M.Sc. (Mathematics) Part II Sem-IV

Paper HCT4.2: Partial Differential Equation

Question Bank

Short Answer Questions

1) Define curve and surface

2) Define partial diff. equation and order of P.D.E

3) Derive the derivation of PDE by elimination of arbitrary constant.

4) Find the first order PDE which represents the set of all spheres with center on the z-axis and radius a

5) Find the first order PDE which represent the set of right circular cones with z axis as the axis of symmetry.

6) Eliminate the arbitrary constant from Z=ax+by

7) Define Euler's equation for Homogeneous function.

8) Define linear PDE, semi linear PDE.

9) Define quasi linear PDE and Nonlinear PDE

10) Define complete integral, general integral, singular integral.

11) Prove that singular integral is also a solution.

12) Define Pfaffian differential equation.

13) Define integral equation of Pfaffian differential equation.

14) What is compatible solution of first order PDE?

15) Give the geometrical interpretation of quasi linear PDE.

16) Define 2^{nd} order semi linear PDE.

17) Write the classification of second order PDE

18) Define family of equipotential surface

19) Define Interior Dirichlet problem and Exterior Dirichlet problem

20) State maximum and minimum principle

Long Answer Question

1) A necessary and sufficient condition that their exist relation between two functions u(x,y),v(x,y) a relation F(u,v)=0 or u=H(v) not involving x,y explicitly is that $\partial(u,v)/\partial(x,y)=0$
2) Define characteristic curve and envelop and give geometrical interpretation of envelope with one example.
3) Show that General integral is also a solution of PDE
4) Prove that singular integral is also solution of PDE
5) Show that the singular integral is obtained by eliminating p and q from the equations F(x,y,z,p,q)=0, fp (x,y,z,p,q)=0 and fq(x,y,z,p,q)= 0
6) Show that $(x-a)^2+(y-b)^2+z^2=1$ is complete integral of $z^2(1+p^2+q^2)=1$ by taking b=2ashow that the envelope of the sub family is $(y-2x)^2+5z^2=5$which is the particular solution.
7) Show that z= ax +(y/b)+ b is the complete integral of pq=1 find the particular solution corresponding to the subfamily b=a and show that it has no singular integral.
8) Show the Geometrical interpretation of solution of quasi linear PDE
9) The general solution of quasi linear pde ,

   P(x,y,z)p+Q(x,y,z)q=R(x,y,z)

   where P,Q,R are continuously differentiable function of x,y,z and is given by F(u,v)=0 where F is arbitrary function of u,v and $u(x,y,z)=c_1$ and $v(x,y,z)= c_2$ are independent solution of the system

   dx/P=dy/Q=dz/Q
10) Find the general integral of $z(xp-yq)=y^2-x^2$
11) Show that there always exist an integrable factor for Pfaffian diiff.equations.in two variables.
12) Prove that Necessary and sufficient condition that the Pfaffian D.E $\bar{x}\,\overline{dr} = 0$ be integrable that $\bar{x}.curl\bar{x}$=0
13) (6x+yz)dx+(xz-2y)dy+(xy+2z)dz=0 check the integrability and find the solutions.
14) Show that the following Pfaffian diff.eq.is integrable and find its integral.
    Ydx+xdy+2zdz=0, ydx+xdy+2zdz=0

15) Show that the equation=$p^2+q^2-1=0$ and $g=(p^2+q^2)x-pz=0$ are compatible and one parameter family common solutions

16) Describe charpits method for solving a first order PDE f(x,y,z,p,q)=0

17) Find the complete integral by charpits method of given pde $z^2(1+p^2+q^2)=1$

18) Describe Jacobi's method for solving a first order PDE

19) Solve by Jacobi's method $z+2u_z-(u_x-u_y)^2=0$

20) Describe Jacobi's method to solve nonlinear pde

21) Solve $p^2x+q^2y=z$.

22) Find the integral surface of given PDE $(2xy-1)p+(z-2x^2)q=2(x-yz)$

23) Find the complete integral of $(p^2+q^2)x=pz$ and hence find the integral surface through the curve x=0 $z^2=4y$

24)State and prove Hernack's theorem

25) Prove that the exterior Dirichlet problem for a circle

26) Show that the necessary and sufficient condition for the existence of the solution of Neumann problem is that $\int_B f$ should vanish.

27) Show that the solution of Neumann problem is either unique or it differs from one another by a constant

28) Prove that the solution of Dirichlet problem if it is exist then it is unique.

29) Show that the surface $x^2+y^2+z^2=cx^{2/3}$ can form an equipotential surface and find the general form of potential function.

30) Obtain D'Alembert solution of the one dimensional wave equation which describes the vibration of a semi finite string.

M.Sc. (Mathematics) Part-I Sem-II

Paper SCT 2.1: Complex Analysis

Question Bank

Short Answer Questions:

1) Evaluate $\oint_{|z|=2} \frac{2z}{z^2+i}\ dz$

2) State and prove Fundamental theorem of algebra

3) If S is a Mobius transformation then S is a composition of translation, dilation and inversion.

4) Prove that every bounded entire function is constant.

5) Find the Laurent series expansion of

$f(z) = \frac{1}{z(z-1)(z-2)}\ \ in\ ann(0; 0,1)$

6) Evaluate:$\int_{|z|=2} \frac{e^z cosz}{(z-1)^3}\ dz$

7) Prove that there are three zeros of $z^3 - 6z + 8 = 0\ in\ B(\ 0; 3)$.

8) Define with one example of each.

i) Singular point of an analytic function ii) Meromorphic function

9) State and prove open mapping theorem.

10) Show that $f(z) = \frac{1}{\cos\left(\frac{1}{z}\right)}$ has countably infinite number of poles and limit of the sequence of poles of f is a non-isolated singularity.

11) Find the fixed points of S(z)=$\frac{3z+2}{2-4z}$

12) Evaluate:$\int_\gamma \frac{cos2z-e^z}{(z+1)^2(z+2)^2} dz$ over $\gamma : |z| = 1.5$.

13) Find Res(f ;1) , Res (f;2) for $f(z) = \frac{z^2}{(z-1)^2(z-2)^2}$

14) Define the following with one example of each.

i) Removable singularity ii) Residue of an analytic function

15) State and prove Hurwitz theorem.

16) Prove that every Mobius map can have at most two fixed points.

17) Evaluate: $\int_\gamma \frac{f'(z)}{f(z)} dz$ where $\gamma : |z - 1 + i| = 2$

Where, $f(z) = \frac{(z+1)^2(z-1)^3(z-2)}{(z-1-i)^3(z-i)^2}$

18) Prove that a Mobius map is uniquely determined by its action on any three distinct points in C_∞.

19) Construct the cross ratio.

20) Prove that a Mobius transformation preserves the cross ratio.

21) If $|z| < 1$ then show that $\int_0^{2\pi} \frac{e^{is}}{e^{is}-z} ds = 2\pi.$

22) If $f: G \to C$ be analytic then prove that f is infinitely differentiable on G.

23) State and prove Cauchy estimate theorem.

24) If f is analytic in $B(a; R)$ and γ is closed rectifiable curve in $B(a; R)$ then, $\int_\gamma f = 0.$

25) If f is analytic on G and $a \in G$ then z=a is a zero of the function f of order m iff $f(a) = f'(a) = f''(a) = - - - = f^{m-1}(a) = 0$ but $f^m(a) \neq 0.$

26) Prove that every non-constant polynomial has a root in C.

27) State and prove Riemann Criteria.

28) Define winding number of a curve around a point with examples.

29) Prove that every non-constant polynomial is an entire function.

30) Define Critical point and Fixed point.

Long Answer Questions:

1) State and prove Morera's therorem.

2) Evaluate $\int_0^\infty \frac{1}{1+x^2} dx$ using Cauchy's residue theorem.

3) If f be an analytic in annulus $n(a\,; R_1, R_2)$. Then prove that

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z-a)^n$$

Where the convergence is absolute and uniform over $ann(a\,; r_1, r_2)$ if

$R_1 < r_1 < r_2 < R_2$. Further prove that the coefficients a_n are given by

$a_n = \int_\gamma \frac{f(z)}{(z-a)^{n+1}} dz$ Where γ is a circle |z-a|=r , for any r, $R_1 < r < R_2$ and this series is unique.

4) Find a Mobius map which maps the points $z_2 = 2, z_3 = i, z_4 = -2$ onto

$w_2 = 1, w_3 = i\,, w_4 = -1$ respectively.

5) State and prove theorem for Cauchy's integral formula .

6) If f and g are meromorphic in a neighborhood of $\bar{B}(a; R)$ with no zeros or poles on the circle = $\{\, z \in \mathbb{C}\, ||z-a| = R\,\}$. If $Z_f\,, Z_g\ and\ P_f\,, P_g$ are the number of zeros and poles of f and g inside γ counted according to their multiplicities and if | f(z) + g(z) | < |f(z) | + | g(z) | on { γ } , then prove that

$$Z_f - Z_g = P_f - P_g$$

7) $|\, f(z)| \le 1$ for $|z| < 1$ and f is analytic, then prove that

$$|f(z)| \le \frac{(|f(0)| + |z|)}{1 + |f(0)||z|} \quad , |\, z\,| < 1$$

8) Define: Analytic function and if f is analytic in B(a;R) then prove that $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ for $|z-a| < R\ \ where\ a_n = \frac{f^{(n)}(a)}{n!}$ and this series has radius of convergence at least equal to R.

9) If γ is a rectifiable curve and φ is a function defined and continuous on

$\{\gamma\}$. For each $m \geq 1$, let $F_m(z) = \int_\gamma \frac{\varphi(w)}{(w-z)^m} dw$ forz $\notin \{\gamma\}$. Then prove that each F_m is analytic on $\mathbb{C} - \{\gamma\}$ and $F'_m(z) = F_{m+1}(z)$.

10) State and prove Schwarz lemma.

11) If $f: G \to \mathbb{C}$ is analytic and suppose that $\bar{B}(a;r) \subset G \ (r > 0)$ and

$\gamma(t) = a + re^{it}, 0 \leq t \leq 2\pi$ then prove that

$f(z) = \frac{1}{2\pi i} \int_\gamma \frac{f(w)}{w-z} dw$, for |w-z|<r .

12) If G is a connected open set and $f: G \to \mathbb{C}$ is analytic function,then prove that the following statements are equivalent.

i) $f \equiv 0$ii) $\{ z \in G \mid f(z) = 0 \}$ has a limit point in G.

iii) there is a point 'a' in G such that $f^{(n)}(a) = 0$ for each $n \geq 0$.

13) Prove that an isolated singularity of f at z=a is removable iff

$\lim_{z \to a}(z-a)f(z) = 0$

14) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} \, dx = \frac{\pi}{\sqrt{2}}$

15) State and prove Montel's theorem.

16) If z_1, z_2, z_3, z_4 are four distinct points in $\mathbb{C}_\infty$, then prove that (z_1, z_2, z_3, z_4) is a real number iff all four points lie on the circle.

17) For a given power series, $\sum_{n=0}^{\infty} a_n(z-a)^n$ define a number $0 \leq R \leq \infty$

by $\frac{1}{R} = \limsup |a_n|^{\frac{1}{n}}$, then prove the following.

i) If |z-a|<R , the series converges absolutely.

ii) if |z-a|>R, the term of the series become unbounded and so the series diverges.

iii) If 0<r<R, the series converges uniformly on $\{z \mid |z-a| \leq r\}$. Further, prove the uniqueness of R having properties (i) and (ii) .

18) State and prove Gaursat's theorem.

19) If z =a is an isolated singularity of a function f and if

$f(z) = \sum_{n=-\infty}^{\infty} a_n(z-a)^n$ is its Laurentz series expansion in $ann(a; 0, R)$.

Then prove the following.

i) z= a is removable singularity iff$a_n = 0$ for $n \leq -1$.

ii) z = a is a pole of order m iff$a_{-m} \neq 0$ and $a_n = 0$ for all $n \leq -(m+1)$

iii) z =a is an essential singularity iff$a_n \neq 0$ for infinitely many negative integral values of n.

20) If G is an open set and $f: G \to \mathbb{C}$ is differentiable function,then prove that f is analytic.

21) Evaluate :$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\ dx$

22) If f is analytic on an open connected set G and f is not indentically zero then for each a in G with f(a) = 0,then prove that there is an integer $n \geq 1$ and an analytic function $g: G \to \mathbb{C}$ such that $g(a) \neq 0$ and $f(z) = (z-a)^n g(z)$

for all z in G.

23) State and prove Argument principle.

24) Show that the set of all bilinear transformation forms a non-abelian group under composition.

25) Explain the following terms with examples.

a) Translation b) Rotation c) Magnification d) Inversion

26) If S is a Mobius transformation then S is the composition of Translation, Dilation and Inversion.

27) State and prove Leibnitz theorem.

28) If $\gamma: [0,1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that $\frac{1}{2\pi i}\int_{\gamma} \frac{dz}{z-a}$ is an integer.

29) State and prove Cauchy’s integral formula first version.

30) Explain Laurent Series Development.

M.Sc. (Mathematics) Part-II Sem-IV

Paper HCT4.4: Operations Research

Question Bank

Short Answer Questions:

1. Define: Slack Variable , Surplus Variable
2. Define : Hyperplanes, Supporting Hyperplane
3. Define : Convex set,Extreme point
4. Prove that: A hyperplane is a convex set .
5. Explain Graphical method of solving Linear Programming Problem.
6. Write a short note on Big-M method.
7. Write a short note on effect of Addition and deletion of variable on optimal solution of LPP.
8. Write a note on Non-linear programming problem.
9. Write disadvantages of Big M method over Two Phase method.
10. What is degeneracy problem?
11. Explain concept of duality in Linear Programming Problem.
12. Write General Rules of Converting any primal into its Dual.
13. Write Matrix form of Symmetric Primal and Dual form.
14. Prove that: The dual of dual of given primal is primal .
15. Write the formulas to find entering vector and outgoing vector in Dual Simplex method.
16. State the advantage of Dual Simplex method over simplex method.
17. State the difference between Dual Simplex method and simplex method.
18. Write short cut method for constructing Gomory's constraint.
19. Write General Quadratic Programming problem.
20. What is Integer Programming problem.
21. Write characteristics of Game theory.
22. What is zero sum game and non-zero sum game?
23. Define payoff matrix.
24. Explain minimax criterion to find optimal strategy.

25. Define: saddle point, Optimal Strategies.
26. Define Value of Game
27. Write rules for determining the saddle point.
28. Explain rectangular games without saddle point.
29. Define : Pure Strategy and mixed Strategy
30. Write short note on Strategic saddle point.

Long Answer Questions:

1. Prove that:The set of all convex combinations of finite number of points is a convex set.
2. Prove that: The collection of all feasible solutions constitutes a convex set whose extreme point corresponds to the basic feasible solutions.
3. Write algorithm of Two Phase method.
4. Write algorithm of Simplex method.
5. Write algorithm of Big M method.
6. Explain the method to resolve degeneracy.
7. State and prove Basic Duality theorem.
8. State and prove Fundamental Duality theorem.
9. If Kth constraint of the primal is an equality then prove that its dual variable wk is unrestricted in sign.
10. If the pth variable is unrestricted in sign then prove that the pth constraint of dual is an equality.
11. State and prove Complementary slackness theorem .
12. State the rules of obtaining dual solution from the primal.
13. Write algorithm of Dual Simplex method.
14. Explain how to construct Gomory's constraint.
15. Write algorithm of Gomory's cutting plane method.
16. Explain geometrical interpretation of Gomory's cutting plane method.

17. Explain Kuhn Tucker necessary conditions for solving Quadratic Programming problem.
18. Write algorithm of Wolfe’s Modified method.
19. Write algorithm of Beale’s method.
20. Write algorithm of Branch and Bound method for solving Integer Programming problem.
21. State and Prove Fundamental theorem of Rectangular games.
22. Explain the solution of m*n games by linear Programming.
23. Explain the Arithmetic method for 2*2 games.
24. Explain the principle of dominance to reduce the size of games.
25. Explain Graphical method for 2*n and m*2 games.
26. Numeric examples on all the methods discussed.

Subject: Applied Mathematics II

Section I

Q.1 Find a root of the following equation $x^3 - x - 11 = 0$ using the regula-falsi method correct to three decimal places.

Q.2 Find a root of the following equation $x^3 - 3x + 1 = 0$ using the regula-falsi method correct to three decimal places.

Q.3. Find a root of the following equation $x^3 + x - 1 = 0$ using the Newton Raphson method correct to three decimal places.

Q.4. Find a root of the following equation $x^3 - 4x - 9 = 0$ using the Newton Raphson method correct to three decimal places.

Q.5. Evaluate $\sqrt{13}$ up to 3 decimal places.

Q.6. Evaluate $\sqrt{7}$ up to 3 decimal places.

Q.7. Find the double root of the eqution $x^3 - x^2 - x + 1 = 0$ given that it is near to 0.8.

Q.8. Find the double root of the eqution $x^3 - 5x^2 + 8x - 4 = 0$ given that it is near to 1.8.

Q.9. Perform two iterations of Newton-Raphson method to find a solution of the system

$x^2 + xy = 6$ & $x^2 - y^2 = 3$ taking $x_0 = y_0 = 1$

Q.10. Solve the system of nonlinear equation $x^2 + y = 11$ & $y^2 + x = 7$ taking $x_0 = 3.5$ & $y_0 = -1.8$ by using Newton-Raphson method.

Q.11. Use Gauss Elimination method to solve x + y + z = 2,x + 2y + 3z = 5,2x + 3y + 4z = 11.

Q.12. Use Gauss Elimination method to solve x + y + z = 9, 2x + 5y + 7z = 52, 2x + y – z = 0.

Q.13. Use Gauss Jordan method to solve x + 3y + 2z = 2 , 2x + 7y + 7z = −1, 2x + 5y + 2z = 7.

Q.14. Use Gauss Jordan method to solve 2x+3y−4z=53, x+4y−5z=−64, x+5y−6z=7

Q.15. Use LU Decomposition method to solve x + y + z = 1,4x + 3y – z = 6,3x + 5y + 3z = 4.

Q.16. Use LU Decomposition method to solve x + 2y + 3z = 9,4x + 5y + 6z = 24,3x + y – 2z = 4.

Q.17. Evaluate $\int_0^1 \frac{dx}{1+x}$ applying Trapezoidal rule.

Q.18. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule.

Q.19. Use Simpson's 1/3rd rule to find $\int_0^{0.6} e^{-x^2} dx$ taking seven ordinates.

Q.20. Compute the value of $\int_{0.2}^{1.4} (\sin x - \log x + e^x) dx$ using Simpson's 3/8th rule.

Q.21. Find an approximate value of $\log_e 5$ by calculating to 4 decimal places, by Simpson's 1/3 rule $\int_0^5 \frac{dx}{4x+5}$, dividing the range into 10 equal parts.

Q.22. Evaluate $\int_0^1 \int_0^1 xe^y dxdy$ using Trapezoidal rule wigh h=k=0.5.

Q.23. Evaluate $\int_0^5 \int_0^5 \frac{dxdy}{\sqrt{x^2 + y^2}}$ taking two subintervals.

Q.24. Evaluate $\int_0^1 \int_0^1 e^{x+y} dxdy$ using Simpson's rule.

Q.25. Evaluate $\int_1^{2.8} \int_2^{3.2} \frac{dxdy}{x+y}$ using Simpson's rule.

Section II

1. Let A and B be fuzzy sets defined on universal set $X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3\}$ as

$$A = \frac{1}{-5} + \frac{0.75}{-4} + \frac{0.20}{-3} + \frac{0.8}{-2} + \frac{0.32}{-1} + \frac{0.28}{0} + \frac{0.9}{1} + \frac{0.65}{2} + \frac{1}{3}$$ and

$$B = \frac{0}{-5} + \frac{0.80}{-4} + \frac{0.20}{-3} + \frac{0.70}{-2} + \frac{0.20}{-1} + \frac{0.15}{0} + \frac{1}{1} + \frac{0.60}{2} + \frac{1}{3}$$

Find $S(A, B)$ and $S(B, A)$

2. Find strong $\alpha-$cuts of the fuzzy set A defined by the membership function

$$A(x)=\begin{cases}\frac{x-10}{20}, & 10\le x\le 30\\ \frac{40-x}{10} & 30<x\le 40\\ 0, & otherwise\end{cases}$$ for $\alpha = 0, 0.3, 0.9$

3. Verify which of the following fuzzy sets are fuzzy numbers

(i) $A=\frac{1}{1}+\frac{0.5}{2}+\frac{0.6}{3}+\frac{0.7}{4}+\frac{0.8}{5}$

(ii) $B(x)=\log x,\ x\in[1,2.72]$

4. Let A be a fuzzy set defined on universal set $X=\{-3,-2,-1,0,1,2,3\}$ by the membership function $A(\mathrm{x})=\frac{x+3}{10}, \forall \mathrm{x}\in X$ and f be a function defined on X as $f(x)=2x^2+10$. Then find $f(A)$

5. Let A,B be the fuzzy numbers defined by the membership functions

$$A(\mathrm{x})=\begin{cases}\frac{x+5}{2}, & -5\le x\le -3\\ \frac{-x}{3}, & -3<x\le 10\\ 0, & otherwise\end{cases}$$ and $$\mathrm{B(x)}=\begin{cases}\frac{\mathrm{x}}{2}, & 0\le x\le 2\\ \frac{5-x}{3}, & 2<x\le 5\\ 0, & otherwise\end{cases}$$ find a fuzzy number $A.B$

6. Solve the fuzzy equation $A+X=B$ where A,B are fuzzy numbers defined by the membership functions $$A(\mathrm{x})=\begin{cases}\frac{x-9}{2}, & 9\le x\le 11\\ \frac{14-x}{3}, & 11<x\le 14\\ 0, & otherwise\end{cases}$$ and $$\mathrm{B(x)}=\begin{cases}\mathrm{x}-5, & 5\le x\le 6\\ \frac{9-x}{3}, & 6<x\le 9\\ 0, & otherwise\end{cases}$$

7. Let A be a fuzzy set defined on universal set $X=\{0,1,2,3,4,5\}$ by the membership function $A(\mathrm{x})=e^{-x}, \forall x\in X$. Then find fuzzy cardinality of A

8. Let A,B be any two fuzzy sets defined on universal set X and $\alpha,\beta\in[0,1]$. Then prove that

(i) $^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$

(ii) If $\alpha \leq \beta$, then $^{\beta}A \subseteq {}^{\alpha}B$

9. Let A be a fuzzy set defined on universal set $[-1,1]$ by the membership function

$A(x) = \begin{cases} x+1, & -1 \leq x \leq 0 \\ 1-x, & 0 < x \leq 1 \end{cases}$ Find (i) Boundary of A (ii) Core of A

10. Let A, B be the fuzzy numbers defined by the membership functions

$A(x) = \begin{cases} \dfrac{x-1}{4}, & 1 \leq x \leq 15 \\ 6-x, & 5 < x \leq 6 \\ 0, & otherwise \end{cases}$ and $\mathrm{B}(x) = \begin{cases} \dfrac{x-6}{4}, & 6 \leq x \leq 10 \\ 11-x, & 10 < x \leq 11 \\ 0, & otherwise \end{cases}$, find $Max(\mathrm{A}, \mathrm{B})$

11. Let A, B be the fuzzy numbers defined by the membership functions

$A(x) = \begin{cases} \dfrac{x+5}{2}, & -5 \leq x \leq -3 \\ \dfrac{-x}{3}, & -3 < x \leq 0 \\ 0, & otherwise \end{cases}$ and $\mathrm{B}(x) = \begin{cases} \dfrac{x}{2}, & 0 \leq x \leq 2 \\ \dfrac{5-x}{3}, & 2 < x \leq 5 \\ 0, & otherwise \end{cases}$, find a fuzzy number A, B

12. Solve using the Simplex method the following problem: Maximize Z= $3x + 2y$

Subject to $2x + y \leq 18$, $2x + 3y \leq 42$, $3x + y \leq 24$, $x \geq 0, y \geq 0$.

13. Solve using the Simplex method the following problem: Maximize = 40x+30y

Subject to $x + y \leq 12$, $2x + y \leq 16$, $x \geq 0$, $y \geq 0$

14. The Funny Toys Company has four men available for work on four separate jobs. Only one man can work on any one job. The cost of assigning each man to each job is given in the following table. The objective is to assign men to jobs in such a way that the total cost of assignment is minimum.

		Job		
Person	1	2	3	4
A	20	25	22	28
B	15	18	23	17
C	19	17	21	24

D	25	23	24	24

M.Sc. (Mathematics) Part I Sem-II

Paper HCT2.1: Algebra II

Question Bank

Short Answer Questions

1) If a and b in K are algebraic over F of degrees m and n resp.if m and n are relatively prime then prove that F(a,b) is of degree mn over F
2) Construct a field with 9 elements.
3) Show that $8x^3$-6x-1 is irreducible over Q
4) Show that it is impossible, by straight edge and compass alone to trisect 60^{0}
5) Let R be the field of real number and Q be the field of rational number then show that $\sqrt{2}, \sqrt{3}$ in R are algebraic over Q
6) What is $[Q(\sqrt{2}, \sqrt{3}):Q]$=?
7) If a in K is a root of P(x) in F[x] where f $\subseteq$ K then in K[x] prove that (x-a)/P(x)
8) Define multiple root of a polynomial.
9) Define Spliting field.
10) There exist a Spliting filed for every f(x) in F[x}
11) Any two Spliting field of the same polynomial over a given field F are isomorphic by an isomorphism leaving every element of F is fixed.
12) Define separable element.
13) Define separable extension
14) Define perfect field
15) Define algebraic element.
16) Define algebraic extension.
17) Define simple extension.
18) Define Characteristic of a ring.
19) Define fixed field.
20) Define galois group.
21) Define Constructible Number.

Long Answer Question

1) Let K be an extension of field Show that the elements in K which are algebraic over F forms a subfield of K

2) If L is an algebraic extension of K and K is an algebraic extension of F, show that L is an algebraic extension of F.

3) Show that Q($\sqrt{2}, \sqrt{3}$) =Q($\sqrt{2} + \sqrt{3}$)

4) Let Q be the field of rational numbers, Determine the degree of Spliting field of the polynomial x^3-2over Q

5) Show that a polynomial of degree n over a field can have at most n roots in any extension field.

6) Show that the polynomial f(x) in F[x] has a multiple roots iff f(x) and f ‘ (x) have a non-trivial common factor.

7) Show that any finite extension of a field of characteristic zero is a simple extension.

8) Show that any field of char.0 is perfect

9) Show that the polynomial x^7-10x^5+15x+5 is not solvable by radicals over Q.

10) if K is a field and σ1,σ2…..σn are distinct automorphism of K then show that they are linearly independent.

11) if L is finite ext. of F and K is subfield of L which contains F then [k:F]/[L:F]

12) Let a in K be alge. Over F then any two monomial monic polynomial for a over F are equal

13) Let a in K be alg. Over F. let P(x) be a minimal polynomial for a over F then prove that P(x) is irreducible over F

14) Let K be ext. of field F and let a in K be algebraic of degree n over F then prove that

F (a) = { b0+b1a+b2a2+……..+bn-1an-1 /b0,b1……bn-1 in F} .

15) Let k be an ext. of a field F and Let a_1, a_2….. a_n be n elements in K algebraic over F then F(a_1,a_2…a_n)is finite ext. of F and consequently an algebraic ext. of F

16) Let K be an ext. of a field F then the elements in K which are algebraic over F forms a subfield of K.

17)If a and b in K are algebraic over F of degrees m & n resp. then a+b,a-b,ab,a\b(b not equal to 0) are algebraic over F of degrees at most mn.

18) State and prove remainder theorem.

19) A polynomial of degree n over a field can have at most n roots in any extension field.

20) If P(x) is an irreducible polynomial in F[x] of deg $n \geq 1$ then there is an ext. E of F such that [E:F]=n in which P(x) has a root

21) let f(x) in F[x] be of degree $n \geq 1$ then there is a finite ext. E of F of degree at most n! in which f(x) has n roots .

22)let ψ be an isomorphism of field F onto a field F' such that $\psi(\alpha) = \alpha'$ for all α in F then there is an isomorphism ψ^* of F[x]ontoF'[t] s.t. $\psi^*(\alpha) = \psi(\alpha) = \alpha'$ for every α in F

23) if f(x) in F[x] is irreducible and if a and b are two roots of f(x) then P.T F(a) is isomorphic to F(b) by an isomorphism which takes a onto b and which leaves every element of F fixed.

24) Let F be a field of rational number. Determine the degree of the Spliting field of the polynomial x^3-2 over F.

26) if P is a prime then Prove that the Spliting field over F the field of rational number of the polynomial x^p-1 is of degree p-1

28) The polynomial f(x) in F[x] has multiple root iff f(x) and f '(x) have a non-trivial common factor.

29) Prove that collection of all automorphism of a field K forms a group w.r.t.composite of two functions.

30) Let G be a subgroup of the group of all automorphism of a field K then Prove that the fixed field of G is subfield of K.

31) Suppose K is finite extension of a field F of char.0 and H is a subgroup of G(k,F)let K_H be the fixed field of H then prove that a) $[K:k_H]=O(H)$ b) $H = G(K,K_H)$

32)A field K is a normal extension of a field F of Char.0 iff K is a Spliting field some polynomial over F

M.Sc. (Mathematics) Part-I Sem-II

Paper OET 2.1: Fundamental in Mathematics

Question Bank

Short Answer Questions:

1) Define Basis & Dimension of vector space.

2) Define Symmetric matrix.

3) Define Solution of system of equation

4) Define Rank of matrix

5) Define a) Linearly dependent vector b) Linearly independent vector

6) Write a note on Types of Matrices

7) Write a note on Null space, Range space

8) Write a note on Linear transformation.

9) If matrix A = $\begin{bmatrix} 6 & 7 \\ -1 & 2 \end{bmatrix}$ then find A^{-1}.

10) Show that inverse of matrix is unique.

11) If D=$\begin{bmatrix} 0 & 3 & 7 \\ 8 & 1 & 4 \\ 7 & 1 & 2 \end{bmatrix}$show that $(5D)^T = 5D^T$.

12) Solve the system of equation

$x - y + z = 9$
$x + y + -2z = 2$
$2x+y+z = -1$

13) Explain Properties of determinant with example.

14) If A = $\begin{bmatrix} 2 & -1 & 0 \\ 4 & 7 & 8 \\ -3 & 1 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 0 & 6 \\ 1 & -8 & 2 \\ -5 & 4 & 7 \end{bmatrix}$ find A+B, A-B.

15) Show that $T(v_1, v_2, v_3) = (v_1 + v_2, v_2 - v_1, v_1 - v_3)$is linear transformation.

16) Find the Transpose of matrix A=$\begin{bmatrix} 2 & 3 & -6 \\ -2 & 6 & 1 \\ 1 & 5 & 2 \end{bmatrix}$.

17) Prove that inverse of the square matrix A exists iff $|A| \neq 0$.

18) a) Define subspace. b)State Rank Nullity theorem

19) Find the solution of $2x-3y+z = 0$ $x+2y=3z = 0$ $4x - y - 2z = 0$.

20) Check whether mapping T: $R^2 \rightarrow R^2$ defined by $T(a_1,a_2) = (a_2, 1+ a_2)$ is linear transformation.

21) Find the inverse of $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$.

22) Find the rank of the matrix $\begin{matrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \\ -2 & -1 & 4 \end{matrix}$

23) Find value of α where (3,1, α) is Linear combination of (1,0,1)and (1,1,2).

24) Show that (1,2,1),(2,1,4),(4,5,0) are Linearly independent

25) If{x, y, z} is basis of R^3 then check whether {x + y, y + z, z + x} is linearly independent?

26) If V(F) be a vector space and $\alpha, \beta \in V \; and \; a, b \in F$. Then prove that

i) $a\alpha = b\alpha \; and \; \alpha \neq 0 \; then \; a = b$

ii) $a\alpha = a\beta \; and \; \alpha \neq 0 \; then \; \alpha = \beta$.

27) Show that the map $T: R^2 \to R^2 \; such \; that \; T(a, b) = (a^2, b)$ is not a linear.

Long Answer Questions:

1) Find inverse of matrix A = $\begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$.

2) Let K be solution set of linear equation AX=B and let K_h be solution set of corresponding homogenous system of linear equations for any solution 'S' to AX=B,

$$K = \{s\}+K_h = \{s+k : k\in K_h\}$$

3) Define vector space with all conditions.

4) Show that $2x^3+x^2+x+1$, $x^3 + 3x^2 + x-2$, $x^3 + 2x^2 - x + 3$ are linearly Independent over R.

5) Show that equations $3x + y + z = 8$, $-x + y -2z = -5$, $2x + 2y + 2z = 12$, $-2x + 2y - 3z = -7$ are consistent and find solution.

6) Prove that intersection of subspace of vector space is subspace.

7) Define Matrix & explain the types of matrices with example.

8) Determine whether the following vectors forms a basis of R^3(R).

i)(1,1,2),(1,2,5),(5,3,4)

ii)((1,2,-1),(1,0,2),(2,1,1)

9) Prove that if a vector space V is generated by finite set S then some subset of S is basis for V hence V has finite basis.

10) Let V& W be a vector spaces & $T:V\rightarrow W$ be linear then prove that T is one- one iff

N (T) = {0}

11) State and prove Rank Nullity theorem.

12) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 7 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

13) Show that $T:R^2\rightarrow R^2$ defined by $T(a_1,a_2) = (a_1+a_2,a_1)$ is one – one and onto.

14) Find the Rank and Nullity of the matrix $\begin{matrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 3 & -1 & 2 \\ 0 & 5 & 2 \\ 2 & 1 & 2 \end{matrix}$ and verify Rank Nullity theorem.

15) Find the matrix representation of following linear operator defined on R^3.

T(x, y, z) = (z, y + z, x +y +z)

16) If T is linear transformation on V and $T^2 - T + I = 0$ then show that T is invertible.

17) Show that $2x^3+x^2+x+1$, $x^3 + 3x^2 + x-2$, $x^3 + 2x^2 - x + 3$ are linearly independent over R.

18) If V is vector space & $S_1 \subseteq S_2 \subseteq V$ and S_1 is linearly dependent then prove that S_2 is linearly dependent.

19) Find the solution of $x -3y -8z +10 = 0$, $3x + y - 4z = 0$, $2x + 5y + 6z-13 = 0$

20) Let $\{e_1= (1,0) ,e_2 = (0,1)\}$ & $\{e_1'= (1,2) ,e_2' = (2,3)\}$ find the transition matrix

i) From $\{e_i\}$ to $\{e_i'\}$

ii) From $\{e_i'\}$ to $\{e_i\}$.

21) Prove that the union of two subspaces is a subspace if one is contained in the other.

22) Prove that the linear span L(S) of any non-empty subset S of a vector space V(F) is a subspace of V(F).

23) If M and N be subspaces of a vector space V(F). Then show that

$i)\ M \subset M + N$ $\quad$ $ii)\ M \subset M + N$ $\quad$ $iii)\ M + N = L(M \cup N)$

24) Show that any superset of linearly dependent vectors is linearly dependent.

25) Prove that the vectors $(1,2,1), (2,1,4), (4,5,0)$ are linearly independent.

26) Determine whether or not the following vectors form a basis of R^3.

(1,1,2), (1,2,5), (5,3,4)

27) Show that the necessary and sufficient condition for a matrix A to be invertible is that $|A| \neq 0$.

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M.Sc. Mathematics Part-II SEMESTER-III

Subject: Differential Geometry SCT 3.1

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Short Question:

1. For a patch X: D→ E^3 , $if\ E = X_u \cdot X_u$, $F = X_u \cdot X_v$, $G = X_v \cdot X_v$, then prove that X is regular iff $EG - F^2 \neq 0$.
2. Find the quadratic approximation near origin for the surface

   M: z= $e^{x^2+y^2} - 1$
3. Define directional derivative $v_p[f]$
4. Covariant derivative of a vector field.
5. Let e_1, e_2, e_3 be frame at a point p of E^3 . If v is any tangent vector to E^3 at p, then show that v = $\sum_{i=1}^{3}(v.e_i)e_i$
6. If X is a patch in M ⊂ E^3 then prove that

   $K(X) = \frac{ln-m^2}{EG-F^2}$ $\quad$ $H(X) = \frac{Gl+En-2Fm}{2(EG-F^2)}$
7. Find the unit speed reparameterization of a circle of radius r and hence compute the tangent vector field of the curve.
8. Show that the rotation is an orthogonal transformation.
9. If T is a translation by $\bar{a}$ then T has an inverse T^{-1} which is translation by $-\bar{a}$.
10. Given any two points $\bar{p}$ and $\bar{q}$ of E^3 there exists a unique translation T such that T($\bar{p}$) $=$ $\bar{q}$.
11. Find the quadratic approximation near origin for the surface M given by M: z=log(cosx)-log(cosy)

Long Question:

12. If $\alpha : I \to E^3$ is a regular arbitrary speed curve in E^3, then derive expressions for Frenet apparatus T, N, B, τ and κ.
13. Find the unit speed reparameterization of a curve helix.

14. Show that the curve $\beta(s) = \left(a\cos\left(\frac{s}{c}\right), a\sin\left(\frac{s}{c}\right), \frac{bs}{c}\right)$, c=$\sqrt{a^2+b^2}$ has unit speed and hence find compute the Frenet apparatus for the unit speed curve β.

15. Show that M: $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is a surface and X(u, v) = (aucos v, businv, u^2) is a parametrization of M.

16. Let V, W, Y and Z be vector fields on E^3 , then prove that

i) $\nabla_V(aY + bZ) = a\nabla_V Y + b\nabla_V Z$ ii)$\nabla_{fV+gW} Y = f\nabla_V Y + g\nabla_W Y \ \ \forall f, g$

iii) $\nabla_V(fY) = f\nabla_V Y + V[f]Y$ $iv)\ V[Y \cdot Z] = Y \cdot \nabla_V Z + \nabla_V Y \cdot Z$

17. Compute the Frenet apparatus for the curve $\alpha(t) = (e^t cost, e^t sint, e^t)$

18. Prove that $u \cdot (v \times w) \neq 0\ iff\ u, v, w\ are\ linearly\ independent.$

19. If $V = y^2U_1 - xU_3\ and\ \ f = x^2y^2\ \ ,\ g = z^3\ ,\ h = x + 2y^2 - 3z^3$ find

20. i) V[f] ii) V[h] iii) V[g] .

21. For a non-unit speed regular curve in E^3 prove that

22. $\begin{bmatrix} \dot{T} \\ \dot{N} \\ \dot{B} \end{bmatrix} = \begin{bmatrix} 0 & \kappa v & 0 \\ -\kappa v & 0 & \tau v \\ 0 & \tau v & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$

23. f $f = x^2cosy - zsiny + x^2y^2z^2\ \ and\ g = xy + y^2z^2 - x^3z^3$ then find and v = (1, 0, -3) and p = (-2,0,3) find i) $v_p[f]\ \ ii\ )\ v_p[g]\ \ \ iii)\ v_p[fg]$

24. Show that M: $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ is a surface and X(u,v) = (aucos v, businv,u^2) is a parametrization of M .

M.Sc. (Mathematics) Part-II Sem-IV

Paper HCT 4.1: Measure and Integration

Question Bank

Short Answer Questions:

1) Let $\{ A_n\}$ be a countable collection of measurable sets, then prove that

$\mu(\bigcup_{i=1}^{\infty} A_i) = \lim_{n\to\infty} \mu(\bigcup_{i=1}^{n} A_i)$

2) Let $(X, \mathfrak{B})$ be a measurable space and f be extended real valued function defined on X. Then prove that the following statements are equivalent.

i) $\{x \in X \mid f(x) < \alpha \}$ is measurable ii) $\{x \in X \mid f(x) \geq \alpha \}$ is measurable .

iii)$\{x \in X \mid f(x) > \alpha \}$ is measurable iv) $\{x \in X \mid f(x) \leq \alpha \}$ is measurable.

3) State Tonelli's theorem .

4) Give an example to show that Hahn decomposition is not unique.

5) Let $\{ A_n\}$ be a countable collection of measurable sets, then prove that

$\mu(\bigcup_{i=1}^{\infty} A_i) = \lim_{n\to\infty} \mu(\bigcup_{i=1}^{n} A_i)$

6) Show that the set of all locally measurable sets form a $\sigma - algebra$.

7) Let $(X, \mathfrak{B})$ be a measurable space and f be extended real valued function defined on X. Then prove that the following statements are equivalent.

i) $\{x \in X \mid f(x) < \alpha \}$ is measurable ii) $\{x \in X \mid f(x) \geq \alpha \}$ is measurable .

iii)$\{x \in X \mid f(x) > \alpha \}$ is measurable iv) $\{x \in X \mid f(x) \leq \alpha \}$ is measurable.

8) Let $(X, \mathfrak{B}, \mu)$ be a measure space and g a non-negative measurable function on X. Define $\nu(E) = \int_E g \, d\mu$. Then show that ν is a measure.

9) Show that the set of all locally measurable sets form a $\sigma - algebra$.

10) State and prove monotone convergence theorem.

11) If E is a measurable subset of $X \times Y$ then prove that

i) $\left(\tilde{E}\right)_x = (\widetilde{E_x})$ $\quad$ $ii)$ $(\cup_i E_i)_x = \cup_i (E_i)_x$

12) Let $\{f_n\}$ be a sequence of non-negative measurable functions. Then prove that

$\int \sum_{n=1}^{\infty} f_i = \sum_{n=1}^{\infty} \int f_i$

13) If $E_1, E_2 \in \mathfrak{B}$ and if $\mu(E_1 \Delta E_2) = 0$ then prove that $\mu(E_1) = \mu(E_2)$.

14) Define σ-finite measure. Prove that if f is integrable then the set $\{x \,|f(x) \neq 0\,\}$ is of σ-finite measure.

15) Prove that every finite measure is a σ-finite measure.

16) Show that every σ-finite measure is saturated.

17) If $(X, \mathfrak{B}, \mu)$ be a complete measure space. If $E_1 \in \mathfrak{B}$ and $\mu(E_1 \Delta E_2) = 0$ then $E_2 \in \mathfrak{B}$.

18) If μ is a complete measure. Let f be a measurable function such that f=g a.e. Then g is measurable.

19) State Generalized Fatous Lemma.

20) Define Positive and Negative set.

Long Answer Questions:

1) Let $Let\ E_i\ \in \mathfrak{B}$ then prove that

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} \mu(E_i)$$

2) Let f and g be two measurable functions and c be any constant . Then prove that $f \pm c, cf, fg$ are measurable functions. Moreover, if $\{f_n\}$ is a sequence of measurable functions, then prove that $\inf\{f_n\}, \sup\{f_n\}, \overline{lim}\ \{f_n\}, \underline{lim}\{f_n\}$ are also measurable.

3) If f and g are non-negative measurable functions and a,b are non-negative constants then prove that

$i)\ \int(af + bg) = a\int f + b\int g \quad ii) \int f \geq 0 \quad ii)\ \int f = 0\ then\ f = 0\ a.e.$

4) State and prove Hahn decomposition theorem.

5) If ν is a signed measure and μ is a measure such that $\nu \perp\ \mu\ and\ \nu \ll\ \mu$ then prove that v = 0.

6) Prove that the class $\mathfrak{B}$ of μ^* measurable sets is a σ $-$algebra .

7) Let $E \in \mathfrak{R}_{\sigma\delta}$ with $(\mu \times \nu)(E) < \infty$. Then prove that the function defined by $g(x) = \nu(E_x)$ is a measurable function on X and $\int_X g d\mu = (\mu \times \nu)(E)$.

8) Let B be a μ^* measurable set with $\mu^*(B) <\ \infty$. Then prove that $\mu_*(B) = \mu^*(B)$.

9) If $v_1 \perp\ \mu\ and\ v_2 \perp\ \mu$ where $v_1, v_2\ and\ \mu$ are measures then show that

$(c_1 v_1 + c_2\ v_2) \perp\ \mu$.

10) Let $\{\ A_i \times B_j\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle A × B. Then show that

$\lambda(A\ \times B\ ) = \sum_{(i,j)} A_i \times B_j$

11) State and prove Lebesgue convergence theorem.

12) If ν is a signed measure and μ is a measure such that $\nu \perp \mu$ and $\nu \ll \mu$ then prove that v = 0.

13) State and prove Lebesgue decomposition theorem.

14) Let $x \in X$ be any element. Then show that for any E $E \in R_{\sigma\delta}, E_x$ is a measurable subset of Y.

15) Let μ be a measure on algebra $\mathcal{A}$, μ^* the measure induced by μ and E be any set. Then prove that for each $\varepsilon > 0$ there exists a set A $\in A_\sigma$ with E ⊆ A and $\mu^*(A) \le \mu^*(E) + \varepsilon$.

16) State and prove Tonelli's theorem.

17) Let $\{A_i\}$ be a disjoint sequence of sets in A then

$$\mu_*[E \cap (\cup_{i=1}^{\infty} A_i)] = \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$$

18) Show that a Hahn decomposition of X is unique except for null sets.

19) Show that $(\mathfrak{R}, \mathcal{M}, \mu)$ is a measure space, where $\mathcal{M}$ is set of Lebesgue measurable sets and μ is defined as

$$\mu(E) = \begin{cases} |E| \ , & if\ E\ is\ finite \\ \infty & ,if\ E\ is\ infinte \end{cases}$$

20) $v_1 \ll \mu$ and $v_2 \ll \mu$ then prove that $\left[\frac{d(v_1+v_2)}{d\mu}\right] = \left[\frac{dv_1}{d\mu}\right] + \left[\frac{dv_2}{d\mu}\right]$

21) State and prove Fubini's theorem .

22) Let $\{A_i\}$ be a disjoint sequence of sets in A then

$$\mu_*[E \cap (\cup_{i=1}^{\infty} A_i)] = \sum_{i=1}^{\infty} \mu_*(E \cap A_i)$$

23) Define inner measure μ_* on an algebra . For any set E , prove that

$\mu_*(E) \le \mu^*(E)$. Further prove that if $E \in \mathcal{A}$ then prove that $\mu_*(E) = \mu^*(E)$.

24) State and prove Fatou's lemma .

25) State and prove Generalized Lebesgue convergence theorem.

26) Define a positive set. Prove that the union of countable collection of positive sets is positive.

27) State and prove Jordan decomposition theorem.

28) State and prove Radon Nikodym theorem.

HCT 1.4 M.Sc. I Mathematics Sem I

Classical Mechanics

Short Questions

1. Define Hamiltonian.
2. Define Unitary matrix.
3. Define Constraints.
4. Define Cyclic coordinates.
5. State isoperimetric problem.
6. Explain Cayley -Klein parameters
7. Write only rotation matrix for rotation through an angle $\varnothing$ about z – axis.
8. State fundamental lemma of calculus of variations.
9. Find Kinetic energy and potential energy for harmonic oscillator.
10. If I = $\int_{x_1}^{x_2} F(y, y')$dx is functional then prove that F- y'$F_{y'}$ = constant.
11. . If q is cyclic in L then show that q is cyclic in H.
12. Write a note on…Non conservative system, Eulerian angles, Infinitesimal Rotations.
13. Find a Lagrangian of a particle of mass m, slides under gravity without friction along the parabolic path $y=ax^2$ where a is constant.
14. Prove that a generalized momentum corresponding to cyclic coordinate is conserved.
15. If a single particle with constant mass m then show that equation of motion implies $\frac{dT}{dt}$ = Fv.

Long Questions

16. Define Eulerian angles and derive expression of rotation matrix in terms of Eulerian angles up to second iterations only.
17. Write the Hamiltonian function and equation of motion for compound pendulum. Find Lagrange's equations of motion for simple pendulum.
18. If A, B, C are three non-collinear points on a rigid body then discuss configuration of a rigid body.
19. Derive an expression of generalized force.
20. Describe the Hamiltonian and Hamilton's equations for an ideal spring mass arrangement
21. Find necessary condition for functional I = $\int_{x_1}^{x_2} F(x, y, y', y'')$ dx to be extremum.
22. Describe the motion of particle of mass m constrained to move on the surface of cylinder of radius a and attracted towards the origin by a force which is proportional to the distance of the particle from the origin.
23. What is Routhian? Explain Routh's procedure in details.
24. Find the Routhian for an Lagrangian L, given by L = (1/2)$\mu(\dot{r}^2+r^2\dot{\theta}^2)+\frac{GMm}{r}$ where $\mu=\frac{mM}{m+M}$.
25. Find necessary condition for extremum of I = $\int_{x_1}^{x_2} F(x, y, y')$dx.
26. Find expression for Kinetic energy of a particle of mass M on the surface of earth.
27. Find extremal of J[y(x)]=$\int_{x_1}^{x_2}$ ¿¿]dx.

28. If a single particle with variable mass M then show that equation of motion implies. $\frac{d}{dt}$[MT]= F.p.
29. Explain holonomic and nonholonomic constraints with example.
30. Derive components of angular velocity vector along the space set of axes.
31. Find extremal of functional I= $\int_{t_1}^{t_2} \left(y' \text{¿¿} 2 - y^2 + 4\,ysinx \right) dx \, \text{¿}$ where y (0) = y (π) = 0.
32. Derive Lagrange’s equation of motion for partially conservative system.
33. A smooth circular wire rotates with constant angular velocity ω, about a vertical axis which lies in plane of circle. A bead slides on wire. Set up equation of motion.
34. If T = (1/2) m $\dot{r}^2$ and δV= -Fδr then using Hamilton’s principle show that m$\ddot{r}$ = F.

M.Sc. I Mathematics Sem I

General Topology HCT 2.3

Short Questions

1. If B is basis for topology τ on X then show that τ equals the collection of all unions of element of B.
2. If x $\in R$ then show that {x} is closed set in usual topology on R.
3. Show that interior of A is contained in A.
4. Prove that being T_o Space is Hereditary property.
5. Define continuous map. Give example of not continuous map.
6. State Urysohn's lemma.
7. Show that being locally compact space is topological property.
8. Prove that every Second axiom space is First axiom space.
9. Prove that usual topology is FAS(First axiom space).
10. Prove that a subset of topological space is open iff it is neighbourhood of each of its points.
11. Define subspace with example.
12. Define derived set.
13. Define closure of set.
14. Define boundary set.
15. Define lindelof space.
16. Define connected space.
17. Define separated set.
18. Define Base subbase with example.

Long Questions

19. If A is connected subspace of topological space X such that A $\subseteq$ B $\subseteq \overline{A}$ then show that B is connected.
20. Prove that closed subspace of lindelof space is lindelof space.
21. In separable space show that any countable family of mutually disjoint open sets is countable.
22. Let <X, τ > be T-space. <X, τ > is regular iff for $\forall$G $\in \tau$, x $\in$ G , $\exists$ open set H in X such that x $\in H \subseteq \overline{H} \subseteq G$.
23. Show that coffinite T-space is compact.
24. whether T_3 space is T_4 space.
25. Prove that subspace of regular space is regular.
26. Prove that every convergent sequence in T_2 space has a unique limit.
27. Prove that every continuous image of second countable space is second countable.
28. Prove that every metric space is T_1 space.
29. Prove that every T_2 space is T_o space.
30. Prove that every completely regular space is regular space.
31. Mapping f from X to Y is continuous iff inverse image of closed set in Y is closed in X.
32. Explain homeomorphism.
33. Let f:<X, τ >$\rightarrow$ Y define $\tau^{¿}=\{G \subseteq Y \vee f^{-1}(G) \in \tau\}$ then show that
    a) $\tau^{¿}$ is topology on Y.
    b) $\tau^{¿}$ is largest topology on Y.
34. Show that every compact space is countably compact.
35. Prove all the properties of interior of set.

36. Prove all the properties of exterior of set.
37. Prove all the properties of boundary of set.
38. Prove all the properties of closed set.
39. Prove all the properties of closure of set.
40. Prove all the properties of derived set.
41. Prove usual topology.
42. Prove co-finite topology.
43. Prove ray topology.
44. Prove p-exclusive topology.
45. Prove p-inclusive topology.
46. Prove co-countable topology.
47. Continuous mapping of compact space into a Hausdorff space is not closed.
48. Prove that every continuous image of second countable space is second countable.

M.Sc. (Mathematics) Part I SEM-I

Paper HCT 1.3: Differential Equation

Question Bank

Short Answer Questions

1) Write the types of linear differential equation
2) Define solution of differential equation and complete or general solution of differential equation
3) Prove that If Φ_1 and Φ_2 are two solutions of second order differential equation with constant coefficient L(Y) then there linear combination is also solution
4) Find the solution of y''+16 y' =0
5) Find the solution of y''+2iy'+y=0
6) Define initial value problem for second order equations
7) Explain linear dependent and independent of solution of two functions
8) Define Wronskian.
9) If $\Phi_1 = x$ and $\Phi_2 = e^{rx}$ then find $W(\Phi_1, \Phi_2)$
10) Derive second order non homogeneous linear differential equation
11) Solve y"+9y= sinx
12) Solve y"+y= tanx
13) Derive n^{th} order homogeneous differential equation
14) Define Initial Value Problem for n^{th} order
15) Define linear differential equation with variable coefficient of order n
16) Define Homogeneous equation with analytic function
17) State existence theorem for analytic coefficient
18) Show that $\int_{-1}^{1} P_n(x) \quad P_m(x)\, dx = 0$ $\quad m \neq n$
19) Define Linear equation with regular singular points
20) Define Euler's Equation
21) Define second order equation with regular singular point
22) Define Bessel's equation of order n.
23) Write the method of finding the appropriate solution of initial value problem.
24) Explain Lipchitz constant
25) Find Wronskian of sinx ,e^{ix}
26) Find Wronskian of cos x , e^{ix}
27) Find the general solution 4y"- y = e^x
28) Find the general solution of y"+ 4y = cos x
29) Find the general solution of y"+ 9y = sin3x
30) Find the general solution of y^{iv}+16y =0

Long Answer Question

1) Let a_1 and a_2 be constant and consider the equation $L(Y)= y'' + a_1y' + a_2y = 0$
   i) If r_1 ,r_2 are distinct roots of char.poly.then the function $\Phi_1(x)=e^{r1x}$, $\Phi_2(x) = e^{r2x}$ is solution of $L(Y) = 0$
   ii) If r1 is repeated root of char poly. Then $\Phi_1(x)=e^{r1x}$ $\Phi_2(x)=x\, e^{r1x}$ is solution of $L(Y) = 0$
2) Consider the equation $y''+y'-6y=0$ i) compute the solution Φ_1, Φ_2 ii) compute the solution Φ when $\Phi(0)=0$ and $\Phi'(0) = 1$
3) State and prove Existence theorem for initial value problem
4) State and prove uniqueness theorem for initial value problem
5) Consider the equation $L(Y)= y'' + a_1y' + a_2y = 0$ then two solution of L(Y) are linearly independent on any interval I
6) Show that two solution of Φ_1, Φ_2 of L(Y) are linearly independent on I iff $W(\Phi_1, \Phi_2)(x) \neq 0$
7) Show that two solution of Φ_1, Φ_2 of L(Y) are linearly dependent on I iff $W(\Phi_1, \Phi_2)(x) = 0$
8) Find the general solution of $4y''-y=e^x$
9) Find the general solution of $4y''-y=e^{-x}$
10) Let Φ be any solution of $L(Y) = y^n+a_1y^{n-1}+\ldots..+a_ny=0$ on an I containing the point x_0 then for all x in I prove that $\| \Phi(x_0)\|\, e^{-k|x-x0|} \leq \|\Phi (x_0)\| \leq \| \Phi(x_0)\|\, e^{-k|x-x0|}$
11) State and prove that existence theorem for IVP for n order
12) State and prove uniqueness theorem for IVP for n order
13) Find the solution of $y''' - y' = x$ such that $\Phi(0)=1$ $\Phi'(0)=0$
14) One solution of $x^2y'' -2y=0$ is x2 find all solution of $x^2y'' -2y=2x-1$ $0<x<$infinity
15) Find a general solution of $y''-(2\backslash x^2)\, y = x$ ($0<x<$infinity)
16) Find the two linearly independent power series solution of $y''-xy=0$
17) Find the two linearly independent power series solution of $y''-xy'+y= 0$
18) State and prove existence theorem for differential equation with analytic coefficient.
19) Define Legendre's polynomial and solution of Legendre's differential equation.
20) Find the all solutions of $x2y''+ xy'' + 4y =1$ for $|x|<0$

M.Sc. (Mathematics) Part II Sem-III

Paper HCT 3.3: Linear Algebra

<u>Question Bank</u>

<u>Short Answer Questions</u>

1) Let V be the finite dimensional vector space then prove that A(A(W))=W where W is subspace of V
2) Let V be the vector space of all polynomial functions from R into R of the form $f(x) = c.+c_1x+c_2x^2+c_3x^3$ The differentiation operator D maps V into V Let B= $\{f_1,f_2,f_3)$ defined by $f_i(x) = x^{i-1}$,i=1,2,3,4. Then find $[D]_B$
3) Let V be a finite dimensional inner product space and T is a linear operator on V If T is invertible show that T* is invertible and $(T^*)^{-1}= (T^{-1})^*$.
4) Define orthonormal set with one example
5) Define Best Approximation
6) Define Orthonormal set
7) Define Annihilator of a subspace.
8) Define T conductor of Subspace if T is linear on V(F)
9) If V be a FDVS over F then show that each basis of V* is dual of some basis for V
10) If f and g are linear functionals on a vector space V, then prove that g is a Scalar multiple of f iff the null space of g contains the null space of f.

    Find the minimal polynomial of $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$
11) Prove that similar matrices have the same characteristic polynomial.

13) Let $\alpha = (1, 2)$, $\beta = (-1, 1)$. If γ is a vector such that $<\alpha, \gamma> = -1$ and $<\beta, \gamma> = 3$, then find γ

14) If the characteristic polynomial of a 5×5 matrix A is $(x-3)^3(x-2)^2$ then find all possible Jordan canonical forms of A.

15) Let dim V (F) is finite and T be a linear operator on V. Then show that T is Diagonalizable if and only if the minimal polynomial for T has the form $p = (x - c_1)(x - c_2) \ldots (x - c_k)$ where $c_1, c_2, \ldots, c_k$ are distinct elements of F.

16) Define Diagonalizable operator
17) Define characteristic polynomial of a linear operator
18) Define annihilating polynomial
19) prove that minimal polynomial of a linear operator is unique
20) Define Double dual of a vector space
21) Define annihilator of an annihilator
22) Define Characteristic values of a linear operator
23) Define singular and non-singular transformation
24) Define invariant direct sum
25) State primary decomposition theorem
26) State and prove Schwartz's inequality

Long Answer Question

1) Let V and W be vector spaces over the field F, and let T be a linear transformation from V into W. The null space of T^t is the annihilator of the range of T. If V and W are finite dimensional, then show that
   i) Rank (T^t) = rank (T)
   ii) Then range of Tt is the annihilator of the null space of T.
2) State and prove Cayley-Hamilton theorem.
3) Let V be a finite dimensional vector space over the field F and let W be a subspace of V. Then show that $\dim W + \dim W_0 = \dim V$
4) For any linear operator T on a finite dimensional inner product space V, show that there exists a unique linear operator T* on V such that <T(a),b>=<a,T*(b)>
5) If f is a non-zero linear functional on the vector space V, then show that the Null space of f is a hyperspace in V. Conversely, prove that every hyperspace in V is the null space of a (not unique) non zero linear functional on V.
6) Let T be a linear operator on an n-dimensional vector space V. Show that the characteristic and minimal polynomial for T have the same roots, except for multiplicities
7) Apply the Gram-Schmidt process to the vectors x= (3, 0, 4), Y = (1, 0, 7), Z= (2, 9, 11), to obtain an orthonormal basis for 3 with the standard inner product.
8) Let V be a complex vector space and f a form on V such that f (α, α) is real For every α. Then show that f is Hermitian
9) If V be a finite dimensional vector space over F then show that the each basis of V* is dual of some basis of V.
10) Prove that the minimal polynomial of a matrix or linear operator is divisor of every polynomial that annihilates the matrix or linear operator
11) If U be an linear operator on an inner product space V then prove that U is unitary iff the adjoint U* of U exist and U.U*= U*.U=I
12) Explain Jordan Canonical form and Nilpotent matrix.
13) Let dim V (F) is finite and W be a subspace of V. Then show that dim W + dim A (W) = dim V.
14) State and prove primary decomposition theorem.
15) Find a 3 × 3 matrix for which characteristic polynomial is x^2. Then find the minimal polynomial of corresponding matrix.
16) Determine all possible Jordan Canonical forms for a linear operator $T: V \to V$ Whose characteristic polynomial is $\Delta(t) = (t-2)^3 (t-5)^2$.
17) If α is a char. vector of T corresponding to the char.value c T then kα is char.vector of T corresponding to the same char.value c where k is any non-zero constant
18) The minimal polynomial of a linear operator is a divisor of every polynomial that annihilates the linear operator.
19) Prove that any orthogonal set of non-zero vectors in an I.P.S V is linearly independent.
20) Explain Gram Schmidt orthogonalization process
21) Every finite dimensional I.P.S has an orthonormal basis
22) Let V be an I.P.S and W be finite dimensional subspace and E the orthogonal projection V on W then the mapping $\beta \rightarrow \beta - E(\beta)$ is the orthogonal projection V on $W^{\perp}$
23) Let V be the finite dimensional I.P.S. suppose W is subspace of V which is invariant under T then orthogonal complement of W is invariant under T*.

M. Sc. Mathematics Part-II Semester-IV
Paper Name: Integral Equations
Question Bank

Short Answer Questions:

1. Define nth iterated kernel.
2. Define symmetric kernel.
3. Define Symmetric kernel.
4. Define Convolution type kernel.
5. Define Fredholm integral equation of first kind.
6. Define Fredholm integral equation of second kind.
7. Define Fredholm integral equation of third kind.
8. Define Homogeneous Fredholm integral equation.
9. Define Volterra integral equation of first kind.
10. Define Volterra integral equation of second kind.
11. Define Volterra integral equation of third kind.
12. Define Homogeneous Volterra integral equation.
13. Define Green's function.
14. Verify that the given function $y(x) = 1 - x$ is solution of $\int_0^x e^{x-t} y(t)dt = x$.
15. Verify that the given function $y(x) = \frac{1}{2}$ is solution of $\int_0^x \frac{y(t)}{\sqrt{x-t}}\, dt = x$.
16. Verify that the given function $y(x) = 3$ is solution of $\int_0^x (x-t)^2 y(t)dt = x^3$.
17. Verify that $y(x) = \cos x$ is solution of $y(x) - \int_0^x (x^2 + t)\cos t\, y(t)dt = \sin x$.
18. Convert the following differential equation into integral equation.
    $y'' + y = 0, y(0) = y'(0) = 0.$
19. Convert the following differential equation into integral equation.
    $y'' + xy = 1, y(0) = y'(0) = 0.$
20. Convert the following differential equation into integral equation.
    $$y'' + xy + y = 0, y(0) = 1, \qquad y'(0) = 1.$$
21. Convert the following differential equation into integral equation.
    $y'' + y = \cos x\,, y(0) = 0, y'(0) = 1.$
22. Convert the following differential equation into integral equation.
    $y'' + y = 0, y(0) = 0, y'(0) = 1.$
23. Convert the following differential equation into integral equation.
    $y'' - 5y' + 6y = 0, y(0) = 0, y'(0) = -1.$
24. Convert the following differential equation into integral equation.
    $y'' + y = 0, y(0) = 1, y'(1) = 0.$
25. Convert the following differential equation into integral equation.
    $y'' + \lambda y = 0, y(0) = y(1) = 0.$
26. Convert the following differential equation into integral equation.
    $y'' + \lambda y = 0, y(0) = y\left(\frac{\pi}{2}\right) = 0.$
27. Solve: $y(x) = \lambda \int_0^1 x^2 t y(t)dt$
28. Solve: $y(x) - \lambda \int_0^1 (3x - 2)t y(t)dt = 0$
29. Find eigenvalues and the corresponding eigenfunctions of $y(x) = \lambda \int_0^1 \sin(\pi x)\cos(\pi t)\, y(t)dt$
30. Find eigenvalues and the corresponding eigenfunctions of $y(x) = \lambda \int_0^1 (6x - t)y(t)dt$
31. Find eigenvalues and the corresponding eigenfunctions of $y(x) = \lambda \int_0^1 (2xt - 4x^2)y(t)dt$
32. Solve: $y(x) = \cos x + \lambda \int_0^\pi \sin x\, y(t)dt$
33. Solve: $y(x) = e^x + \lambda \int_0^1 2e^x e^t y(t)dt$
34. Solve: $y(x) = f(x) + \lambda \int_0^1 xt y(t)dt$
35. Find iterated kernel for $K(x, t\,) = (x - t); a = 0, b = 1.$

36. Find iterated kernel for $K(x,t) = x + \sin t\,; a = -\pi, b = \pi$.
37. Find iterated kernel for the kernel $K(x,t) = \frac{1+x^2}{1+t^2}$ of Volterra integral equation.
38. Find iterated kernel for the kernel $K(x,t) = \frac{1+e^x}{1+e^t}$ of Volterra integral equation.
39. Find the resolvent kernel for the kernel $K(x,t) = \sin x \cos t\,; a = 0, b = \frac{\pi}{2}$.
40. Find the resolvent kernel for the kernel $K(x,t) = a^{x-t}, a > 0$.
41. Solve by the method of successive approximation: $y(x) = x + \int_0^{\frac{1}{2}} y(t)dt$.
42. Solve by the method of successive approximation: $y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xty(t)dt$
43. Solve by the method of successive approximation: $y(x) = e^{x^2} + \int_0^x e^{x^2-t^2} y(t)dt$
44. Solve by the method of successive approximation: $y(x) = x + \int_0^x (t-x)y(t)dt$
45. Solve by the iterative method: $y(x) = x3^x - \int_0^x 3^{x-t} y(t)dt$
46. Solve by the method of successive approximations: $y(x) = 1 + x - \int_0^x y(t)dt\,, y_0(x) = 1$.
47. State Hilbert-Schmidt thereom.
48. Solve: $\int_0^t \frac{Y(x)dx}{(t-x)^{\frac{1}{3}}} = t(1+t)$.
49. Solve: $\int_0^x F(x)\cos px\, dx = \begin{cases} 1, 0 \le p < 1 \\ 2, 1 \le p < 2 \\ 0, \quad\quad p \ge 2 \end{cases}$

Long Answer Questions:

1. If $R(x,t;\ \lambda)$ is the resolvent kernel of a Fredholm integral equation, $y(x) = f(x) + \lambda \int_0^x K(x,t)\, y(t)dt$, then prove that the resolvent kernel satisfies the integral equation $R(x,t;\ \lambda) = K(x,t) + \lambda \int_0^x K(x,z)\, R(z,t;\ \lambda)dt$.
2. If $R(x,t;\ \lambda)$ is the resolvent kernel of a Fredholm integral equation, $y(x) = f(x) + \lambda \int_a^b K(x,t)\, y(t)dt$, then prove that the resolvent kernel satisfies the integral equation $R(x,t;\ \lambda) = K(x,t) + \lambda \int_a^b K(x,z)\, R(z,t;\ \lambda)dt$.
3. Prove that if a kernel is symmetric, then all its iterated kernel are also symmetric.
4. Prove that the eigen functions of a symmetric kernel, corresponding to distinct eigenvalues are orthogonal.
5. Convert $y''(x) - 3y'(x) + 2y(x) = 4\sin x\,, y(0) = 1, y'(0) = -2$ into integral equation and also recover the initial value problem from the integral equation obtained.
6. Convert $y''(x) - 2xy'(x) - 3y(x) = 4\sin x\,, y(0) = 1, y'(0) = 0$ into integral equation and also recover the initial value problem from the integral equation obtained.
7. Convert $y''(x) + xy(x) = 1, y(0) = 0, y(1) = 0$ into integral equation and also recover the boundary value problem from the integral equation obtained.
8. Convert $y'' + \lambda y = 4\sin x\,, y(0) = 0, y\left(\frac{\pi}{2}\right) = 0$ into integral equation and also recover the initial value problem from the integral equation obtained.
9. Solve: $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)dt$.
10. Find the eigenvalues and eigen functions of $y(x) = \lambda \int_1^2 \left(xt + \frac{1}{xt}\right) y(t)dt$.
11. Find the eigenvalues and eigen functions of $y(x) = \lambda \int_0^\pi K(x,t)y(t)dt$ where $K(x,t) = \begin{cases} \cos x \sin t\,, 0 \le x \le t \\ \cos t \sin x\,, t \le x \le \pi \end{cases}$.
12. Find the eigenvalues and eigen functions of $y(x) = \lambda \int_0^1 K(x,t)y(t)dt$ where $K(x,t) = \begin{cases} -e^{-t} \sinh x\,, 0 \le x \le t \\ -e^{-x} \sinh t\,, t \le x \le 1 \end{cases}$.
13. Solve: $y(x) = x + \lambda \int_0^1 (xt^2 + x^2 t)y(t)dt$
14. Solve: $y(x) - \lambda \int_{-\pi}^{\pi} (xcos\ t + t^2 \sin x + \cos x \sin t)y(t)dt$
15. Solve: $y(x) = 1 + \lambda \int_0^\pi \sin(x+t)y(t)dt$.
16. Solve: $y(x) = 1 + \int_0^x (t-x)y(t)dt$.
17. Solve the following symmetric integral equation with the help of Hilbert-Schmidt theorem: $y(x) = 1 + \lambda \int_0^\pi \cos(x+t)y(t)dt$.
18. Solve: $Y(t) = e^{-t} - 2\int_0^t \cos(t-x)Y(x)dx$

19. Examine whether the Green's function exists for the following BVP. If it does, then construct it. $y'' = 0, y(0) = y'(1), y'(0) = y(1)$.
20. Examine whether the Green's function exists for the following BVP. If it does, then construct it. $y'' - k^2y = 0, y(0) = y(1) = 0, k \neq 0$.
21. Examine whether the Green's function exists for the following BVP. If it does, then construct it.$x^2y'' + xy' - y = 0, y(x)$ is bounded as $x \to 0, y(1) = 0$.
22. Solve by the method of successive approximation: $y(x) = 1 + \lambda \int_0^{\pi} \sin(x+t)\, y(t)dt$
23.

M.Sc. (Mathematics) Part-II Sem-III

Paper HCT 3.2: Advanced Discrete Mathematics

Question Bank

Short Answer Questions:

1) Define isomorphism of graph with two examples .

2) Show that every totally ordered set is lattice .

3) Show that every Boolean ring is a commutative ring.

4) Write the short note on fusion of two vertices in a graph .

5) Draw all the spanning trees of K_4 graph .

6) Give the short note on isomorphism of graph .

7) If five points are choosen randomly in the interior of equilateral triangle with each side of two units . Then show that atleast one pair of point has separation less than one unit .

8) Give the short note on complete graph with examples .

9) Let A and B be any two finite sets which are disjoint then prove that $|A \cup B| = |A| + |B|$

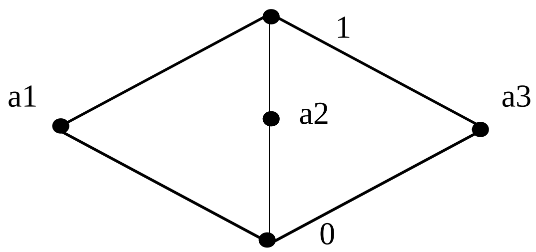
10) Prove that in a group of n people , there are two persons having the same number of friends.

11) State and prove Hand shaking lemma.

12) In a distributive lattice (L,$\preccurlyeq$) if $a \wedge b = a \wedge c$ & $a \vee b = a \vee c$ then prove that b = c

13) Show that there is no simple graph with 10 vertices and 46 edges.

14) Show that the lattice given by the following diagram is modular lattice.

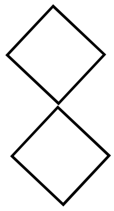


15) Write note on Fusion of graph

16) Write note on Isomorphism of graphs

17) Write note on Bounded lattice with example.

18) Determine whether the posets represented by each of the hasse diagram in the following lattices

a)  b) 

19) Explain pigeonhole principle.

20) Prove that any tree with at least two vertices is a bipartite graph.

21) Show that an acyclic graph with n vertices is tree iff it contains precisely (n-1) edges.

22) Explain complete bipartite graph with example.

23) Let G be a connected weighted graph in which no two edges have the same weight then show that G has unique minimal spanning tree.

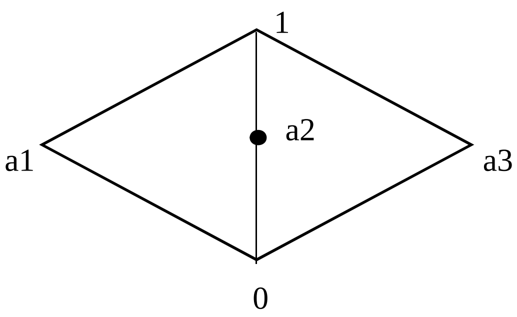
24) Using generating functions solve the recurrence relation $a_n = 2a_{n-1}$; $\forall\, n \geq 1$ and $a_0 = 3$.

25) Prove that in a tree addition of any new edge creates exactly one circuit.

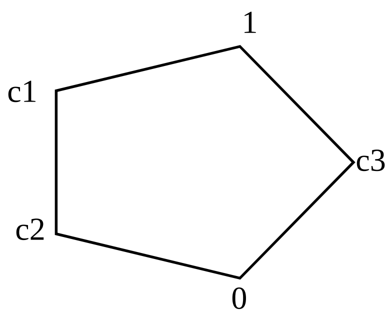
26) Find all sub-lattices of D_{24} that contains five or more elements.

27) Show that the lattices given in the following figure are not distributive.

(1)



(2)



28) Show that in any of five integers from 1 to 8 are chosen then at least two of them will have sum equals to 9.

29) Prove that if G_1 and G_2 are disjoint simple graphs then the complement of their join is the union of their complements.

30) Among the integers 1 to 300 find how many are not divisible by 7 but divisible by 3.

Long Answer Questions:

1) State and prove bridge theorem .

2) Let (B ,+, * , 0 ,1) is a Boolean ring . Then show that (B,∧,∨ ,0,1) is a Boolean ring .

3) Give the short note on matrix representation of graph with two examples .

4) Show that a graph G is connected if and only if it has a spanning tree .

5) Determine the number of integers between 1 to 2000 both inclusive which are divisible by 10,11 ,12 .

6) Find the general solution of a_r - 5 a_{r-1} + 6 a_{r-2} = 8 r + 5

7) Calculate the generating function of $\frac{1}{1-5\,x+6x^2}$.

8) Let L and L' be any two lattices . A bijective function f: L → L' is lattice isomorphism if and only if both f and f^{-1} preserves order .

9) Show that an edge e of a graph G is a bridge if and only if e is not a part of any cycle in G .

10) Show that the lattice of submodules of a module is a modular lattice .

11) Show that a graph G is connected if and only if given any pair u and v of vertices there is a path from u to v .

12) Let (L, ∧,∨) be a triplet with a non empty set L and meet & join are binary operations on L which satisfy associative , commutative , idempotent and absorption law then show that L is lattice .

13) Show that the lattice of normal subgroups of a group is a modular lattice .

14) If G is tree with vertices then show that it has precisely (n-1) edges .

15) Let a graph G be a non empty graph with atleast two vertices then G is bipartite graph if and only If it has no odd cycle.

16) Find the coeffiocient of x^{27} in $(x^4 + x^5 + x^6 + --------)^5$.

17) Find the general solution of a_r = - 4 a_{r-1} - 3 a_{r-2} ; r ≥ 2 with the conditions a_0 = 2 and a_1 = 8

18) Let G be a connected graph . Then show that G s a tree if and only if for every edge e of G the subgraph G –e has two components .

19) Find the general solution of a_r – 3 a_{r-1} - 4 a_{r-2} = 4^r

20) Calculate the generating function of $\frac{1}{1-7x+10\,x^2}$.

21) Find the number of integers between 1 to 1000 both inclusive which are divisible by none of 2,4,8.

22) Let (B, ∧,∨ ,0,1) is a boolean algebra . Then prove that (b, +,*,0,1) is a Boolean ring .

Where x+y = (x∧ y') ∨ (x'∧ y) and x * y = x ∧ y for all x,y ϵ B.

23) Let G be a graph with n vertices and q edges . w(G) dentes the number of connected components of G . Then show that G has atleast (n-w(G)) edges.

24) Given any two vertices u and v of a graph G show that every u-v walk contains u-v path.

25) Find the coefficient of x^{20} in the expansion of $(x^3 + x^4 + x^5 + \cdots)^5$.

26) Find the general solution of the given recurrence relation $a_r - 5\, a_{r-1} + 6\, a_{r-2} = 6$.

27) Let G be a k-regular graph, where k is an odd number. Prove that the number of edges in G is a multiple of k.

28) Let G be a connected graph. Show that G is tree iff for every edge e of G , the subgraph G-e has two components.

29) Among the integers 1 to 1000. Find how many of them are not divisible by 3, nor by 5, nor by 7.

30) Let T be a tree. Let u and v be the two vertices of T which are not adjacent. Let G be a subgraph of T obtained from T by joining u and v by an edge. Prove that G contains cycle.

31) Let G be a graph with exactly one spanning tree. Prove that G is a tree.

32) Let G be graph with n vertices and e edges & let m be the smallest positive integer such that m ≥ $2e/n$ then prove that G has a vertex of degree at least m.

33) Let G be a graph with n vertices $v_1, v_2, v_3, \ldots v_n$ & let A denote the adjacency matrix of G wrt this listing of vertices. Let B = [$b_{i,j}$] be the matrix B = A + $A^2 + A^3 + \cdots + A^{n-1}$

Then show that G is connected graph iff for every pair of distinct indices i, j we have $b_{i,j} \neq 0$

34) Show that in a complemented distributive lattice, the followings are equivalent

1. a ≼ b
2. a ∧ b' = 0
3. a' ∨ b = 1
4. b' ≤ a'

35) Prove that the product of two lattices is a lattice.

36) Let G be a simple graph with n vertices and let $\bar{G}$ be its complement.

(a) Prove that for each vertex v in G , $d_G(v) + d_{\bar{G}}(v) = n - 1$

(b) Suppose that G has exactly one even vertex. Find how many odd vertices does $\bar{G}$ have?

37) Prove that an edge e of a graph G is a bridge iff e is not a part of any cycle in G.

38) Show that in any lattice the distributive inequalities holds:

(a) a $\wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(b) a $\vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c)$

Punyashlok Ahilyadevi Holkar Solapur University, Solapur
M.Sc.-I
Real Analysis-II
Question Bank

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1. Prove that outer measure of an interval is its length.
2. If $\{A_n\}$ be a countable collection of sets of real numbers then
$$m^*\left(\bigcup_n A_n\right) \leq \sum_n m^*(A_n).$$
3. Prove that outer measure m^* is countably sub additive.
4. If A is countable set then prove that $m^*(A) = 0$.
5. Prove that Cantor's set C is an uncountable set with outer measure zero.
6. Prove that outer measure m^* is translation invariant.
7. Let any set A and any $\epsilon > 0$, there is an open set O such that $A \subset O$ and $m^*(O) \leq m^*(A)+\epsilon$ then prove that there is a set $G \in G_\delta$ such that $A \subseteq G$ and $m^*(A) = (m^*(G)$.
Where G_δ is set which is Countable intersection of open sets.
8. If E is measurable then $\tilde{E}$ is also measurable.
9. If $m^*(A) = 0$ then A is measurable.
10. If E_1 and E_2 are measuable sets then prove that $E_1 \cup E_2$ is measurable.
11. If A be any set and $E_1, E_2, ..., E_n$ be a finite sequence of disjopint measurable sets then show that
$$m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i).$$
12. Prove that collection M of all measurable sets is σ-algebra.
13. Prove that inteval (a, ∞) is measurable.
14. Show that Every Borel set is measurable.
15. If $\{E_i\}_{i=1}^{\infty}$ be a sequence of measurable sets then prove that
$$m\left(\bigcup_i E_i\right) \leq \sum_i m(E_i).$$
Also deduce that if sets E_i's are pairwise disjoint then
$$m\left(\bigcup_i E_i\right) = \sum_i m(E_i).$$
16. If $\{E_n\}_{n=1}^{\infty}$ be an infinite decreasing sequence of measurable sets and $m(E_1) < \infty$ then prove that
$$m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n).$$

17. If $\{E_n\}_{n=1}^{\infty}$ be an infinite increasing sequence of measurable sets then prove that

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n).$$

18. If E be a measurable set then prove that translation $E+y$ is a measurable set and $m(E+y) = m(y)$.

19. If E be a measurable set then prove that there exist a Borel set B_1 and B_2 such that, $B_1 \subseteq E \subseteq B_2$ and $m(B_1) = m(E) = m(B_2)$.

20. If E_1 and E_2 are measurable sets then prove that,

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = m(E_1) + m(E_2)$$

21. If $E \subset [0,1)$ be a measurable set and for each $y \in [0,1)$ then show that $E + y$ is measurable and $m(E + y) = m(E)$.
Where $E + y = \{x + y : x \in E\}$.

22. Show that there exist a non-measurable set in the interval $[0,1)$.

23. If a function f is measurable then show that set $\{x|f(x) = \alpha\}$ is measurable for all $\alpha \in \mathbb{R}$.

24. If f and g are the two measurable functions on the same domain.Then prove that functions $f + c, cf, f + g, f - g$ and $f.g$ are also measurable where c is constant.

25. Prove that the sum,product and difference of two simple function are simple.

26. If f is measurable function and $f = g$ a.e., then prove that g is measurable.

27. Show that continuous function defined on a measurable set is measurable.

28. State and Prove Egoroff's theorem.

29. If $\phi = \sum\limits_{i=1}^{n} a_i \chi_{E_i}$ where $E_i \cap E_j = \phi$ for $i \neq j$ and each E_i is measurable set with finite measure then prove that

$$\int \phi = \sum_{i=1}^{n} a_i \; m(E_i).$$

30. If ϕ and ψ be the simple function which vanishes outside a set o finite measure E. Then prove the following results:
    (a) $\int a\phi + b\psi = a \int \phi + b \int \psi$
    (b) $\phi \geq \psi$ a.e $\implies \int \phi \geq \int \psi$

31. If f be a bounded function defined on a measurable set E of finite measure then

$$\inf_{f\leq\psi} \int_E \psi(x)dx = \sup_{f\geq\phi} \int_E \phi(x)dx$$

for all simple functions ϕ and ψ if and only if f is measurable.

32. State and prove Bounded Convergence Theorem.

33. If f and g are two non negative measurable function then,

   (a) $\int\limits_E cf = c\int\limits_E f, c > 0$

   (b) $\int\limits_E f + g = \int\limits_E f + \int\limits_E g$

   (c) $f \leq g$ a.e. then $\int\limits_E f \leq \int\limits_E g$

34. State and prove Fatou's Lemma.

35. State and prove Monotone Convergence Theorem.

36. If $\{u_n\}$ be a sequence of non negative measurable functions and $f = \sum\limits_{n=1}^{\infty} u_n$ then prove that $\int f = \sum\limits_{n=1}^{\infty} \int u_n$.

37. If f be a non negative measurable function and $\{E_i\}$ be a disjoint sequence of measurable sets and $E = \cup E_i$ then prove that $\int\limits_E f = \sum\limits_i \int\limits_{E_i} f$.

38. Let f and g are two non negative measurable function. If f is integrable over E and $g(x) < f(x)$ on E then prove that g is also integrable and $\int\limits_E f - g = \int\limits_E f - \int\limits_E g$.

39. If f be non negative measurable function which is integrable over E.Then prove that $\epsilon > 0, \exists\delta > 0$ such that for every set $A \subseteq E$ with measure $m(A) < \delta$ we have,$\int\limits_A f < \epsilon$.

40. If f and g are two integrable functions over E then,

   (a) cf is integrable over E, and $\int\limits_E cf = c\int\limits_E f$

   (b) $f + g$ is integrable over E, and$\int\limits_E f + g = \int\limits_E f + \int\limits_E g$

   (c) $f \leq g$ a.e. then $\int\limits_E f \leq \int\limits_E g$

   (d) If A and B are disjoint measurable sets contained in E, then $\int\limits_{A\cup B} f = \int\limits_A f + \int\limits_B f$

41. If f is integrable function, prove that $|f|$ is also integrable and $\left|\int f\right| \leq \int\limits_E |f|$.Does integrability $|f|$ implies that of f.

42. State and Prove Lebesgue Convergence Theorem.

43. State and Prove Generalized Lebesgue Convergence Theorem.

44. If f_n is sequence of measurable functions defined on a measurable set E of finite measure and $f_n \to f$ a.e.then f_n convergence to f in measure.
    i.e. Convergence a.e. implies convergence in measure.

45. If f_n be sequence of measurable functions that converges to f in measure, prove that there is a sub-sequence $\{f_{n_k}\}$ which converges to$f$ a.e.

46. If $f(x) = |x|$, $x \in [-1, 1]$ find $D^+f(x), D_+f(x), D^-f(x)$, and $D_-f(x)$ at$x = 0$.Whether f is differentiable at $x = 0$?

47. If f be an increasing real valued function on the interval $[a, b]$. Then f is differentiable almost everywhere and the derivative f' is Lebesgue measurable and also $\int_a^b f'(x)dx \leq f(b) - (a)$.

48. If f is function of bounded variations on $[a, b]$ then prove that

    (a) $P_a^b - N_a^b = f(b) - f(a)$

    (b) $T_b^a = P_a^b + N_a^b$

49. A f is function of bounded variations on $[a, b]$ if and only if f is difference of two monotone real valued functions on $[a, b]$.

50. If f is function of bounded variations on $[a, b]$ then f is differentiable a.e. and f' is measurable.

51. If f is integrable on $[a, b]$ then the function F defined by

$$F(x) = \int_a^x f(t)dt$$

    is a continuous function of bounded variations.

52. If f is integrable on $[a, b]$ and $\int_a^x f(t)dt = 0$ for all $x \in [a, b]$ then prove that $f = 0$ a.e. on $[a, b]$.

53. If f is bounded and measurable on $[a, b]$ and if $F(x) = F(a) + \int_a^x f(t)dt$ then prove that $F'(x) = f(x)$ a.e. on $[a, b]$.

54. If f is integrable on $[a, b]$ and if $F(x) = F(a) + \int_a^x f(t)dt$ then prove that $F'(x) = f(x)$ a.e. on $[a, b]$.

55. If f is absolutely continuous on $[a, b]$ then f is a function of bounded variations on $[a, b]$ and hence f is differentiable a.e. on $[a, b]$

56. A function F is an indefinite integral of some integrable function if and only if F absolutely continuous on $[a, b]$

57. Prove that every absolutely continuous function is indefinite integral of its derivative.

M.Sc. (Mathematics) Part II Sem-IV

Paper HCT4.2: Partial Differential Equation

Question Bank

Short Answer Questions

1) Define curve and surface

2) Define partial diff. equation and order of P.D.E

3) Derive the derivation of PDE by elimination of arbitrary constant.

4) Find the first order PDE which represents the set of all spheres with center on the z-axis and radius a

5) Find the first order PDE which represent the set of right circular cones with z axis as the axis of symmetry.

6) Eliminate the arbitrary constant from Z=ax+by

7) Define Euler's equation for Homogeneous function.

8) Define linear PDE, semi linear PDE.

9) Define quasi linear PDE and Nonlinear PDE

10) Define complete integral, general integral, singular integral.

11) Prove that singular integral is also a solution.

12) Define Pfaffian differential equation.

13) Define integral equation of Pfaffian differential equation.

14) What is compatible solution of first order PDE?

15) Give the geometrical interpretation of quasi linear PDE.

16) Define 2^{nd} order semi linear PDE.

17) Write the classification of second order PDE

18) Define family of equipotential surface

19) Define Interior Dirichlet problem and Exterior Dirichlet problem

20) State maximum and minimum principle

Long Answer Question

1) A necessary and sufficient condition that their exist relation between two functions u(x,y),v(x,y) a relation F(u,v)=0 or u=H(v) not involving x,y explicitly is that $\partial(u,v)/\partial(x,y)=0$
2) Define characteristic curve and envelop and give geometrical interpretation of envelope with one example.
3) Show that General integral is also a solution of PDE
4) Prove that singular integral is also solution of PDE
5) Show that the singular integral is obtained by eliminating p and q from the equations F(x,y,z,p,q)=0, fp (x,y,z,p,q)=0 and fq(x,y,z,p,q)= 0
6) Show that $(x-a)^2+(y-b)^2+z^2=1$ is complete integral of $z^2(1+p^2+q^2)=1$ by taking b=2ashow that the envelope of the sub family is $(y-2x)^2+5z^2=5$which is the particular solution.
7) Show that z= ax +(y/b)+ b is the complete integral of pq=1 find the particular solution corresponding to the subfamily b=a and show that it has no singular integral.
8) Show the Geometrical interpretation of solution of quasi linear PDE
9) The general solution of quasi linear pde ,

   P(x,y,z)p+Q(x,y,z)q=R(x,y,z)

   where P,Q,R are continuously differentiable function of x,y,z and is given by F(u,v)=0 where F is arbitrary function of u,v and $u(x,y,z)=c_1$ and $v(x,y,z)= c_2$ are independent solution of the system dx/P=dy/Q=dz/Q
10) Find the general integral of $z(xp-yq)=y^2-x^2$
11) Show that there always exist an integrable factor for Pfaffian diiff.equations.in two variables.
12) Prove that Necessary and sufficient condition that the Pfaffian D.E $\bar{x}\,\overline{dr} = 0$ be integrable that $\bar{x}.curl\bar{x}$=0
13) (6x+yz)dx+(xz-2y)dy+(xy+2z)dz=0 check the integrability and find the solutions.
14) Show that the following Pfaffian diff.eq.is integrable and find its integral. Ydx+xdy+2zdz=0, ydx+xdy+2zdz=0

15) Show that the equation=$p^2+q^2-1=0$ and $g=(p^2+q^2)x-pz=0$ are compatible and one parameter family common solutions

16) Describe charpits method for solving a first order PDE f(x,y,z,p,q)=0

17) Find the complete integral by charpits method of given pde $z^2(1+p^2+q^2)=1$

18) Describe Jacobi's method for solving a first order PDE

19) Solve by Jacobi's method $z+2u_z-(u_x-u_y)^2=0$

20) Describe Jacobi's method to solve nonlinear pde

21) Solve $p^2x+q^2y=z$.

22) Find the integral surface of given PDE $(2xy-1)p+(z-2x^2)q=2(x-yz)$

23) Find the complete integral of $(p^2+q^2)x=pz$ and hence find the integral surface through the curve x=0 $z^2=4y$

24)State and prove Hernack’s theorem

25) Prove that the exterior Dirichlet problem for a circle

26) Show that the necessary and sufficient condition for the existence of the solution of Neumann problem is that $\int_B f$ should vanish.

27) Show that the solution of Neumann problem is either unique or it differs from one another by a constant

28) Prove that the solution of Dirichlet problem if it is exist then it is unique.

29) Show that the surface $x^2+y^2+z^2=cx^{2/3}$ can form an equipotential surface and find the general form of potential function.

30) Obtain D’Alembert solution of the one dimensional wave equation which describes the vibration of a semi finite string.

M.Sc. (Mathematics) Part-I Sem-II

Paper OET 2.1: Fundamental in Mathematics

Question Bank

Short Answer Questions:

1) Define Basis & Dimension of vector space.

2) Define Symmetric matrix.

3) Define Solution of system of equation

4) Define Rank of matrix

5) Define a) Linearly dependent vector b) Linearly independent vector

6) Write a note on Types of Matrices

7) Write a note on Null space, Range space

8) Write a note on Linear transformation.

9) If matrix A = $\begin{bmatrix} 6 & 7 \\ -1 & 2 \end{bmatrix}$ then find A^{-1}.

10) Show that inverse of matrix is unique.

11) If D=$\begin{bmatrix} 0 & 3 & 7 \\ 8 & 1 & 4 \\ 7 & 1 & 2 \end{bmatrix}$show that $(5D)^T = 5D^T$.

12) Solve the system of equation

$$x - y + z = 9$$
$$x + y + -2z = 2$$
$$2x+y+z = -1$$

13) Explain Properties of determinant with example.

14) If A = $\begin{bmatrix} 2 & -1 & 0 \\ 4 & 7 & 8 \\ -3 & 1 & 3 \end{bmatrix}$ B = $\begin{bmatrix} 2 & 0 & 6 \\ 1 & -8 & 2 \\ -5 & 4 & 7 \end{bmatrix}$ find A+B, A-B.

15) Show that $T(v_1, v_2, v_3) = (v_1 + v_2, v_2 - v_1, v_1 - v_3)$is linear transformation.

16) Find the Transpose of matrix A=$\begin{bmatrix} 2 & 3 & -6 \\ -2 & 6 & 1 \\ 1 & 5 & 2 \end{bmatrix}$.

17) Prove that inverse of the square matrix A exists iff $|A| \neq 0$.

18) a) Define subspace. b)State Rank Nullity theorem

19) Find the solution of $2x-3y+z = 0$ $x+2y=3z = 0$ $4x - y-2z = 0$.

20) Check whether mapping $T: R^2 \rightarrow R^2$ defined by $T(a_1,a_2) = (a_2, 1+ a_2)$ is linear transformation.

21) Find the inverse of $\begin{bmatrix} 1^2 & 2^2 & 3^2 \\ 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \end{bmatrix}$.

22) Find the rank of the matrix $\begin{matrix} 2 & 1 & 1 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \\ -2 & -1 & 4 \end{matrix}$

23) Find value of α where (3,1, α) is Linear combination of (1,0,1)and (1,1,2).

24) Show that (1,2,1),(2,1,4),(4,5,0) are Linearly independent

25) If{x, y, z} is basis of R^3 then check whether {x + y, y + z, z + x} is linearly independent?

26) If V(F) be a vector space and $\alpha, \beta \in V\ and\ a, b \in F$. Then prove that

i) $a\alpha = b\alpha\ and\ \alpha \neq 0\ then\ a = b$

ii) $a\alpha = \alpha\beta\ and\ \alpha \neq 0\ then\ \alpha = \beta$.

27) Show that the map $T: R^2 \to R^2\ such\ that\ T(a, b) = (a^2, b)$ is not a linear.

Long Answer Questions:

1) Find inverse of matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & -2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$.

2) Let K be solution set of linear equation AX=B and let K_h be solution set of corresponding homogenous system of linear equations for any solution 'S' to AX=B,

$$K = \{s\}+K_h = \{s+k : k \in K_h\}$$

3) Define vector space with all conditions.
4) Show that $2x^3+x^2+x+1$, $x^3 + 3x^2 + x-2$, $x^3 + 2x^2 - x + 3$ are linearly Independent over R.

5) Show that equations 3x + y + z = 8, -x + y -2z = -5, 2x + 2y + 2z = 12, -2x + 2y – 3z = -7 are consistent and find solution.

6) Prove that intersection of subspace of vector space is subspace.

7) Define Matrix & explain the types of matrices with example.

8) Determine whether the following vectors forms a basis of $R^3(R)$.

i)(1,1,2),(1,2,5),(5,3,4)

ii)((1,2,-1),(1,0,2),(2,1,1)

9) Prove that if a vector space V is generated by finite set S then some subset of S is basis for V hence V has finite basis.

10) Let V& W be a vector spaces & $T:V\rightarrow W$ be linear then prove that T is one- one iff

N (T) = {0}

11) State and prove Rank Nullity theorem.

12) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 7 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$.

13) Show that $T:R^2\rightarrow R^2$ defined by $T(a_1,a_2) = (a_1+a_2,a_1)$ is one – one and onto.

14) Find the Rank and Nullity of the matrix $\begin{matrix} 1 & 3 & 2 \\ -1 & 2 & 0 \\ 3 & -1 & 2 \\ 0 & 5 & 2 \\ 2 & 1 & 2 \end{matrix}$ and verify Rank Nullity theorem.

15) Find the matrix representation of following linear operator defined on R^3.

T(x, y, z) = (z, y + z, x +y +z)

16) If T is linear transformation on V and $T^2 - T + I = 0$ then show that T is invertible.

17) Show that $2x^3+x^2+x+1$, $x^3 + 3x^2 + x-2$, $x^3 + 2x^2 - x + 3$ are linearly independent over R.
18) If V is vector space & $S_1 \subseteq S_2 \subseteq V$ and S_1 is linearly dependent then prove that S_2 is linearly dependent.

19) Find the solution of x -3y -8z +10 = 0, 3x + y – 4z = 0, 2x + 5y + 6z-13 = 0
20) Let $\{e_1= (1,0) ,e_2 = (0,1)\}$ & $\{e_1'= (1,2) ,e_2' = (2,3)\}$ find the transition matrix

i) From $\{e_i\}$ to $\{e_i'\}$

ii) From $\{e_i'\}$ to $\{e_i\}$.

21) Prove that the union of two subspaces is a subspace if one is contained in the other.

22) Prove that the linear span L(S) of any non-empty subset S of a vector space V(F) is a subspace of V(F).

23) If M and N be subspaces of a vector space V(F). Then show that

$i)\ M \subset M + N$ $\quad ii)\ M \subset M + N$ $\quad iii)\ M + N = L(M \cup N)$

24) Show that any superset of linearly dependent vectors is linearly dependent.

25) Prove that the vectors $(1,2,1), (2,1,4), (4,5,0)$ are linearly independent.

26) Determine whether or not the following vectors form a basis of R^3.

(1,1,2), (1,2,5), (5,3,4)

27) Show that the necessary and sufficient condition for a matrix A to be invertible is that $|A| \neq 0$.

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M.Sc.-I
Functional Analysis
Question Bank

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1 Short answer question

1. If V be a normed linear space and defined $d(x, y) = ||x - y||$, for all $x, y \in V$ then prov that $< V, d >$ is metric space.
2. Show that $\Bigg| ||x|| - ||y|| \Bigg| \leq ||x - y||, \forall x, y \in V$
3. Show that the real linear space and complex linear space are Banach spaces under the norm, $||x|| \leq |x|, x \in \mathbb{R}$ or $\mathbb{C}$.
4. Show that every convergent sequence in a norm linear space X is Cauchy.
5. State and prove Cauchy's Inequality.
6. Show that every complete subspace of normed linear space is closed and what about converse?
7. If $T : X \to Y$ be any linear transformation . T is Continuous on X if and only if T is Continuous at any point of X.
8. If $T : X \to Y$ be any linear transformation .T is Continuous on X if and only if T is bounded X.
9. If $T : X \to Y$ be any linear transformation .T is bounded if and only if T maps bounded sets in X. Let N and N' be normed linear space and T be linear transformation of N to N'. Then T^{-1} exists and is continuous on its domain of definition if and only if there exists a constant $m > 0$ such that, $m||x|| \leq ||T(x)||, \forall x \in N$.
10. If $T : X \to Y$ be bounded linear transformation then $||T||$ can be expressed by any one of the following formula,
    (a) $||T|| = sup\{||T(x)|| : x \in X, ||x|| \leq 1\}$
    (b) $||T|| = sup\{||T(x)|| : x \in X, ||x|| = 1\}$
    (c) $||T|| = sup\{\frac{||T(x)||}{||x||} : x \in X, x \neq 0\}$
    (d) $||T|| = inf\{k : k > 0, ||T(x)|| \leq k||x|| for x \in X\}$
11. Let X be any normed linear space and M be a closed subspace of X.Let $T : X \to \frac{X}{M}$ be defined by ,$T(x) = x + M, x \in X$ then prove that T is bounded linear operator and $||T|| \leq 1$.
12. If $T : X \to Y$ be bounded linear transformation $N(t)\{x \in X : T(x) = 0\}$ where $N(T)$ is null space or kernal of T then show that the null space $N(T)$ of T is closed subspace of X.
13. Let N and N' be any normed linear space and let $T : n \to N'$ be a bounded linear transformation of N to N'. If M is kernal of T, then prove that
    (a) M is closed subspace of N.
    (b) T induces a natural linear transformation T' of $\frac{N}{M}$ onto N' such that, $||T'|| = ||T||$.
14. Prove that $B(X, Y)$ is subspace of $L(X, Y)$.
15. Prove that $B(X, Y)$ is normed linear space, Where,
    $||T|| = sup\{||T(x)|| : x \in X, ||x|| \leq 1\}$.

16. If X be a normed linear space and $S, T : X \to x$ be bounded linear operators, $ST(x) = SoT(x) = S[T(x)]$ then prove that ST is also bounded linear operators.

17. If X and Ybe a normed linear spaces and let T be a linear transformation of X into Y then show that the following statement are equivalent

    (a) T is bounded.

    (b) T is uniformly continuous on X.

    (c) T is continuous at some point of X.

18. Let X and Ybe a normed linear spaces then X and Y are topologically isomorphic if and only if there exists a linear transformation T of X To Y and positive constants m and M such that $m||x|| \leq ||T(x)|| \leq M||x||$.

19. Let Nbe a normed linear space and two norms $||.||_1$ and $||.||_2$ are defined on N.Then these two norms are equivalent if and only if there exists a positive real numbers m and M such that $m||x||_1 \leq ||x||_2 \leq M||x||_1, \forall x \in N$.

20. If n is positive integer, ℓ_p^n represents different normed linear spaces for different values of $p(1 \leq p \leq \infty)$ on the same set of all n-tuples $(x_1, x_2, ..., x_n)$ with ,

$$||x||_p = \left[\sum_{i=1}^{n} |x_i|^p\right]^{\frac{1}{p}} \qquad ; 1 \leq p \leq \infty$$

    then show that all these norms are equivalent to one another.

21. State and Prove Riesz Lemma.

22. If Nbe a normed linear space over $K(\mathbb{R}/\mathbb{C})$ and x_0 be any non-zero vector in N.Then prove that there exists a functional F in N^* such that, $F(x_0) = ||x_0||$ and $||F|| = 1$

23. If M be closed linear subspace of a normed linear space N and x_0 be vector not in M and d is distance form from x_0 to M. Then show that there exists a functional F in N^* such that, $F(M) = \{0\}, F(x_0) = 1$ and $||F|| = \frac{1}{d}$.

24. Prove that the normed linear space $\ell_p, 1 \leq p < \infty$ are separable.

25. Show that the normed linear space $N = \{x = (x_n)/x_n \to \ell\}$ with norm $||x|| = \sup\limits_{1 \leq n < \infty} |x_n|$ is separable.

26. Show that every subset of separable normed linear space is separable.

27. Let $S(x, r)$ be an open sphere in B with centre at x and radius r, S_r is the open with centre at origin and radius r then prove that following results,

    (a) $S(x, r) = x + S(0, r)$ or $x + S_r$

    (b) $S_r = r.S_1$ or $S(0, r) = rS(0, r)$

28. State and prove Banach's theorem.

29. Let N and N' be two normed linear spaces and D a subspace of N. Then a linear transformation $T : D \to N'$ is closed if and only if its graph T_G is closed.

30. Let T be projection on V. Then prove that $R(T) =$ Range of T=$\{z \in V/T(z) = z\}$

31. If P is projection on Banach space B and If M and N are its range nd null spaces respectively then prove that M and N are closed linear subspaces of B such that $B = M \oplus N$.

32. Define Inner Product (IP) and examples on that.

33. If X is a complex IPS then Prove that following

    (a) $< ax - by, z >= a < x, z > -b < y, z >$
    (b) $< x, ay + bz >= \bar{a} < x, y > +\bar{b} < x, z >$
    (c) $< x, ay - bz >= \bar{a} < x, y > -\bar{b} < x, z >$
    (d) $< x, 0 >= 0$ and $< 0, x >= 0, \forall x \in X$

34. State and prove Schwarz inequality.

35. If X is an inner product space. Then prove that $||x|| =< x, x >^{\frac{1}{2}}$ is a norm on X.

36. Define Hilbert space and also show that inner product in a Hilbert space is jointly continuous.

37. If x and y are two vectors in a Hilbert space then prove that $||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2)$.

38. If x and y are two vectors in a Hilbert space then prove that $4 < x, y >= ||x + y||^2 - ||x - y||^2 + i||x + iy||^2 - i||x - iy||^2$.

39. If x and y are two vectors in a Hilbert space then prove that following

    (a) $||x + y||^2 + ||x - y||^2 = 4Re < x, y >$
    (b) $< x, y >= Re < x, y > +iRe < x, iy >$

40. Let X be a IPS over K and if $\{x_n\}$ and $\{y_n\}$ are cauchy sequences in X then show that $\{< x_n, y_n >\}$ is cauchy in K.

41. Let X be a IPS over K and if $\{x_n\}$ and $\{y_n\}$ are cauchy sequences in X then show that $\{< x_n, y_n >\}$ is convergent sequence in K.

42. Let X be a IPS over K and if $\{x_n\}$ is cauchy sequences in X then prove that $\{||x_n||\}$ is convergent.

43. If x and y are two orthogonal vectors in a Hilbert space H then prove that $||x + y||^2 = ||x - y||^2 = ||x||^2 + ||y||^2$.

44. Consequences and remark on orthogonal complement.

45. If S is a non-empty subset of a Hilbert space H. Then show that $S^{\perp}$ is a closed linear subspace of H and hence Hilbert space.

46. Let$M$ be a linear subspace of a Hilbert space H then M is closed if and only if $M = M^{\perp\perp}$.

47. If M and N be a closed linear subspace of a Hilbert space H such that $M \perp N$ then prove that linear space $M + N$ is closed.

48. Let M be a closed linear subspace of a Hilbert space H then prove that $H = M \oplus M^{\perp}$.

49. Show that in all IPS, $x \perp y$ if and only if $||x + ay||^2 = ||x - ay||^2$, for any $a \in K$.

50. Show that An orthonormal set in an IPS X is linearly independent.

51. An orthonormal set S in a Hilbert space H is Complete if and only if for any $x \in H$ is such that $x \perp S$ then x must be zero.

52. Let $\{e_i\}$ is complete orthonormal set if and only if $x = \sum < x, e_i > e_i$.
53. Show that every orthonormal set in a Hilbert space is contained in some complete orthonormal set.
54. Prove that every non-zero Hilbert space contains a complete orthonormal set.
55. A Hilbert space is finite dimensional if and only if complete orthonormal set is basis.
56. Show that the adjoint operation preserves addition, reverse the product and it is conjugate linear.
57. If $\{T_n\}$ is sequence of bounded linear operators on Hilbert space and $T_n \to T$ then prove that $T_n^* \to T^*$.
58. Show that the adjoint operator on $B(H)$ is one-one and onto and if T is non-singular operator on H the prove that T^* is non-singular operator and deduce that $(T^*)^{-1} = (T^{-1})^*$.
59. Let S be the set of all self-adjoint operators in $B(H)$ then show that S is closed linear subspace of $B(H)$ and therefore S is real Banach space containing identity transformation.
60. Let A_1 and A_2 are two self-adjoint operators on H then their product $A_1.A_2$ is self-adjoint if and only if they commute.
61. Let T is an arbitrary operator on Hilbert space H then $T = 0$ if and only if $< Tx, y >= 0, \forall x, y \in H$.
62. Let T is an operator on Hilbert space H then $< Tx, x >= 0, \forall x \in H$ if and only if $T = 0$.
63. An operator T on complex Hilbert space H is self-adjoint if and only if $< Tx, x >$ is real for all x.
64. For any arbitrary operator T on H then show that TT^* and T^*T are positive operators.

2 Long answer question

1. Define Banach space and If p be a real number such that such that $1 \leq p < \infty$ and ℓ_p^n the space of all n-tuples $x = (x_1, x_2, ..., x_n)$ of scalars then show that ℓ_p^n is a Banach space under the norm $||x||_p = [\sum_{i=1}^{n} |x|^p]^{\frac{1}{p}}$

2. If x be normed linear space over the field F and M be closed subspace of x.
   Define $||.||_1 : \frac{X}{M} \to \mathbb{R}$ by,
   $||x + M||_1 = inf\{||x + m||/m \in M\}$ then prove that $||.||_1$ is a norm on $\frac{X}{M}$.and also prove that if X is banch space then $< fracXM, ||.||_1 >$ is also Banach Space.

3. Let X be a normed linear space and $S = \{x \in X : ||X|| \leq 1\}$ be subspace of x such that Xbe Banach space if and only if S is cpmplete.

4. Suppose that a Banach space B is a direct sum of linear space M and N (i.e.$B = M \oplus N$).Let $z = x + y$ be a unique representation of the vector $z \in B$ where $x \in M, y \in N$.
   Define the function $||.||_1$ on B by,
   $$||z||_1 = ||x|| + ||y||$$
   then show that
   (a) $||.||_1$ is norm on B
   (b) $< B, ||.||_1 >$ is Banach space if M and N are the closed subspaces of B.

5. If Y is complete then $B(X, Y)$ is complete.

6. Let $||.||_1$ and $||.||_2$ equivalent norms are defined on linear space X.Then prove that
   (a) $< X, ||.||_1 >$ is Banach space if and only if $< X, ||.||_2 >$ is Banach space
   (b) A is bounded in $< X, ||.||_1 >$ if and only if A is bounded in $< X, ||.||_2 >$.

7. Show that any two n-dimensional normed spaces over same scalar field are topologically isomorphic.

8. State and prove Hahn-Banach Theorem.

9. If M be closed linear subspace of a normed linear space N and x_0 be vector not in M and d is distance form from x_0 to M. Then show that there exists a functional F in N^* such that, $F(M) = \{0\}, F(x_0) = d$ and $||F|| = 1$.

10. If N normed linear space then prove that each vector x in N induces functional F_x on N^* defined by, $F_x(f) = f(x)$ for every $f \in N^*$ such that $||F_x|| = ||x||$.
    And also prove that the mapping $J : N \to N^{**}$ defined by $J(x) = F_x$ for every $x \in N$ given an isometric isomorphism of N into N^{**}.

11. State and prove Open Mapping Theorem.

12. Let N and N' be two normed linear spaces then prove that $N \times N'$ is a normed linear space with the co-ordinatewise operations and the norm $||(x, y)|| = ||x|| + ||y||$ where $x \in N, y \in N'$ and show this norm induces a product topology on $N \times N'$ and If $N \times N'$ is complete if and only if both N and N' are complete.

13. State and prove Closed graph theorem.

14. State and Prove Uniform Boundedness Principle or Banach-Steinhaus Theorem.

15. Let $||.||$ be a norm on a Banach space B. Then there exists an inner product $<>$ on B such that, $< x, x >= ||x||^2, \forall x \in B$ if and only if the norm satisfied the parallelogram law.

16. Let M be a closed linear subspace of a Hilbert space H. Let x be a vector not in M and $d = d(x, M)$ then prove that there exists a unique vector y_0 in M such that $||x - y_0|| = d$.

17. Let M be a closed linear subspace of a Hilbert space H then show that there exists a non-zero vector z_0 in H such hat $z_0 \perp M$.

18. Let S be non empty subset of Hilbert space Hthen prove that $< S >$ is dense in H if and only if $S^{\perp} = \{0\}$.

19. Bessel's Inequality for Finite orthonormal sets.

20. Bessel's Inequality for infinite orthonormal sets.

21. If $\{e_i\}$ is an orthonormal set in a Hilbert space H and x is an arbitrary vector in H then prove that $[x - \sum < x, e_i > e_i] \perp e_j$ for each j

22. A Hilbert space H is separable if and only if every orthonormal set in H is countable.

23. Let y be a fixed vector in a Hilbert space H and let f_y be a fuction defined as, $f_y(x) =< x, y >$ for every $x \in H$ then prove that f_y is functional on H and $||y|| = ||f_y||$.

24. State and prove Riesz-Representation theorem.

25. Prove that the mapping $\phi : H \to H^*$ defined by, $\phi(y) = f_y$ where $f_y(x) =< x, y >$ for every $x \in H$ is an additive, one-one, onto isometry but not linear.

26. If H is Hilbert space then prove that H^* is also Hilbert space with the inner product defined by, $< f_x, f_y >=< y, x >$.

27. Prove that every Hilbert space is reflexive.

28. If T be an operator on a Hilbert space H then prove that there exists a unique operator T^* ON H such that for all $x, y \in H$,$< Tx, y >=< x, T^*y >$.

29. If H Hilbert space and T^* be adjoint of the operator T.Then prove that T^* is bounded linear transformation and T determines T^* uniquely.

30. If H Hilbert space then show that the adjoint operator $T \to T^*$ on $B(H)$ has the following properties.
    (a) $T^{**} = T$
    (b) $||T^*|| = ||T||$
    (c) $||T^*T|| = ||T||^2$

31. Show that the real Banach space of all self-adjoint operators on a Hilbert space H is partial ordered set.Whose linear structure and ordered structure are related by following properties,
    (a) If $A_1 \leq A_2$ then $A_1 + A \leq A_2 + A$ for every $A \in S$.
    (b) If $A_1 \leq A_2$ and $\alpha > 0$then $\alpha A_1 \leq \alpha A_2$.