between them.

64 Define almost sure convergence. Prove that almost sure convergence implies convergence in probability.

65 Define almost sure convergence and convergence in r^{th} mean. If $X_n \xrightarrow{r} X$ then prove that $X_n \xrightarrow{P} X$

66 If $X_n \xrightarrow{r} X$ show that $E|X_n|^r \longrightarrow E|X|^r$

67 Prove that $X_n \xrightarrow{P} 0$ if and only if $E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$ as $n \to \infty$.

68 Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$.

Show that i) $X_n + Y_n \xrightarrow{P} X + Y$ ii) $X_n Y_n \xrightarrow{P} XY$

69 Let $\{X_n\}$ be a sequence of random variables such that $X_n \xrightarrow{L} X$ and c be a constant. Show that i) $X_n + c \xrightarrow{L} X + c$ ii) $c\,X_n \xrightarrow{L} cX$, $c \neq 0$

70 Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$. Prove that :

i) $aX_n \xrightarrow{P} aX$ (a is real). ii) $X_n + Y_n \xrightarrow{P} X + Y$.

71 If $X_n \xrightarrow{r} X$ show that $E|X_n|^r \longrightarrow E|X|^r$

72 If $X_n \xrightarrow{a.s.} X$ then for continuous function f, show that $f(X_n) \xrightarrow{a.s.} f(X)$.

73 Let $\{X_n, Y_n\}, n = 1, 2, ...$ be a sequence of pairs of random variables and c be a constant. Prove that:

i) $X_n \xrightarrow{L} X, Y_n \xrightarrow{P} c$ then $X_n + Y_n \xrightarrow{L} X + c$.

ii) $X_n \xrightarrow{L} X, Y_n \xrightarrow{P} c$ then $X_n Y_n \xrightarrow{L} cX, c \neq 0$.

74 Let $\{X_n, Y_n\}, n = 1, 2, ...$ be a sequence of random variables such that $|X_n - Y_n| \xrightarrow{P} 0$, $Y_n \xrightarrow{L} X$ then prove that $X_n \xrightarrow{L} X$.

75 Let $\{X_n, Y_n\}, n = 1, 2, ...$ be a sequence of pairs of random variables and C be a constant. Prove that

(i) If $X_n \xrightarrow{L} X$, $Y_n \xrightarrow{P} C$ then $\begin{cases} X_n Y_n \xrightarrow{L} CX, & C \neq 0 \\ X_n Y_n \xrightarrow{P} 0, & C = 0 \end{cases}$

(ii) If $X_n \xrightarrow{L} X$ then $\frac{X_n}{Y_n} \xrightarrow{L} \frac{X}{C}$, if $C \neq 0$.

76 Let $|X_n| \leq Y$ a.s. and Y is integrable.

Show that $X_n \xrightarrow{P} X \Rightarrow E(X_n) \to E(X)$

77 If $Y \leq X_n$ and Y is integrable then show that $E(\underline{\lim}\, X_n) \leq \underline{\lim}\, E(X_n)$.

78 If $X_n \leq Y$ and Y is integrable then show that $E(\overline{\lim}\, X_n) \geq \overline{\lim}\, E(X_n)$.

79 Describe WLLN for a sequence of random variables. Prove that WLLN holds for sequence of Bernoulli random variables.

Explain the weak and strong law of large numbers.

80 Describe weak law of large numbers (WLLN) for sequence of independent random variables. Prove that WLLN holds for the sequence of Bernoulli random variables

81 State Lindberg-Feller central limit theorem (CLT) and deduce Liapunov's CLT as a special case.

82 State i) Liapounov's CLT ii) Lindberg-Feller CLT. Show that Liapounov's condition for CLT implies Lindberg's condition.

PROBLEMS

1 Let A_n be set of points (x, y) of Cartesian plane lying within the rectangle bounded by two axes and the lines x = n and $y = \frac{1}{n}$. Find $\lim A_n$ if exists**.**

2 Let $A_n = \begin{cases} A, \text{ if } n \text{ is even} \\ B, \text{if } n \text{ is odd} \end{cases}$. Find i) $\underline{\lim}\, A_n$ ii) $\overline{\lim}\, A_n$

3 If $A_n = (1-\frac{1}{n},\ 2+\frac{1}{n}], n \geq 1$. Find $\lim\limits_{n\to\infty} A_n$.

4 Find $\lim\inf$ and $\lim\sup$ of following sequence of sets.

(i) $A_n = \left(1+\frac{1}{n}, 2\right)$ (ii) $A_n = \left[b, b+\frac{1}{n}\right]$.

5 Let $A_n = \{\omega : 0 < \omega < 1+\frac{1}{n}\}$. Find $\lim A_n$.

6 Find $\lim\inf$ and $\lim\sup$ of following sequence of sets.

(i) $A_n = \left(1+\frac{1}{n}, 3+\frac{2}{n}\right)$ (ii) $A_n = \left(0,\ a+b\,(-1)^n\right), a > b > 0$.

7 Let $A_n = \begin{cases} A, \text{ if } n \text{ is even} \\ B, \text{if } n \text{ is odd} \end{cases}$. Find limit of A_n if exists.

8 Find $\lim\inf$ and $\lim\sup$ of following sequence of sets.

(i) $A_n = \left[3, 3+\frac{5}{n}\right]$ (ii) $A_n = \left(0,\ b+\frac{(-1)^n}{n}\right), b > 0$.

9 Find $\lim\inf$ and $\lim\sup$ of following sequence of sets.

(i) $A_n = \left(0,\ 1+\frac{1}{n}\right)$ (ii) $A_n = \left(0,\ 3+(-1)^n\left(1+\frac{1}{n}\right)\right)$.

10 Find $\lim\inf$ and $\lim\sup$ of following sequence of sets.

(i) $A_n = \left(1+\frac{1}{n},\ 2+\frac{1}{n}\right)$ (ii) $A_n = \left[a-\frac{1}{n}, a\right]$

11 Find $\lim\inf$ and $\lim\sup$ of following sequence of sets.

(i) $A_n = \left(0,\ 1+\frac{1}{n}\right)$ (ii) $A_n = \left[\frac{-1}{n}, 3-\frac{1}{n}\right]$

12 Let $A_n = \left(0,\ 1+\frac{1}{n}\right)$. Find $\lim A_n$ if exist.

13 Find $\lim A_n$ if exist. $A_n = \left(1+\frac{1}{n}, 2+\frac{1}{n}\right)$

14 Define Lebesgue measure and Lebesgue-Steltjes measure. Let μ be the measure induced by the function g,

$$g(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } 0 \le x < 3 \\ 4, & \text{if if } x \ge 3 \end{cases}$$

Find $\mu[-1, 1]$ and $\mu[2, 6]$

15 Define a measurable function. Suppose $X(\omega)$ takes three different values such that

$$X(\omega) = \begin{cases} C1, & \omega \in A_1 \\ C2, & \omega \in A_2 \\ C3, & \omega \in A_3 \end{cases}$$

and $C_i \in IR$, $i = 1, 2, 3$. Discuss measurability of X.

16 Let E be an experiment having two outcomes 'success' S and 'failure' F respectively. Let $\Omega = \{S, F\}$ and $IF = \{\phi, S, F, \Omega\}$. Define $X(\omega) = \begin{cases} 1, & \text{if } \omega = S \\ 0, & \text{if } \omega = F \end{cases}$

Examine whether X is random variable with respect to IF.

17 Let $X : \Omega \to \Omega'$. Let A and B are subsets of Ω'.

(i) If $A \cap B = \phi$ then show that $X^{-1}(A) \cap X^{-!}(B) = \phi$

(ii) If $A \subset B$ then show that $X^{-!}(A) \subset X^{-1}(B)$

18 Consider the function defined by

$$X(\omega) = \begin{cases} C_0, & \text{if } \omega \in A_0 \\ C_1, & \text{if } \omega \in A_1 \\ C_2, & \text{if } \omega \in A_2 \end{cases}$$

where C_0, C_1 and C_2 are distinct. Obtain minimum σ-field induced by X.

19 Obtain the distribution of random variable corresponding to each characteristic

function (i) $\phi_x(t) = e^{-|t|}$ (ii) $\phi_x(t) = \dfrac{1}{1+t^2}$ (iii) $\phi_X(t) = e^{\frac{-t^2}{2}}$

20 Obtain the characteristic function if the distribution of random variable X is

$$f(x,\theta) = \tfrac{1}{\theta} e^{-\frac{x}{\theta}}, x \geq 0$$

21 Obtain characteristic function when distribution of random variable is:

i) $f(x) = \theta e^{-\theta x}$, $x \geq 0$, $\theta > 0$.

ii) $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $-\infty < x < \infty$, $-\infty < \mu < \infty$, $\sigma^2 \geq 0$.

Punyashlok Ahilyadevi Holkar Solapur University, Solapur.

Department of Statistics

M.Sc. Part-I Sem-II

OET 2.1: Statistical Methods

Question Bank

Short answers

1. Define any two i) Arithmetic mean ii) Median iii) mode.
2. Write a note on Chi-square test for independence.
3. Write a note on Regression.
4. Define any two i) Sample space ii) Probability of an event
   iii) Mutually exclusive events.
5. Explain addition and multiplication rules of probability.
6. Explain the Signed-rank test.
7. What is Central tendency?
8. Define probability mass function. Explain Binomial and Poisson distributions.
9. Define uniform distribution over (a, b). If X follows $U(0, 1.5)$, then find
   $i)P\ (X > 0.6)$ $ii)P\ (X < 0.3)$.
10. Write a note on Spearman's rank correlation coefficient.
11. In a moderately skewed distribution, the values of mode and mean are 32.1 and 35.4 respectively. Find the value of median.
12. Define standard deviation and quartile deviation. State with reasons which is better**?**
13. Write a note on critical region.
14. Define any two i) Mode ii) Geometric Mean iii) Harmonic Mean
15. Define any three i) Classical definition of Probability ii) Random experiment
    iii) Sample space iv) Trial
16. If X has Poisson distribution such that P(X=1) = 2P(X=2), then find P(X=0). Also find mean and variance of X.
17. Give real life examples of Poisson distribution
18. Define Alternative hypothesis with an example
19. Write a note on regression coefficient

20. Write a note on Karl Pearson's coefficient of correlation

Long answers

1. Define binominal distribution. Give its mean and variance. If random variable X follows binomial distribution with parameters n=12 and p = 0.4,then find $P(X = 2), P(X < 2)\ and\ P(X > 2)$.
2. Define Normal distribution and exponential distribution. Also give their mean and variance.
3. Describe a large sample test procedure for testing the proportion Pin the population has specified value P_0. In a random sample from a college, there are 48 boys and 42 girls. At 5% level of significance, do these figures confirm that male and female students are equal in proportion in the college?
4. Describe the chi-square test for independence of attributes.
5. Describe the sign test for testing that the population median is M_0 against The alternative that the median is $M_1 > M_0$.
6. Calculate the mean, mode and median for the following frequency distribution

Class	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	12	14	17	18	12	10

7. The certain objects were ranked by two persons (A and B) according to the quality perspective. Following data provides the rankings given by them.

$Object\ No.$	1	2	3	4	5	6	7	8	9	10
A	7	6	2	1	3	10	4	9	5	8
B	10	5	1	2	3	8	6	7	4	9

Find Spearman's rank correlation coefficient and interpret the results

8. Following are the marks of students in two subjects

$Physics$	35	23	47	17	10	43	9	6	28
$Mathematics$	30	33	45	23	8	49	12	4	21

Compute the Karls Pearson's correlation coefficient and interpret the results

9. The marks of 25 students are given below.

Marks	0-10	10-20	20-30	30-40	40-50
students	3	4	10	------	1

Calculate the missing frequency and hence obtain the values of mean, median and mode.

10. Define Karl Pearson's correlation coefficient and regression coefficients. State the relation between correlation coefficient and regression coefficients. If sign of one regression coefficient is known, how will you decide the sign of correlation coefficient and other regression coefficient?

11. Define probability mass function of random variable X. A discrete random variable X has the following probability distribution.

X	-2	-1	0	1	2	3
$P(X = x)$	0.1	k	0.2	$2k$	0.3	$3k$

1) Find the value of k.

2) Evaluate $P(X \geq 2)$ and $P(-2 < X < 2)$.

12. Define binomial distribution with parameters n and p. If X is a binomial vitiate with $n = 5$ and $p = 0.3$, Find 1) $P(X = 3)$ 2) $P(X < 3)$

13. Define exponential distribution with parameter λ. Suppose life time of automobile batteries is exponentially distributed with mean 1000 days. What is the probability that such a battery will last more than 1200days?

14. Following data shows the performance of two batsmen A and B

	$Batsman A$	$Batsman B$
$Number\ of\ innings$	50	40
$Mean\ runs$	55	50
$Standard\ deviation$	8	10

Which batsman is more consistent in score? Which batsman is better in scoring the runs.

15. What do you mean by testing of hypothesis? State simple and composite hypothesis. Explain the term Test Statistic?

16. What do you understand by skewness and kurtosis? Explain the types of skewness and kurtosis by suitable diagrams.

17. Define binomial distribution with parameters n and p. For a binominal distribution find n and p if mean is 4 and standard deviation is$\sqrt{3}$.

18. Define probability mass function (pmf) and probability density function (pdf) of a random variable X. Determine k such that the following function are pdf/pmf

$$P(X = x) = \frac{k\,2^x}{x!}, \quad x = 0,1,2,3$$

$$f\,(x) = \frac{k(3-x)}{x!}, 0 \le x \le 3$$

19. Define normal distribution. Discuss the properties of normal distribution.

20. A life time of a certain battery is a random variable which has an exponential distribution with mean 320 hours. Find the probability that such a battery will last at most 160 hours. Also find the probability that a battery will last between640 and 960 hours.

21. Describe a test procedure for testing hypothesis $H_0: \mu = \mu_0$ against alternative $H_1: \mu \neq \mu_0$ for a large sample. The mean life time of 100 electric blubs produced by a manufacturing company is estimated to be 1570 hours with standard deviation of 120 hours. If μ is the mean life time of all blubs produced by the company, test the hypothesis μ = 1600 against the alternative μ ≠1600 hours using level of significance 0.05.

22. Define uniform distribution over (a, b). The radius X of a ball bearing has uniform distribution over (0, 1.5). Find $i)\ P\ (X > 0.5)$ $ii) P\ (X < 0.4)$ $iii) P(0.3 < X < 1.2)$.

23 Define the contingency table? How do you test independence of attributes with (2×2) contingency table?

Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-I Sem-II
Statistics Paper: HCT2.2
Title: Stochastic Processes
<u>Question Bank</u>

Short Answers

1. Define and illustrate Markov chain. Show that initial distribution and TPM specifies the Markov chain completely.
2. State and prove Chapman-Kolmogorov equations.
3. Give classification of Stochastic processes according to state space and time domain.
4. Let {Xn} be a stochastic process with state space= {1,2,3} and initial distribution [1/2,1/4,1/4]' and tpm P as

$$P = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 1/6 & 5/6 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

   Then find : i) $P(X_3 = 2 / X_1 = 1)$ ii) $P(X_2=1)$ iii) $P(X_1=2)$
5. State and illustrate : i) State space ii) Stochastic Process iii) TPM
6. Define and explain Markov property.
7. Explain the concept of first return and probability of ultimate return to a state i.
8. Define and illustrate Persistent and transient state.
9. For a Markov chain $\{X_n\}$, tpm is as given below. Classify the states as persistent or transient.

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/7 & 2/7 & 4/7 \\ 1/6 & 1/3 & 1/2 \end{bmatrix}$$

10. What is transition probability matrix?
11. Write a note on Gambler's ruin problem
12. Simulation of Possion process
13. State and prove first entrance theorem

14. Define processes with independent increments. Give an example whereassumption of processes with independent increment is not suitable withjustification.
15. Explain Yule-Furry process.
16. State key renewal theorem.
17. Prove that, Markov chain is completely specified by one step t.p.m. and initial distribution
18. Let $\{N(t),\ t \geq 0\}$ be a Poisson process with rate λ and s, t be two points such that $s < t$ then find the conditional distribution of $N(s)$given $N(t) = n$.
19. Write an algorithm for simulation of Markov chain.
20. Write a note on Poisson Process.
21. Write a short note on Mean recurrent time of a state.
22. Discuss probability of first return for a state.
23. Discuss stationary distribution of a Markov chain.
24. Write a note on counting process.
25. Describe Poisson Process. State postulates of this process.
26. Define periodicity of Markov chain and give an example of Markov chain which is periodic. Also give example of aperiodic Markov chain.
27. State Markov properly for stochastic process. State and prove Chapman-Kolmogorov equation for Markov chain.
28. Suppose, for a branching process, the offspring distribution is geometric distribution with probability of success 0.2. Find probability of extinction.
29. Write a short note on Delayed renewal process.

Long Answers

1. Prove that, Markov chain is completely specified by one step t.p.m. and initial Distribution
2. Describe gambler's game. If a gambler starts the game with initial amount 'i', find his winning probability.

3. Classify the states of random walk model.

4. Give two definitions of Poisson Process. Show that addition of two Poison processes is a Poisson process.
5. Establish the equivalence between two definitions of Poisson process.

6. Define stationary distribution of a Markov chain. Find the same for a Markov chain with state space {1,2,3}, whose tpm is

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/3 & 1/3 & 1/3 \\ 2/5 & 1/5 & 2/5 \end{bmatrix}$$

7. Define branching process. Derive expression for the mean of the population sizeat n^{th} generation.
8. Define pure birth process and obtain its probability distribution.
9. Describe M/M/S queuing model.
10. Describe birth and death process and obtain its Kolmogorov differentialequations.
11. Define branching process. With usual notations, obtain its mean and variance.
12. Stating the postulates, derive the probability distribution of a Poisson process with rate λ.
13. A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or badly. Let p_i denote theprobability that the class does well on a type i exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type i, i = 1, 2, 3?
14. Describe M/M/1 queuing model.
15. Discuss the classification of stochastic processes according to state space and index set.
16. Let $\{X_n, n \geq 0\}$ be a Markov chain with state space S={0, 1, 2 },t.p.m$P = \begin{bmatrix} 0.6 & 0 & 0.4 \\ 0 & 0.6 & 0.4 \\ 0.4 & 0 & 0.6 \end{bmatrix}$ and initial distribution (0.5, 0.5, 0).

Compute

i. $P(X_2 = 1,\ X_0 = 1)$

ii. $E(X_2)$

17. Consider that a store stocks a certain items, the demand for which is given by given by $\mathrm{p_k} = \mathrm{P}(\text{k demands of the item in a week})$, $\mathrm{p_0} = 0.2, \mathrm{p_1} = 0.7, \mathrm{p_2} = 0.1$. Stocks are replenished at weekends according to the policy: not to replenish if there is any stock in store and to obtain 2 new items if there is no stock. Let $\mathrm{X_n}$ be the number of items at the end of the n^{th} week, just before weeks replenishment, if any and $\mathrm{P}(\mathrm{X_0} = 3) = 1$. Here$\{\mathrm{X_n}, \mathrm{n} \geq 0\}$be a Markov chain. Then

i. Write down the transition probability matrix

ii. What is the probability that at the end of second week, just before weeks replenishment, if any, there are 3 items in store?

If at the end of second week, just before weeks replenishment, if any, there is 1 item in store, what is the probability that at the end of fourth week, just before weeks replenishment, if any, there will be 0 items in store?

18. Describe birth and death process and obtain its Kolmogorov differential equations.

19. Verify the states of random walk model for persistency as well as for periodicity

20. Consider the problem of sending a binary message, 0 or 1, through a signal channel consisting of several stages, where transmission through each stage is subject to a fixed probability of error α. Suppose that X_0 = 0 is the signal that is sent and let X_n be the signal that is received at the n^{th} stage. Assume that $\{X_n\}$ is a Markov chain.

i) Determine the transition probability matrix of $\{X_1\}$

ii) Determine the probability that no error occurs up to stages n = 2

iii) Determine the probability that a correct signal is received at stage 2.

21. Prove that persistency is a class property

22. Discuss Gambler's ruin problem in detail.

23. Calculate the extinction probability for branching process.

24. Explain Gamblers ruin problem. Obtain the probability that starting with i units the Gamblers fortune will reach N before reaching zero.

25. If {N(t)} is a Poisson process, then for s < t, obtain the distribution of N(s),

if it is already known that N(t)=k.

26. Show that recurrence is a class property.

27. A Markov chain with state space S={1,2,3} has tpm $\begin{bmatrix} 0.2 & 0.4 & 0.4 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$. It is known that the process has started with the state $X_0=2$

i) $P(X_1 = 2)$

ii) $P(X_2 = 3)$

iii) $P(X_0=1)$

iv) $P(X_3=2/X_1=1)$

28. Prove that a state j of a Markov chain is recurrent if and only if $\sum p_{jj}{}^{(n)} = \infty$.

29. State and prove class property of periodicity.

30. Write down the algorithm for the simulation of Poisson process and branching process.

Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-II Sem-IV
Statistics Paper HCT4.4: Optimization Techniques
Question Bank

Short answers

1. Explain a dynamic programming problem.
2. Describe two persons zero sum game.
3. Define general linear programming problem. Also explain the terms: solution and feasible solution.
4. Write a short note on Big-M method.
5. Write a short note on effect of Addition and deletion of variable on optimal solution of LPP.
6. Write a short note on Recursive equation approach.
7. Develop necessaryKKT conditions for an optimal solution to a quadratic programming problem.
8. Write down graphical procedure to solve two persons zero sum game.
9. State and prove basic duality theory.
10. Explain Branch and Bound method to solve integer linear programming.
11. Write down characteristics of dynamic programming.
12. Write a note on Dominance property.
13. Write a note on Non-linear programming problem.
14. Find the maximum value of $Z = 50x_1 + 60x_2$, subject to constraints
$$2x_1 + 3x_2 < 1500,\ \ 3x_1 + 2x_2 \leq 1500,\ \ 0 \leq x_1 \leq 400,\ \ \ 0 \leq x_2 \leq 400$$
15. Solve the following game with payoff matrix of player A
$$\begin{array}{cc} & \text{Player B} \\ \begin{array}{c}\text{Player}\\ \text{A}\end{array} & \begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix} \end{array}$$
16. Write a note on Sensitivity analysis.
17. State and prove weak duality theorem.
18. State and prove strong duality theorem.

    Explain the following terms
    a) Two persons zero sum game
    b) Pure and mixed strategies

Long answers

1. Explain Gomory's fractional cut method to solve integer programming problem.

2. Use dynamic programming to solve the following LPP

$$\text{Max } Z = 3x_1 + 5x_2$$

Subject to

$x_1 \leq 4, x_2 \leq 6, 3x_1 + 2x_2 \leq 18, \; x_1, x_2 \geq 0$

3. Use Branch and Bound method to solve following integer programming problem
$Maximize\ Z = 7x_1 + 9x_2$, subject to constraints
$-x_1 + 3x_2 < 6, \;\; 7x_1 + x_2 \leq 35, \quad x_2 \leq 7, \;\; x_1, x_2 \geq 0$ *and integers*

4. Obtain optimum strategies and value of the game with payoff matrix of player A is given below,

$$\begin{bmatrix} 2 & 3 & 11 \\ 7 & 5 & 2 \end{bmatrix}$$

5. Write down dual simplex algorithm.
6. Write down simplex algorithm to solve linear programming problem.
7. Explain the terms convex set and convex combinations. Also show that set of all feasible solutions is convex.
8. Use simplex method to solve following game

$$\begin{array}{cc} & \text{Player B} \\ \begin{array}{c}\text{Player} \\ \text{A}\end{array} & \begin{pmatrix} 4 & 2 & 4 \\ 2 & 4 & 1 \\ 4 & 1 & 8 \end{pmatrix} \end{array}$$

9. Solve following LPP using dynamic programming
$Miaximize\ Z = 3x_1 + 7x_2$, subject to constraints $x_1 + 4x_2 < 8, \quad 0 \leq x_2 \leq 2, x_1 \geq 0$
10. Obtain the range of change in $b_i$values to maintain feasibility of the optimal solution.
11. Describe effect of change in coefficients of objective function $c_j's$ in sensitivity analysis.
12. Solve the following quadratic problem using Beal's method

$$\text{Max } Z = 2x_1 + x_2 - x_1^2$$

Subject to

$2x_1 + 3x_2 \leq 6, \; 2x_1 + x_2 \leq 4, \; x_1, x_2 \geq 0$

13. Solve the following LPP

$$Maximize\ Z = -x_1 + 2x_2 - x_3$$

sub to

$$3x_1 + x_2 - x_3 \leq 10$$
$$-x_1 + 4x_2 + x_3 \geq 6$$
$$x_2 + x_3 \leq 4$$
$$x_1, x_2, x_3 \geq 0$$

14. Use two phase method to solve following

$$Maximize\ Z = 5x_1 + 3x_2,$$

subject to,

$$2x_1 + x_2 \leq 1,$$
$$x_1 + 4x_2 \geq 6,$$
$$x_1, x_2 \geq 0$$

Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-I (Sem-II)
Statistics Paper: HCT-2.3
Title of Paper: Theory of Testing of Hypotheses
<u>Question Bank</u>

Short Answer

1	Explain the terms: i) Test function ii) Power of test
2	Define simple hypothesis and composite hypothesis. Give one example of each.
3	Define null hypothesis and alternative hypothesis. Give one example of each.
4	Explain the terms: i) Randomized test ii) Non-randomized test. Give one example for each.
5	Distinguish between randomized and non-randomized tests.
6	Explain size and power of a test.
7	Define two kinds of errors and power of a test. Which error is minimized in statistical test? Why not both errors?
8	Define two kinds of errors and power of a test.
9	Explain probabilities of type I and type II errors.
10	Define critical region and power function of a test.
11	What do you understand by randomized test procedure?
12	Define non-randomized test. Give an example.
13	Define level of significance and size of a test.
14	Show that power of MP test is greater than α, if the size of test is α.
15	If β is power of MP test ϕ of size α then prove that $\beta \geq \alpha$.
16	Define monotone likelihood ratio (MLR) property.
17	Show that family of Cauchy densities does not possess MLR. Property.
18	Show that one parameter exponential family has monotone likelihood ratio.
19	Define unbiased test. Prove or disprove: A UMP test is unbiased.
20	Define the following: i) UMP test ii) UMP unbiased Test
21	Explain unbiased test. Give one example of the same.
22	Show that every UMP test is UMPU of same size.
23	Define $(1-\alpha)$ level confidence set.

24 Investigate the relationship between confidence estimation and hypothesis testing.

25 Define uniformly most accurate confidence region.

26 Define uniformly most accurate unbiased family of confidence sets.

27 State the relation between UMP size α acceptance region and UMA family of confidence sets at level $(1-\alpha)$.

28 State the relation between UMP unbiased size α acceptance region and UMAU family of confidence sets at level $(1-\alpha)$.

29 Describe a test with Neyman structure.

30 Define similar test and α-similar test.

31 State the relationship between a similar test and a test with Neyman structure.

32 State a necessary and sufficient condition for a similar test to have Neyman structure.

33 Define pivotal quantity. Describe pivotal quantity method to obtain confidence interval of parameter θ.

34 What is goodness of fit test? Give its application.

35 Distinguish between parametric and nonparametric tests.

36 Describe sign test in brief.

37 Describe signed-rank test in brief.

38 Describe Wald-Wolfowitz run test.

39 State one sample and two sample U statistic theorems.

40 Describe Mood's test for two sample problem of scale.

Short Notes

1 Generalized Neyman-Pearson lemma.

2 Likelihood ratio test.

3 Randomized and Non-randomized tests.

4 Shortest length confidence interval.

5 Test for independence of attributes.

6 Testing independence in contingency tables.

7 One sample U statistics.

8 Type I and type II errors.

9 Kendall's rank correlation test.

10 Test with Neyman-structure.

11 Shortest length confidence interval.

12 Relationship between confidence estimation and hypothesis testing.

13 Pivotal quantity method for construction of confidence interval.

14 Non-existence of UMP test

15 Chi-square test for goodness of fit.

16 Chi-square test for contingency tables

17 Nonparametric tests.

18 Goodness of fit problem.

19 Mann-Whitney test.

20 MLR property

21 Neyman-Pearson lemma

Long Answer

1 Define the two kinds of errors and the power of a test. Which error is minimized in a statistical test? Why not both the errors?

2 Which of the two types of error is considered to be more serious in testing statistical hypothesis? Explain with appropriate examples.

3 Define simple and composite hypotheses. State and prove (necessary part only) N-P lemma for testing simple hypothesis against simple alternative.

4 State Neyman-Pearson lemma and prove sufficient condition for a test to be most powerful.

5 State Neyman Pearson lemma for randomized tests and prove the sufficient part of the lemma.

State Neyman Pearson lemma. Show that test given by N-P lemma is unique (except on null set).

6 State Neyman Pearson lemma. Show that power of MP test given by N-P lemma is at least its size.

7 Define size and power of a test. Show that power of N-P test exceeds its size.

8 Let ϕ be a MP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$. Prove that ϕ is necessarily unbiased.

9 Define most powerful (MP) test. Explain the method of obtaining MP test of size α for testing simple hypothesis against simple alternative.

10 Define most powerful (MP) test. Show that MP test need not be unique using suitable example.

11 Illustrate with an example that that MP test need not be unique.

12 Show that if a sufficient statistic T exists for the family $\{f_\theta(x), \theta \in \Theta)$, $\theta = (\theta_0, \theta_1)$, the MP test is a function of T.

13 Show that for a family having MLR property, there exists UMP test for testing one sided hypotheses against one sided alternative.

14 Define monotone likelihood ratio (MLR) property of a family of distributions. Explain the use of MLR in the construction of UMP test with the help of suitable example.

15 State monotone likelihood ratio (MLR) property of a family of distributions. Use the property to test $H_0 : \theta = \theta_0$ against $H_1 : \theta > \theta_0$ for a random variable having distribution function $F(x; \theta)$.

16 When a family of densities is said to have monotone likelihood ratio? Show that the one-parameter exponential family of densities belongs to this class of MLR densities.

17 Define MLR property of a family of distributions. Give an example of a distribution which does not have MLR property.

18 When is *pdf* (or *pmf*) $f(x; \theta)$ said to possess monotone likelihood ratio (MLR) property? Show that exponential distribution with mean θ possess MLR property. Give an example of density which does not have MLR property.

19 Define unbiasedness of test and explain why such test is more desirable.

20 Define unbiased test. Show that MP level α test is always an unbiased test.

21 Define UMP test. Show that UMP level α test is always an unbiased test.

22 Let X has a distribution belonging to one parameter exponential family. Show that there exist a UMP test of level α for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$. State the property of power function of this test.

23 Let X has a density $f(X;\theta), \theta$ is real and the family of densities has MLR property. Derive UMP test for $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

24 Define UMP unbiased test. Show that every UMP test is UMP unbiased of the same size.

25 State the generalized Neyman-Pearson lemma. Also explain in detail any one of its application.

26 Show that no UMP test exists for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$ in one parameter exponential family of distributions.

27 Define i) Unbiased test ii) UMPU test. Prove that every UMP test is UMPU of same size.

28 Define unbiased test. In case of one parameter exponential family of distributions derive suitable optimal test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0 (\theta_0$ specified).

29 Define UMPU test. Prove that every UMP test is UMPU of same size.

30 Explain the concepts of UMPU tests and show that MP and UMP tests of size α are unbiased.

31 Define a family of uniformly most accurate (UMA) confidence sets for an unknown parameter θ, with confidence level $(1-\alpha)$, where $0 < \alpha < 1$.

32 Define (i) UMA confidence interval and (ii) UMAU confidence interval. State and prove the result useful in obtaining UMA confidence interval using suitable test.

33 Define confidence set and UMA confidence set of level$(1-\alpha)$. Derive the relationship between UMA confidence set and UMP test.

34 Define confidence set and UMA confidence set. Derive the relation between UMA confidence set and UMP test.

35 Define UMA confidence intervals. How are they obtained from UMP tests?

36 Define UMA confidence interval. Obtain one sided UMA confidence interval for θ based on a sample of size n from exponential distribution with mean θ.

37 Define a family of UMAU confidence sets with a given confidence coefficient $1-\alpha$ for a parameter θ.

38 Derive a UMAU confidence set for σ^2 with confidence coefficient $1-\alpha$ in sampling from $N(\mu, \sigma^2)$ with μ unknown. Show that this set is actually an interval.

39 Define (i) similar test and (ii) test having Neyman structure. State the result connecting similar test with Neyman structure.

40 Define (i) similar test and (ii) test having Neyman-structure. State and prove the conditions under which a UMP size α similar test is UMPU test. Give one example of the same.

41 Discuss the relationship between a similar test and a test with Neyman structure.

42 State and prove a necessary and sufficient condition for a similar test to have Neyman structure.

43 Prove that a test with Neyman-Structure is similar. Is the converse true?

44 Define a similar test. Obtain similar test for $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 < \sigma_0^2$ when samples are drawn from $N(\mu, \sigma^2)$ with μ unknown.

45 Let power function of every test ϕ of $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$ be continuous in θ. Then show that a UMP $\alpha -$similar test is UMP unbiased provided that its size is α for testing H_0 against H_1.

46 Define shortest length confidence interval. Describe the pivotal quantity method to obtain the shortest length confidence interval.

47 Describe likelihood ratio test procedure for testing $H_0 : \theta \in \Theta_0$ against $H_1 : \theta \in \Theta_1$ and state large sample properties of the test.

48 Describe likelihood ratio test (LRT). Show that LRT for testing simple hypothesis against simple alternative is equivalent to Neyman-Pearson test.

49 Explain the likelihood ratio (LR) test for testing hypothesis. Show that LR tests are consistent.

50 Prove that for a given α $(0 \le \alpha \le 1)$, nonrandomized *N-P* and *LR* tests exists for a simple against simple alternative are equivalent.

51 Prove that likelihood ratio test for testing simple hypothesis against simple alternative is most powerful.

52 Discuss the use of chi-square test in goodness of fit problem.

53 Describe a goodness of fit test based on chi-square distribution.

54 Describe clearly (i) one-sample sign test and (ii) paired-sample sign test. State the assumptions and nature of hypothesis being tested in each case.

55 Discuss Wilcoxon Signed-Rank test?

56 Stating the hypothesis, explain two-sample Wilcoxon-Mann-Whitney test and state mean of the test statistic.

57 Discuss Wald-Wolfowitz Runs test to examine if two samples of sizes n_1 and n_2 come from an identical population, against the alternative that the two populations from which the two samples have been taken, differ in any respect whatsoever.

58 Explain the run test to test randomness.

59 Define U statistic. State one sample and two sample U statistic theorems.

60 Describe Kolmogorov-Smirnov test for the two sample problem. Consider both one sided and two sided hypotheses.

61 Describe Ansari-Bradley and Siegel-Tukey tests for a two sample scale problem.

62 Describe Mood and Sukhatme tests for a two sample scale problem.

63 Describe Kruskal-Wallis test to test whether k independent samples have been drawn from identical populations.

Problems

1. A sample of size one is taken from Poisson distribution with parameter λ. Let $H_0 : \lambda = 1$ *and* $H_1 : \lambda = 2$. Consider the test function

$$\phi(x) = \begin{cases} 1, & x > 3 \\ 0, & otherwise \end{cases}$$

Find probability of Type-I error and power of the test.

2. Let $X \sim B(6, \theta)$. For testing $H_0 : \theta = 1/2$ against $H_1 : \theta = 3/4$, the test is given as, reject H_0 if $x = 0, 6$. Compute the probabilities of type I and type II errors and power of test.

3. Let $X \sim B(n, p)$. For testing $H_0 : p = 1/4$ against $H_1 : p = 1/2$, consider the test function $\phi(x) = \begin{cases} 0.3, & \text{if } x = 0 \\ 0.2, & \text{if } x = 1 \\ 0, & \text{otherwise} \end{cases}$

Find the probability of type I error of the test function ϕ.

4. Let X have $B(n, p)$ distribution with $n = 10$ and p in parameter set $\{2/3, 1/3\}$. A test rejects the null hypothesis $H_0 : p = 2/3$ for the alternative $H_1 : p = 1/3$, if observed value of X, a random sample of size 1, is less than or equal to 2. Find power function of the test if observed value is 3.

5. A single observation of a random variable having geometric distribution with *pmf*

$$f(x;\theta) = \begin{cases} \theta(1-\theta)^{x-1}, & x = 1, 2, ... \\ 0, & \text{otherwise} \end{cases}$$

The null hypothesis $H_0 : \theta = 0.5$ against alternative $H_1 : \theta = 0.6$ is rejected if the observed value of random variable is ≥ 5. Find probability of type I and type II errors.

6. Let $f_0(x) = 2x$, $0 < x < 1$ and $f_1(x) = x$, $0 < x < 1$. Find the power of MP test of size α.

7. To test $H_0 : \theta = 1$ against $H_1 : \theta = 0$ for a single observation from following distribution is used

$$f(x;\theta) = \begin{cases} (2\theta x + 1 - \theta), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Find MP test of level α and find its power.

8 Let X be a single observation from exponential distribution with mean θ. For testing $H_0: \theta \le 1$ against $H_1: \theta > 1$, examine whether the test

$$\phi(x) = \begin{cases} 1, & x > c \\ 0, & otherwise \end{cases}$$

is unbiased, where c is such that the test has given size α.

9 Let X be exponential with mean θ and $H_0: \theta = 1$ against $H_1: \theta = 2$ is to be tested. Suppose the test ϕ rejects H_0 for $x > 2$. Obtain size and power of test ϕ.

10 Find the MP level α test of $H_0: \theta = 1$ against $H_1: \theta = 2$ based on a sample of size n from $f(x, \theta) = \theta\, x^{\theta - 1}$, $0 \le x \le 1$, $\theta > 0$.

11 Let $X_1, X_2, ..., X_n$ be *iid* Bernoulli $B(1, \theta)$ random variables. Obtain most powerful size α test of $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$.

12 Let $X_1, X_2, ..., X_n$ be iid *Poisson* (θ) random variables. Obtain most powerful size α test of $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$.

13 Obtain MP test of level α for testing $H_0: \mu = \mu_0$ against $H_1: \mu = \mu_1 (> \mu_0)$ based on a random sample of size n from $N(\mu, \sigma^2)$, where σ^2 is known.

14 Let X be normally distributed with $\sigma = 10$ and it is desired to test $H_0: \mu = 100$ against $H_1: \mu = 110$. How large should a sample be taken so that $\alpha = 0.05$ and $\beta = 0.02$.

15 Use Neyman-Pearson lemma to test $H_0: \mu = 0$ against $H_1: \mu = 1$ on the basis of a random sample $X_1, X_2, ..., X_n$ of size n from $N(\mu, 1)$ distribution.

16 Obtain a most powerful test of size α for testing $H_0: \sigma = \sigma_0$ against $H_1: \sigma = \sigma_1 (> \sigma_0)$ based on a random sample of size n from $N(\mu, \sigma^2)$, where μ is known.

17 Obtain a most powerful test of size α for testing $H_0: X \sim N(0, 1)$ against $H_1: X \sim$ Cauchy $(0, 1)$.

18 Let $0 < \alpha < 1$ and ϕ^* be a MP size α test for H_0 against H_1. Let $\beta = E_{H_1}[\phi^*(\underline{x})]$. Show that $1 - \phi^*$ is MP test for testing H_1 against H_0 at level $1 - \beta$.

19 Let α be a real number, $0<\alpha<1$ and ϕ be a MP test of size α for testing H_0 against H_1. Also let $\beta = E_{H0}[\phi(x)] < 1$. Show that $(1-\phi)$ is an MP test for testing H_1 against H_0 at level $(1-\beta)$.

20 Obtain a most powerful test of size α for testing $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1 (> \sigma_0)$ based on a random sample of size *n* from $N(\mu, \sigma^2)$, where μ is known.

21 Examine whether the following families of densities have MLR property or not.

i) $f(x, \theta) = \dfrac{1}{2}\, e^{-|x-\theta|}$, $-\infty < x < \infty$ ..

ii) $f(x, \theta) = \dfrac{1}{\pi}\, \dfrac{1}{1+(x-\theta)^2}$, $-\infty < x < \infty$.

22 Let *X* have a Cauchy distribution with *pdf*

$$f(x,\theta) = \frac{1}{\pi} . \frac{1}{1+(x-\theta)^2} , -\infty < x < \infty .$$

Examine whether $\{ f_\theta \}$ has MLR.

23 Examine whether $U(\theta, \theta+1)$ distribution has monotone likelihood ratio.

24 Let X have hypergeometric distribution given by *pmf*

$$P_M(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} ,\ x = 0,1,...,M$$

where *N* is known and *M* is unknown. Show that the distribution has MLR in *x*.

25 Let $X_1, X_2, ..., X_n$ be a sample of size n from the distribution having the *pdf*

$$f(x,\theta) = \frac{\theta}{x^2} , 0 \le \theta \le x < \infty .$$

Examine whether joint density of these observations has MLR property.

26 Let $X_1, X_2, ..., X_n$ be *iid* $U(0, \theta)$ distribution. Consider following test for $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$

$$\phi(x) = \begin{cases} 1,\ x_{(n)} > \theta_0 \ or\ x_{(n)} < \theta_0\, \alpha^{\frac{1}{n}} \\ 0 \text{ , otherwise} \end{cases}$$

Examine whether ϕ is UMP.

27 Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, 1)$. Obtain UMP level α test for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

28 Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, \sigma^2)$, where θ is known. Obtain UMP level α test for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$.

29 Let $X_1, X_2, ..., X_n$ be a random sample from $N(0,\ \sigma^2)$ distribution. Obtain UMP level α test for testing $H_0 : \sigma^2 \le 1$ against $H_1 : \sigma^2 > 1$.

30 Let $X_1, X_2, ..., X_n$ be a random sample from Poisson(θ). Obtain UMP level α test for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

31 Let $X_1, X_2, ..., X_n$ are *iid* $N(\theta, \sigma^2)$, where σ^2 is known. Show that UMP test does not exists for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$.

32 Let $X_1, X_2, ..., X_n$ be a random sample drawn from $U(0, \theta)$ distribution. Find UMP size α test for testing $H_0 : \theta \le \theta_0$ against $H_1 : \theta > \theta_0$.

33 Let $X_1, X_2, ..., X_n$ be a random sample drawn from uniform distribution $U(0, \theta)$. Find UMP size α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \ne \theta_0$.

34 Let $X_1, X_2, ..., X_n$ be a random sample from $N(\mu_0, \sigma^2)$. Obtain UMP level α test for testing $H_0 : \sigma^2 \le \sigma_0^2$ against $H_1 : \sigma^2 > \sigma_0^2$.

35 To test $H_0 : \theta \le 1/2$ against $H_1 : \theta > 1/2$, using single observation of X whose density is

$$f(x;\, \theta) = \begin{cases} 2x\,, & \text{if } 0 < x < \theta \\ 1+\theta\,, & \text{if } \theta \le x < 1 \\ 0\,, & \text{otherwise} \end{cases}.$$

The test used is $\phi(x) = \begin{cases} 1, \text{ if } x > c \\ 0, \text{ otherwise} \end{cases}$

where c is a constant selected such that the test has size $\alpha = 0.1$. Examine whether test is unbiased.

36 A single observation x is available on a random variable X whose *pdf* is

$$f(x;\, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}},\ x > 0\,,\ \theta > 0,$$

for testing $H_0 : \theta \le 1$ against $H_1 : \theta > 1$. Examine the test which rejects H_0 when $x > c$ or else accept is unbiased, where c is such that the test has given size α.

37 Obtain UMPU level α test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ based on $N(\theta, \sigma^2)$, σ^2 is known for a sample of size *n*.

38 Obtain UMPU level α test for testing $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ based on $N(\mu, \sigma^2)$ and its power function (μ is unknown) for a sample of size *n*.

39 Let $X_1, X_2, ..., X_n$ be a random sample from exponential distribution with mean θ. Consider the testing of hypothesis problem $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$. Find UMA $(1-\alpha)$ level family of confidence sets corresponding to size α UMP test.

40 Let $X_1, X_2, ..., X_n$ be a random sample of size *n* from $U(0,\theta)$ distribution. Consider the testing problem $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. Find $(1-\alpha)$ level UMA confidence sets for θ.

41 Let $X_1, X_2, ..., X_n$ be a sample from $U(0,\theta)$. Consider the testing problem $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$. Find UMA family of confidence intervals for θ at level $(1-\alpha)$.

42 Let $X_1, X_2, ..., X_n$ be a random sample drawn from $N(\mu_0, \sigma^2)$, where σ^2 is unknown and $\mu = \mu_0$. Consider the testing of hypothesis problem $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma \neq \sigma_0$. Find UMAU family of confidence intervals for σ^2 at level $(1-\alpha)$.

43 Using approximate pivotal quantity, derive $100(1-\alpha)\%$ confidence interval for μ of $N(\mu, \sigma^2)$, σ^2 is unknown based on sample of size *n*.

44 Let $X_1, X_2, ..., X_n$ be a random sample of size *n* from $N(\mu, \sigma^2)$ population, when both μ and σ^2 are unknown. Obtain $100(1-\alpha)\%$ confidence interval for σ^2.

45 Let $X_1, X_2, ..., X_n$ be a random sample of size *n* from $N(\mu, \sigma^2)$ population. Obtain $100(1-\alpha)\%$ confidence interval for:

(i) μ when σ^2 is known.

(ii) σ^2 when is μ known.

46 A sample of size *n* from $N(\theta, \sigma^2)$ with $\sigma^2 = 4$ was observed. A 95% confidence interval for θ was constructed from the above sample. Find the value of *n* if confidence interval is $(9.02, 10.98)$.

47 Obtain the shortest length confidence interval for θ based on a sample of size n exponential distribution with mean θ.

48 Let $X_1, X_2, ..., X_n$ be a random sample of size n from $U(0,\theta)$ distribution. Obtain shortest length confidence interval for θ.

49 Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\theta,1)$ distribution. Obtain shortest length confidence interval for θ.

50 Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\theta,\sigma^2)$, σ^2 is known. Obtain shortest length confidence interval for θ.

51 Let $X_1, X_2, ..., X_n$ be a random sample of size n from $N(\theta,\sigma^2)$, θ is known. Obtain shortest length confidence interval for σ^2.

52 Derive LRT for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ based on a sample of size n from $N(\mu,1)$ distribution.

53 Let $X_1, X_2, ..., X_n$ be a random sample from normal density with variance 1 and unknown mean μ. Find out likelihood ratio test of hypothesis $H_0 : \mu = 3$ against the alternative $H_1 : \mu \neq 3$.

54 Let a random sample $X_1, X_2, ..., X_n$ has been drawn from $N(\mu,\sigma^2)$ distribution. Obtain likelihood ratio test of $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$. Is the desired test UMPU? Justify.

55 Let $X_1, X_2, ..., X_n$ be a random sample from $N(\theta, \sigma^2)$ distribution where both θ and σ^2 are unknown. Find the likelihood ratio test of $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma \neq \sigma_0$.

56 Let X be a binomial $B(n,\theta)$ random variable. Develop a level likelihood ratio test of $H_0 : \theta \leq \theta_0$ against $H_1 : \theta > \theta_0$.

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Punyashlok Ahilyadevi Holkar Solapur University, Solapur

Department of Statistics

M.Sc. (Statistics) Part-II Sem-IV

Statistics Paper SCT4.2: Clinical Trials

Question Bank

Short answers

1. Explain the concept of Stratified randomization.
2. Write advantages and disadvantages of crossover design.
3. Pharmacokinetic parameters used in bioequivalence.
4. Endpoints in clinical trials.
5. Explain the following terms:
   i) Full analysis set/cohort.
   ii) Completers set/cohort.
   iii) Per-protocols set/cohort
6. Explain the role of Bio-statistician in the planning and execution of CTs. Also state some sources of bias in CTs.
7. Write any two definitions of clinical trials. Explain the terms in it.
8. Write notes on run in and washout period.
9. Describe Balanced Incomplete Block Design (BIBD).
10. Explain Cox's proportional hazard model for assessment of test drug based on censored data.
11. Explain the difference between: Statistically significant difference and clinically significant difference.
12. Explain the difference between: Interaction and confounding.
13. Explain the four phases involve in development of clinical trials.
14. Write the note on: Investigation New Drug Application (INDA).
15. Write the note on: Abbreviated New Drug Application (ANDA).
16. What are the major objectives behind conduction of the clinical trials (CTs)
17. Explain the following terms related with CTs
    i) Subject
    ii) Treatment
    iii) Clinical Endpoints.
    iv) Placebo
18. Write notes on: Superiority trials
19. Write notes on: Equivalence or non-inferiority trials

Long answers

1. Explain the overall clinical drug development process.
2. Define Blinding. Explain the various types of blinding methods used in clinical trials.
3. Discuss role of ethics in clinical trials.
4. What are clinical trials? Explain the why clinical trials are essential in the development of new interventions.
5. Explain the permuted randomization and its advantages over complete randomization.
6. Explain the concept of sample size. Discuss are the factors necessary to calculate the appropriate sample size.
7. Explain the role of Good clinical practice in clinical trials.
8. Explain the Cox's proportional hazard model for assessment of test drug based on censored data.
9. Explain the concept hypothesis of superiority and hypothesis of non-inferiority
10. What is patient compliance? What is difference between missing value and drop outs?
11. Classify the clinical trials depending upon their functioning. Explain their respective functions in brief.
12. Explain the role of sampling distribution for the valid and unbiased assessment of true efficiency and safety of the study medication.
13. Explain the patient selection process for clinical trials.
14. What is randomization? Why randomization is needed? What are the types of randomizations involved in clinical trials?
15. For the selection of appropriate design for clinical trials, which issues must be considered?
16. a) Define:
    i) Clinical Trials
    ii) Experimental unit
    iii) Treatment
    iv) Evaluation.
17. Write a note on

i) Active control and Equivalence trials.

ii) Combination trials.

18. Explain the assumption for using the placebo control and No treatment control.
19. Explain the dose response concurrent control.
20. Write the short note on Washout period and Carryover effect in crossover design.
21. Discuss the parallel design useful in clinical trials and its advantages over the crossover designs.
22. Explain the method of block randomization and its advantages over complete randomization.
23. What is meaning of blinding? Way it is used in Clinical trials? Explain the type of bindings.
24. A pharmaceutical company is interested in conducting a clinical trial to compare two cholesterol lowering agents. Suppose that a difference of 8% in the percent change of LDL-cholesterol is considered a clinically meaningful difference and that standard deviation is assumed to be 15%. Find the required sample size for having an 80% power and $\alpha = 0.05$.
25. Explain the difference between the Multicenter trails and Meta analysis.
26. Explain concept of protocol and process of protocol developments in clinical trials.
27. Discuss the concept of bioequivalence study.
28. Write note on Protocol in Clinical Trials (CTs).
29. What is safety report in CT? Explain Treatment IND and termination of IND.
30. Explain the method of Permuted block randomization and its advantages over complete randomization.
31. What are controls? Explain active concurrent control.
32. what are crossover designs? In which situations crossover designs are useful?
33. Write a note on interim analysis and data monitoring.
34. List out the different kinds of CTs. Also discuss brief objectives of each of them.
35. Give two examples of response variables where categorical data are generated.
36. List out the kinds of uncertainty in CTs.

Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-II Sem-IV
Statistics Paper HCT4.1
Title: Data Mining

Question Bank

Short answers

1. Write a short note on Imbalanced data.
2. Describe CRISP process in brief.
3. Describe SEMMA process in brief.
4. Distinguish between training data and testing data.
5. Describe supervised learning.
6. Describe Gini's method to obtain information gain in decision tree.
7. Discuss loss functions used in SVM.
8. Discuss unsupervised learning. Also give illustrations.
9. What is market basket analysis?
10. Discuss association rules and prediction.
11. Write a note on problem of classification.
12. Why kNN classifier is also called as lazy classifier?
13. Explain the steps Involved in Supervised Learning.
14. What are the advantages and disadvantages of supervised learning?
15. What are the advantages and disadvantages of unsupervised learning?
16. Write a short note on clustering.
17. Write a short note on association rules.

18. Discuss the drawbacks of kNN classifier.
19. Explain how to deal with missing values in kNN classifiers.
20. Write a short note on confusion matrix.
21. Discuss sensitivity and specificity of a model.
22. Discuss accuracy and precision of a classifier.

Long answers

1. Describe k-nearest neighbor classifier in detail.
2. Describe decision tree classifier in detail.
3. Describe naive Bayesian classifier in detail.
4. Describe logistic regression classifier in detail.
5. Describe Artificial Neural Network (ANN).
6. Distinguish between single layer and multi-layer neural network.
7. Describe how regression is used in ANN?
8. Discuss Support Vector Machine (SVM) in detail.
9. Discuss density based methods for unsupervised learning.
10. Discuss McCulloch-Pitts AN model in detail.
11 Write down the algorithm for decision tree classifier.
12. Write down the algorithm for Bayesian classifier.
13. Write down the algorithm for kNN classifier.
14. What are the different metrics for Evaluating Classifier Performance?
15. Discuss information gain in decision tree.
16. Write a note on confusion matrix. Also explain different metrics for evaluating classifier performance.
17. Discuss accuracy and precision of a classifier. Describe how accuracy can mislead with respect to performance of the data.

Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-II (Sem-IV)
Statistics Paper: HCT-4.3
Title of Paper: Reliability and Survival Analysis
<u>Question Bank</u>

Short Answers

1 Define:
i) Reliability of a component ii) Failure rate function iii) Coherent structure

2 Define reliability of component. Obtain the reliability of series system of n independent components.

3 Define reliability of a system. Obtain the reliability of parallel system of n independent components.

4 Define dual of a system. Obtain dual of a 2-out-of-3 system.

5 Define: i) Structure function ii) Coherent structure. Illustrate giving one example each.

6 Define k out of n system. Obtain the reliability function of this system.

7 Define minimal path sets and minimal cut sets. Illustrate the same by example.

8 Define dual of a structure function. Show that dual of dual is primal.

9 Define coherent system. Give an illustration of a system which is not coherent.

10 Define irrelevant component. Give an illustration.

11 Define associated random variables. Show that ordered statistics are associated.

12 Define IFR and DFR class of life distributions.

13 Define IFRA and DFRA classes of distributions.

14 Give two definitions of star shaped function and prove their equivalence.

15 Define Polya function of order 2.

16 Define the following terms: i) survival function ii) Random censoring

17 Define the following terms: i) Type-II censoring ii) Cumulative hazard function

18 Define TTT transform and show that it is concave if F is IFR.

19 Explain the concept of random censoring giving one example.

20 Describe type-I censoring and type-II censoring.

21 Describe situations where random censoring occurs naturally.

22 State the properties of order statistics from an exponential distribution.

23 Show that for exponential distribution normalized spacings are independently distributed.

24 Give two real life examples where both left and right censoring occurs.

Short Notes

1 Pivotal decomposition of structure function
2 Cumulative hazard function
3 Mean residual life function
4 Hazard transform.
5 Graphical estimation technique for Weibull distribution
6 Polya function of order 2.
7 Relevant and irrelevant component.
8 Star shaped function.
9 Birnbaum's measure of structural importance.
10 Path and cut sets of the dual of the system.
11 Log-rank test
12 Proportional Hazard model
13 Random censoring
14 Estimation of survival function under uncensored data.
15 Type-I censoring
16 Type-II censoring
17 Empirical survival function and its properties.
18 Graphical estimation technique for Weibull distribution
19 Generalized maximum likelihood estimator
20 Tarone-Ware Tests

Long Answers

1 Define reliability of a component and reliability of a system. Obtain the reliability of series and parallel systems of n independent components.
2 Define reliability of the system. Obtain expression for reliability of a coherent system in terms of reliabilities of components. Also obtain reliability of series system if each component has reliability p.
3 Define dual of a structure function. Obtain the dual of k-out-of- n system.
4 Define k-out-of-n system. Obtain reliability of this system.
5 Define coherent system. Show that k-out-of-n system is coherent system.
6 Define associated random variables. If $X_1, X_2, \ldots, X_n$ are binary associated random variables then prove that $1-X_1, 1-X_2, \ldots, 1-X_n$ are also binary associated.
7 Obtain a structure function of system in terms of i) minimal path ii) minimal cut.
8 Obtain structure function of a coherent system using minimal cut sets. Illustrate the same by an example.

9 Obtain the structure function by using minimal path sets. Illustrate the same by an example.

10 Show that minimal path vector is a minimal cut vector of its dual.

11 A system consists of 4 components. System functions when both components 1 and 4 functions and at least one of the remaining two functions. Find the reliability of system.

12 For a coherent system with n components prove that:

i) $\phi(0)=0$ and $\phi(1)=1$ ii) $\prod_{i=1}^{n} X_i \leq \phi(X) \leq \coprod_{i=1}^{n} X_i$

13 If $X_1, X_2, ..., X_n$ are associated binary random variables then prove that

i) $P\left(\prod_{j=1}^{n} X_j = 1\right) \geq \prod_{j=1}^{n} P(X_j = 1)$ ii) $P\left(\coprod_{j=1}^{n} X_j = 1\right) \leq \coprod_{j=1}^{n} P(X_j = 1)$

14 If $X_1, X_2, ..., X_n$ are associated state variables of coherent system then prove that

$$\prod_{i=1}^{n} P(X_i = 1) \leq P(\phi(X) = 1) \leq \coprod_{i=1}^{n} P(X_i = 1)$$

15 Define associated random variables. If $X_1, X_2, ..., X_n$ are binary associated random variables then prove that $E\left[\prod_{i=1}^{n} X_i\right] \geq \prod_{i=1}^{n} E(X_i)$.

16 Define associated random variables. If $X_1, X_2, ..., X_n$ are binary associated random variables then prove that $P\left[\coprod_{j=1}^{n} X_j = 1\right] \leq \coprod_{j=1}^{n} P(X_j = 1)$.

17 Define mean time to failure (MTTF) and mean residual life (MRL) function. Obtain the same for exponential distribution.

18 If failure time of an item has gamma distribution obtain the failure rate function

19 Define mean residual life function and obtain the same for exponential distribution.

20 Define IFR and IFRA class of distributions. If $F \in IFR$ then show that $F \in IFRA$.

21 Define IFR and IFRA classes of distributions. Prove that $\text{IFR} \subset \text{IFRA}$.

22 State and prove IFR closure property under convolution.

23 Show that IFRA class of life distribution is closed under convolution.

24 State and prove IFRA closure theorem.

25 Define NBU and NBUE classes of distributions. Prove that $F \in IFRA \Rightarrow F \in NBU$.

26 Define NWU and NWUE class of distributions. Prove that if $F \in DFRA$ then $F \in NWU$.

27 Define NBU and NBUE class of distributions. Prove or disprove: $\text{NBUE} \Rightarrow \text{NBU}$.

28 Give two definitions of star shaped function and prove their equivalence.

29 Define star shaped function. Prove that $F \in IFRA$ if and only if $-\log R(t)$ is star shaped.

30 State three equivalent definitions of a PF_2 function and describe how this concept is useful in examining whether a given distribution is IFR.

31 Define Poly function of order 2 (PF_2). Prove that if $f \in PF_2$ then $F \in IFR$.

32 Define PF_2 function and describe how it is useful in examining whether a given distribution is IFR.

33 Define totally positive function of order n. Give an example.

34 Let $F(x) = \int_{\alpha} F_{\alpha}(x)\, d\, G(\alpha)$ be a mixture of $\{F_{\alpha}\}$ with mixing distribution $G(\alpha)$. Prove that if each F_{α} is DFR then F is DFR.

35 Let $F(x) = \int_{\alpha} F_{\alpha}(x)\, d\, G(\alpha)$ be a mixture of $\{F_{\alpha}\}$ with mixing distribution $G(\alpha)$. Prove that if each F_{α} is DFRA then F is DFRA.

36 If failure time of item has Weibull distribution with distribution function

$$F(t) = \begin{cases} 1 - e^{-(\lambda t)^{\alpha}} & , t > 0 \\ 0, otherwise \end{cases}.$$

Examine whether it belongs to IFR or DFR.

37 If failure time of an item has the distribution

$$f(t) = \frac{\lambda^{\alpha}}{\Gamma\alpha} t^{\alpha-1} e^{-\lambda t} \quad ,t > 0, \quad \lambda, \alpha > 0.$$

Examine whether it belongs to IFR or DFR.

38 Obtain the reliability function and hazard function for the Weibull distribution.

39 Define gamma distribution as failure time model. Discuss the monotonicity property of its hazard rate.

40 If f(t), F(t) and h(t) are the density, distribution and hazard functions of a random variable T, then show that $\text{h(t)} = \frac{\text{f(t)}}{\bar{\text{F}}\text{(t)}}$, $\bar{\text{F}}\text{(t)} = 1 - \text{F(t)}$. Also establish a suitable relationship between h(t) and reliability function.

41 Define Hazard function and survival function. Obtain the same for an exponential distribution.

42 Describe the need of censoring experiment. Describe Type-I and Type-II censoring with suitable examples.

43 Define type-I censoring and obtain the likelihood corresponding to a parametric model for lifetime distribution.

44 Describe various censoring schemes.

45 Describe each of the following with one illustration:

a) Type-I censoring

b) Type-II censoring

c) Random censoring

46 Describe situations where random censoring occurs naturally. Obtain actuarial estimate of survival function and derive Greenwood's formula for the estimate of variance of the estimator,

47 Under time censoring with replacement obtain likelihood function. When the observations are taken from exponential distribution with mean θ, does MLE for θ exist? Justify.

48 Discuss maximum likelihood estimation of parameters of a gamma distribution under uncensored data.

49 Discuss maximum likelihood estimation of parameters of a Weibull distribution based on uncensored data.

50 Describe graphical method of estimating the parameters of Weibull distribution based on complete data.

51 Obtain moment estimator of the parameters of lognormal distribution based on random sample of size n.

52 Derive the likelihood function of observed data under type I censoring.

53 Obtain the likelihood function under random censoring set up when observations come from a distribution F with density F.

54 Describe Type-I censoring. Obtain MLE of mean of exponential distribution under Type I censoring.

55 Obtain maximum likelihood estimator of the mean of exponential distribution under type I censoring.

Obtain maximum likelihood estimate of mean of the exponential distribution under type II censoring.

56 Obtain MLE of the mean (θ) of an exponential distribution based on complete sample and type I censoring.

57 Obtain MLE of the mean (θ) of an exponential distribution based on type I and type II censoring.

58 Obtain the nonparametric estimator of survival function based on complete data. Also obtain confidence band for the same using Kolmogorov-Smirnov statistic.

59 Describe actuarial method of estimation of survival function, with suitable illustration.

60 Derive Greenwood's formula for an estimate of variance of actuarial estimator of survival function.

61 Obtain the actuarial estimator of the survival function. Clearly state the assumption that you need to make. State Greenwood's formula for the variance of the estimator.

62 Describe Kaplan-Meier estimator and derive an expression for the same.

63 Show that Kaplan-Meier estimator of survival function is the generalized likelihood estimator of the survival function.

64 Describe two sample problem under randomly censored set up and develop Gehan's test for the same.

65 Describe Gehan's test for two sample testing problem in presence of censoring.

66 Describe Mantel-Haenzel test. Indicate the null distribution of test statistic.

67 Describe Mantel's technique of computing Gehan's statistics for a two-sample problem for testing equality of two life distributions.

68 Describe Deshpande's test for exponentiality against IFRA.

69 Develop Hollander-Proschan test for exponentiality against NBU.

70 Develop a test for exponentiality against NBU.

71 Define TTT transform. Obtain relation between TTT transform and failure rate function of a survival distribution.

72 Define TTT transform. Show that for an IFR distribution TTT transform is a convex function.

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Punyashlok Ahilyadevi Holkar Solapur University, Solapur
Department of Statistics
M.Sc. (Statistics) Part-II (Sem-IV)
Statistics Paper: HCT-4.2
Title of Paper: Industrial Statistics
<u>Question Bank</u>

Short Answer

1 What is statistical basis of control chart?
2 Distinguish between process control and product control. Discuss the situations where they are used.
3 Distinguish between process control and product control. What are the statistical techniques to achieve these?
4 Explain the basic differences among the chance and assignable causes of variation that affect process results.
5 Explain the importance of SQC in industry.
6 Explain how control charts are useful in achieving process control.
7 Discuss relation between testing of hypothesis and control chart.
8 Define type I and type II errors relative to control charts.
9 Explain Ishikawa diagram with suitable example.
10 Explain the use of Pareto chart with suitable example.
11 Distinguish between tolerance limits and specification limits. Interpret the situation when tolerance limits are not included within the specification limits.
12 Define ARL and OC function of control chart.
13 Define ARL of control chart. State the distribution of ARL.
14 Explain the construction of moving average control chart.
15 Describe conforming run length chart.
16 Obtain the control limits for $(\overline{X}, R)$ and $(\overline{X}, S)$ charts when standards are known and unknown.
17 State the control limits of $\overline{X}$ and R charts when i) standards are known ii) standards are unknown.
18 Explain the need of acceptance sampling in industry.
19 Describe a single sampling plan for attributes.
20 Distinguish between defect and defective.
21 Define producer's risk and consumer's risk.
22 Define single and double sampling for attributes.
23 Distinguish between acceptable quality level (AQL) and average outgoing quality level (AOQL).

24 Explain the following terms:

i) Consumer's risk

i) Producer's risk

iii) Acceptance Quality Level.

25 Explain the following terms: (i) OC function (ii) ATI (iii) LTPD

26 Discuss the importance of control charts for variables.

27 Compare the acceptance sampling by variables and attributes.

28 Define terms ASN, AOQ and ATI for an acceptance sampling plan.

29 Define the control limits for the control chart for defects in a constant and varying number of sample units.

Short Notes

1 Producer's risk and Consumer's risk

2 DMAIC Cycle

3 Pareto chart

4 Demerit system

5 Curtailed sampling plan

6 S^2 control chart

7 Conforming run length (CRL) chart

8 Continuous sampling plan.

9 Six-sigma methodology

Long Answer

1 Discuss various definitions of 'Quality' and various dimensions of quality.

2 List seven SPC tools and explain in detail any two of off-line tools.

3 Explain Deming's PDCA cycle for continuous improvement.

4 Discuss the various sources of assignable causes and chance causes of variation. Also state how they are detected in a manufacturing process.

5 Define control chart. Discuss $\overline{X}$ and R charts for controlling the quality of product.

6 Discuss the various steps involved in the construction of $\overline{X}$ and R charts.

7 Outline the steps involved in the construction of $\overline{X}$ and S charts.

8 Define ARL and OC function of a control chart. Obtain the same for $\overline{X}$ chart assuming normality of process with known standards.

9 How does control chart help in detecting lack of control in a process? Indicate how to derive OC curve of $\overline{X}$ control chart given that standard deviation is known and constant.

10 Distinguish between variable and attribute control charts. Also give one real life situation of each type of chart, where they are applicable.

11 Discuss the control chart for fraction nonconforming when sample size is fixed**.**

12 Discuss the control chart for fraction nonconforming when the sample size is i) fixed and ii) variable.

13 Describe the development and implementation of control chart for fraction nonconforming when sample size is not fixed and standards are not given.

14 Discuss various steps involved in the construction of p chart with fixed sample size and variable sample size.

15 Discuss in detail np chart. Obtain OC function of the same.

16 Distinguish between defect and defective. Give some examples of defects for which c chart is applicable. How do you calculate control limits for c chart? State assumptions made.

17 Discuss the development and operation of demerit control chart.

18 What is CUSUM chart? Explain the methodology of using it for process control.

19 What is CUSUM chart? Explain its construction and operation.

20 What is CUSUM chart? Explain tabular CUSUM procedure for monitoring process mean.

21 Describe construction and operation of tabular CUSUM for monitoring process mean.

22 Explain V-mask method of implementing CUSUM chart for mean.

23 What is basic difference between $\overline{X}$ chart and CUSUM chart? Explain V-mask CUSUM procedure.

24 Explain exponentially weighted moving average (EWMA) chart with at least one example of its potential application.

25 What is an EWMA control chart? In which situation it is preferred to $\overline{X}$ chart? Explain the procedure of obtaining control limits for the same.

26 State the importance of exponentially weighted moving average (EWMA) charts. How are these used in practical situations?

27 Explain the construction of nonparametric control chart based on sign test for monitoring process location.

28 Discuss nonparametric signed-rank control chart for process location.

29 Stating the assumptions, explain the construction and operations of the Hotelling's T^2 chart to monitor process mean vector.

30 Describe the development and operation of Hotelling's T^2 chart to monitor process mean vector.

31 Explain the assumptions, construction and operation of Hotelling's T^2 chart.

32 Define process capability index C_p. Stating the underlying assumption clearly, establish relationship between C_p and probability of nonconforming item.

33 Define the process capability index C_P. Give its interpretation in terms of probability of non-conformance.

34 Define process capability indices C_p and C_{pk}. Obtain $(1-\alpha)$ level confidence interval for C_p.

35 Explain the process capability index C_{pk}. Also explain the procedure of estimating the same.

36 Define process capability index C_p. Stating the underlying assumption clearly, establish relationship between C_p and probability of nonconforming item.

37 Stating the underlying assumptions, define process capability indices C_P and C_{PK}. Derive the relationship between them.

38 Define process capability indices C_P and C_{PK} and C_{PM}. Describe a situation where C_{PK} is more suitable than C_P.

39 Stating the assumptions clearly, define index $C_{p.}$ Interpret $C_p=1$. Obtain $(1-\alpha)$ level confidence interval for C_p.

40 Explain the basic concepts of six-sigma methodology. Also explain the benefits of implementing the same.

41 Explain SIX SIGMA methodology and DMAIC cycle in detail

42 Describe DMAIC with reference to six-sigma.

43 Define acceptance sampling plan for attributes. What do you mean by ASN and ATI in acceptance sampling plan?

44 Explain the need of acceptance sampling in improving the quality of product.

45 Describe single sampling plan for attributes. Give an algorithm to design the single sampling plan.

46 Describe a single sampling plan for attributes. Obtain OC function of the same.

47 Discuss in detail double sampling plan (N, n_1, c_1, n_2, c_2) stating the assumptions followed in both stages. Hence or otherwise obtain the ASN function of this sampling procedure.

48 Describe double sampling plan for attributes and obtain ASN of the same.

49 Describe double sampling plan for attributes. Obtain AOQ and ASN for the same.

50 What do you mean by ASN and ATI in acceptance sampling plans? Obtain ASN and ATI functions of double sampling plan by attributes.

51 Describe curtailed sampling plan. In what respect it differs from usual sampling plan?

52 What are sampling inspection plans? Stating clearly assumptions, explain the construction and implementation of sequential sampling plan.

53 Describe a variable sampling scheme and compare it with the attribute sampling scheme.

54 Explain the variable sampling plan when lower specification is given and standard deviation is known.

55 Explain the variable sampling plan when upper specification is given and standard deviation is known.

56 Describe variable sampling plan when both lower and upper specifications are given and standard deviation is known.

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Punyashlok Ahilyadevi Holkar Solapur University, Solapur

M. Sc. Statistics Part I Semester –II
SCT 2.1 Sampling theory
<u>Question Bank</u>

SHORT ANSWER

1 Explain the term sampling design with suitable design.

2 Define and illustrate the concept of sampling design.

3 Explain sampling method and census method.

4 Discuss the census survey and sampling survey.

5 Define a random sampling? Provide an example.

6 Give advantages of sampling method over census method.

7 Describe a procedure for obtaining a sample of size n from a population of size N using SRSWOR method

8 Describe procedure for obtaining a sample of size n from a population of size N using SRSWR method.

9 In SRSWR, show that sample mean is an unbiased estimator of population mean.

10 In SRSWR sampling procedure, show that sample mean is an unbiased estimator of population mean.

11 In SRSWOR, show that the probability of drawing a specified unit at every draw is same.

12 Obtain the difference between the variances of sample means in SRSWR and SRSWOR.

13 State motivations for stratifying a population before sampling.

14 Discuss the concept of stratification.

15 Define proportional allocation with illustration.

16 Define equal allocation. Give an illustration.

17 Describe circular method for drawing a systematic sample. Give an example.

18 Explain linear systematic sampling with an example.

19 Describe cumulative total method for PPS sampling.

20 Explain Lahiri's method for PPSWR sampling.

21 Define ordered and unordered estimators.

22 Describe the method of cluster sampling.

23 Explain the purpose of double sampling.

24 Explain the similarities between stratified sampling and cluster sampling.

25 Give an example of almost ratio type estimator.

26 Define a linear regression estimator for a population mean. Is it unbiased?

27 Distinguish between sampling and non-sampling errors.

28 What is the difference between sampling and non-sampling errors?

29 What do you understand by non-response error? Illustrate.

30 Explain the Deep stratification.

SHORT NOTES

1 Cumulative total method for PPSWR sampling

2 Midzuno system of sampling.

3 Inclusion probabilities.

4 Method of collapsed strata.

5 Deep stratification.

6 Circular systematic sampling.

7 Neyman allocation

8 Proportional allocation

9 Deming’s technique.

10 Two stage sampling

11 Unbiased ratio type estimator

12 Almost unbiased ratio type estimator.

13 Ordered and unordered estimators

14 Horvitz- Thompson estimator

15 Des Raj Estimator

16 Murthy’s estimator

17 Cluster sampling.

18 Ratio estimation in stratified sampling

19 Sample size determination in surveys

20 Rao-Hartley-Cochran Scheme.

LONG ANSWER

1 Distinguish between sampling with and without replacement. Clearly mention the advantages of one over the other.

2 What are basic principles of sample survey? Write in brief the advantages of sampling over complete census.

3 Describe SRSWR and SRSWOR schemes. Obtain unbiased estimator of population mean in both of these sampling schemes.

4 For SRSWOR show that sample proportion p is unbiased estimator for population proportion P.

5 Consider a population of N= 6 units with values 1, 2, 3,4, 5 and 6.

(i) Write down all possible samples of size 2 drawn by SRSWOR scheme. Verify that the sample mean is unbiased for the population mean.

(ii) Also compute the sampling variance of the sample mean.

6 In SRSWR show that $E(s^2) = \sigma^2$

7 In SRSWOR examine whether sample mean is an unbiased estimator of population mean. Derive its variance.

8 In SRSWR, suggest an unbiased estimator of population total Y and derive its sampling variance.

9 In SRSWR, suggest an unbiased estimator of the population mean and derive its sampling variance.

10 In SRSWOR, derive an unbiased estimator of a population mean and its sampling variance.

11 Describe simple random sampling. In SRSWOR, show that the probability of drawing a specified unit at every draw is the same.

12 What is stratified sampling? When it is useful? Obtain the sizes of sample from various strata under optimum allocation. Find the corresponding variance of sample mean.

What is stratified sampling? Obtain the sizes of sample from various strata under proportional allocation. Find the corresponding variance of sample mean.

13 Describe stratified random sampling. Explain various sample allocation criteria in stratified sampling.

14 Discuss the following allocations of the sample size in stratified random sampling:

(i) Proportional allocation

(ii) Neyman allocation

(iii) Optimum allocation with a linear cost function

Explain the practical implications of these methods.

15 Explain and illustrate the benefits of stratifying a population before sampling.

16 Explain the problem of allocating the sample size in stratified random sampling. Describe any two methods for allocating a sample size n to different strata of a population.

17 Explain the problem of allocating the sample size in stratified random sampling. Derive the optimum allocation.

18 If in every stratum, the sample estimator $\bar{y}_h$ is unbiased, then show that $\bar{y}_{st} = \sum_{h=1}^{L} W_h \bar{y}_h$ is unbiased estimator of population mean $\bar{Y}$, where W_h is the proportion of population units in the strata and L denotes the total number of strata in the population. Derive the sampling variance of $\bar{y}_{st}$ and state how you would unbiasedly estimate the same.

19 What is proportional allocation? Derive the variance of the estimator of the population mean under this allocation.

20 What is optimum allocation? Derive the optimum allocation for sample size

assuming a linear cost function.

21 Describe the method of collapsed strata of variance estimation.

22 What do you understand by allocations in stratified sampling? Define Neyman's optimum allocation and obtain variance of the estimator of population mean under this allocation.

23 With usual notations prove that $V_{opt} \leq V_{prop} \leq V_{ran}$

24 Explain the concept of construction of strata. Derive the expression for best value of the boundary point (Y_h) of the h^{th} stratum under proportional allocation.

25 Explain the concept of systematic sampling. Derive the sampling variance of unbiased estimator of population mean under the linear systematic sampling.

26 Define linear systematic sampling. Derive sampling variance of systematic sampling in terms of intra class correlation.

27 Give an illustration for linear systematic sampling. Show that, under this method, a positive correlation between units in the same sample inflates the sampling variance of the estimator of population total.

28 Define linear systematic sampling. Derive the sampling variance of the traditional unbiased estimator of a population mean under this scheme.

29 Describe the linear systematic sampling procedure. Obtain the variance of sample mean in presence of linear trend in a population of size $N = (n \times k)$ where n and k are positive integers.

30 Define linear systematic sampling. When the units in the population have only a linear trend, compare the performance of systematic sampling with SRSWOR sampling.

31 Define cluster sampling. Bring out the similarities and differences between cluster sampling and stratified sampling.

32 Explain cluster sampling and clearly specify the advantages of the scheme.

33 Define cluster sampling. Develop a basic theory for single stage cluster sampling for estimating a population mean assuming SRSWOR of clusters.

34 Define cluster sampling and clearly specify the advantages of the scheme. When this method is better than SRSWOR?

35 If clusters are selected using SRSWR from a population of k clusters of size M each, then obtain the variance of an unbiased estimator of population mean in terms of the population intra-class correlation coefficient.

36 In cluster sampling, obtain an unbiased estimator of population mean when the clusters (of unequal sizes) are selected using simple random sampling. Compute also the variance of estimator.

37 Bring out similarities and differences between cluster sampling and stratified sampling.

38 In what sense cluster sampling is different from simple random sampling? Define an unbiased estimator for population mean in case of cluster sampling with equal

cluster size. Compare the efficiency of cluster sampling in terms of intra-class correlation coefficient with respect to simple random sampling without replacement.

39 In simple random sample of n clusters each containing M elements from a population of N clusters. Obtain an unbiased estimator of Population total. Also derive its variance.

40 Explain a cluster sampling. In SRSWOR of n clusters each containing M elements from a population of N clusters. Show that sample mean is unbiased estimator of population mean.

41 In SRSWOR of n clusters each containing M elements from a population of N clusters. Obtain mean and variance of estimator of sample mean.

42 What is double sampling? Explain any one practical situation where double sampling is appropriate.

43 Define one-stage and two-stage cluster sampling. How do cluster sampling and stratified sampling differ, both in construction and use? Give an example of a survey that uses both stratification and clustering in the sample design.

44 Define a two stage sampling design and give a practical situation where such a design can be used.

45 Explain and illustrate the following ;

(i) Two-stage sampling

(ii) Two-phase sampling

Pinpoint the difference between the two types of sampling schemes.

46 Write short notes on: Two stage sampling and two-phase sampling with their merits and demerits.

47 Give a practical example where two-stage sampling scheme may be adopted. For equal size first-stage units, obtain an estimator for population mean in two-stage sampling and its variance. Discuss the problem of allocation of first and second-stage sample sizes for a fixed cost.

48 In two-stage sampling, obtain an unbiased estimator of population mean if SRSWR is used at both stages. Also derive the variance of the estimator.

49 Define probability proportional to size (PPS) sampling. Discuss cumulative total method of sample selection for PPSWR scheme.

50 Giving examples, explain motivations for unequal probability sampling. Explain Lahiri's method for PPSWR sampling.

51 Discuss the need of PPS sampling illustrating with an example from real life.

52 Describe the Lahiri's method of selecting a probability proportional to size sample from a finite population of size N.

53 Describe Lahiri's method, with justification, of drawing a sample of size n from a population of size N with probabilities proportional to the sizes of the respective units.

54 Describe the cumulative total method for drawing PPSWR samples. What are its limitations?

55 Define PPSWR sampling design. Obtain an unbiased estimator of the population mean and its variance when a PPSWR sample of size n is drawn from a population of size N.

56 Define PPSWR sampling design. Obtain an unbiased estimator of population total and its variance when PPSWR sample of size n is drawn from a population of size N.

57 Define Horvitz-Thompson estimator of population mean and establish its unbiasedness under an arbitrary sampling design. Also derive its sampling variance.

58 Define Horvitz-Thompson estimator for population total. Show that it is unbiased. Obtain its variance.

59 Define Horvitz-Thompson estimator and show that it is unbiased for population total.

60 Define Des Raj ordered estimator for population total. Examine it for unbiasedness and obtain its variance.

61 Define Des Raj's ordered estimator for population mean on the basis of a sample of size 2 and show that it is unbiased.

62 Develop Des Raj's ordered estimator of population mean based sample size 2. Show that it is unbiased.

63 Define ordered and unordered estimators. Develop Murthy's unordered estimator for n = 2.

64 Define Midzuno sampling design. Obtain single and double inclusion probabilities under this sampling design.

65 Describe Midzuno scheme of sampling. Develop an estimator of population total Y for Midzuno scheme and derive the expected value.

66 Let π_i and π_{ij} $(i \neq j)$ be the inclusion probabilities of first and second order respectively in a simple random sample of size n, selected from a finite population of size N. Then show that

(i) $\sum_{i=1}^{N} \pi_i = 1$ (ii) $\sum_{(j \neq i)=1}^{N} \pi_{ij} = n(n-1)$

67 Explain the ratio and regression methods of estimation. When are these methods considered to be efficient?

68 Discuss when the ratio estimator of population mean under simple random sampling is preferred to the conventional unbiased estimator.

69 Explain the ratio method of estimation for estimating a population total. Show that it is generally biased. Evaluate the mean squared error of the estimator to the first order of approximation. Assume SRSWOR of n units from the population.

70 Explain the ratio and regression methods of estimation.

71 Make a comparison between the ratio and regression estimators in terms of MSE and state when the ratio estimator can be more efficient than regression estimator. Justify your answer.

72 Define unbiased and almost unbiased ratio-type estimators.

73 What are almost unbiased-type ratio estimators? Provide one example.

74 Define ratio estimator of population mean and obtain its expected value and mean square error when underlying design is SRSWOR. State the underlying assumptions.

75 Explain ratio method of estimation. Assuming SRSWOR, derive approximate expression for bias of ratio estimator.

76 Discuss the need of ratio estimation and find the bias and the variance of the sample estimator $\hat{R}$ of population ratio R.

77 Explain briefly the circumstances under which the ratio estimator of a population mean will be less precise than the sample mean of a simple random sample of the same total size.

78 Define the ratio method of estimation of population total. Assuming SRSWOR, derive approximate expression for bias of ratio estimator.

79 Define linear regression estimator for population mean. Investigate its properties under SRSWOR scheme.

80 Define a regression estimator of population mean. Show that this estimator is more efficient than the sample mean under SRSWOR using large sample approximation.

81 Define linear regression estimator for population mean. Is it unbiased? Assuming SRSWOR, derive MSE of the estimator.

82 Outline regression method of estimating a population mean. Assuming SRSWOR, derive the MSE of estimator.

83 Explain the regression method for estimation. Derive the approximate expression for bias of the estimator.

84 Define linear regression estimator for population mean. Derive an approximate expression for bias of an estimator.

85 What is the problem of non-response? Discuss Hansen-Hurwitz technique for dealing this problem.

86 Explain the problem of non-response and any one technique to deal with the non-response.

87 Define non sampling errors. Describe main sources of these errors.

88 Explain the problem of non-response and any one technique to deal with the non-response.

89 What is the problem of non-response? Discuss Hansen-Hurwitz techniques of tackling this problem giving all the details.

90 Describe Hansen-Hurwitz technique for dealing with non-response in postal surveys.

91 Discuss the problem of non-sampling errors and methods of dealing with this problem.

Punyashlok Ahilyadevi Holkar Solapur University, Solapur

M. Sc. Statistics Part II Semester – IV

SCT 4.1 Time Series Analysis

Question Bank

Short Answer Questions:

1) Define an invertible process. What is necessary and sufficient condition for invertibility?

2) Write a short note Double exponential smoothing?

3) Define Weak and Strict Stationarity. Give one example each.

4) Define AR(1) process. Obtain its autocovariance function.

5) Define causal process. What is necessary and sufficient condition for causality?

6) Define autocovariance function (ACVF). State the characterizing properties of ACVF.

7) Explain two-sided moving average method.

8) Define MA(1) process. Obtain its partial autocovariance function.

9) Define ARCH and GARCH model.

10) What are diagnostic checking methods of time series models? Explain one.

11) What is difference in ARMA(p, q) and ARIMA(p, d, q)?

12) What is method of differencing for estimation of seasonality?

Short notes

1) Estimation of trend using lease square method.

2) Double exponential smoothing

3) Single exponential smoothing.

4) Moving average for estimation of seasonal component.

5) Weak stationarity and strong stationarity

6) Partial autocorrelation function

7) Autocovariance function

8) Estimation of π_j weights.

9) Forecasting using single exponential smoothing.

10) ARCH and GARCH models

11) Causality

12) Invertability

Long Answer Questions

1) Write the procedure of obtaining ψ_j weights and hence to obtain the autocovariance function.

2) Obtain the Autocovariance function of ARMA(1,1) process.

3) Explain second method of obtaining autocovariance function of causal ARMA process

4) Discuss in brief about Yule-Walker equations.

5) Describe the test based on turning points for testing randomness of residuals.

6) Define the ARIMA model. Discuss the problem of forecasting ARIMA models.

7) Describe the main components of time series. Discuss any one method of trend removal in the absence of a seasonal component.

8) What do you mean by smoothing of a time series? Also explain Holt-Winter exponential smoothing.

9) What are the different methods of diagnostic checking in time series? Explain the role of residual analysis in model checking.

10) Define the causal ARMA(p, q) process. Examine the causality of the process

$X_t - 0.5 X_{t-1} + 0.3X_{t-2} = Z_t + 0.2Z_{t-1}$, where $\{Z_t\}$ is WN(0, σ^2).

11) Outline a procedure for model selection of an observed time series.